Under consideration for publication in Geoph. J. Int.

Temperature and heat flux scaling laws for isoviscous, infinite Prandtl number mixed heating convection.

(Received 18 December 2017)

Comparison with previous scaling laws

Several previous studies have established empirical scaling laws for mixed heating convection in a Cartesian geometry (Sotin & Labrosse 1999; Moore 2008; Choblet & Parmentier 2009) or in a spherical geometry (Jarvis 1993; Shahnas *et al.* 2008; Deschamps *et al.* 2010; Choblet 2012; Weller *et al.* 2016). Although based on the same convective system, these empirical scaling laws are different from the theoretical ones developed in this study. The differences may be caused by the numerical model, e.g., 2D vs 3D geometry, the explored parameter space, i.e., the ranges of values for the heating rate (H) and Rayleigh number (Ra) considered, or by different methods used to measure the properties, in particular different definitions of the thermal boundary layer (TBL). In this later case, previous studies used the horizontally averaged temperature profile to define the TBL, this approach relying on a balance between heat transport mechanisms. By contrast, our approach is based on the hot temperature profile and relies on a balance between forces. These frameworks are fundamentally different and lead to significant differences (Vilella & Kaminski 2017). Here, we only compared our results with the scaling laws proposed by Moore (2008), since this study is in Cartesian geometry and provides scaling laws for the top and bottom TBLs. Moreover, we plot our dataset against the scaling laws of Moore (2008) because we cannot test his dataset against our scaling laws, since this would require full distributions of temperature. Note that the study of Moore (2008) also involves 2D-Cartesian geometry, i.e., discrepancies between both studies may not be used to evaluate geometrical effects. However, the scaling laws inferred by Moore (2008) have been confirmed in 3D spherical geometry (Weller *et al.* 2016), and Choblet & Parmentier (2009) have shown that there is no systematic differences between numerical simulations conducted in 2D- and 3D-Cartesian geometry.

Previous scaling laws were all determined for heating rates lower than its critical threshold (H_{crit}), i.e., heat is supplied from the bottom. We therefore begin by considering only cases where $H \leqslant H_{crit}$. Figure 1a shows the predicted value for the dimensionless surface heat flux (Nu_t) by Moore (2008) as a function of the measured values in our numerical results. Our results are in excellent agreement with this scaling law ($R^2 = 0.9957$). This result is not surprising, because both the scaling law proposed by Moore (2008) and ours are based on the same theoretical arguments. The measurements of the temperature jump across the top ($\Delta T_{TBL,t}$) and bottom ($\Delta T_{TBL,b}$) TBL are more challenging. To be consistent with Moore (2008), we follow the definition used in his study (see details therein). Surprisingly, the fit for $\Delta T_{TBL,t}$ ($R^2 = 0.8197$) is close to the one obtained with our scaling law (fig. 1b, $R^2 = 0.7880$). More importantly this fit is much poorer than that obtained by Moore (2008) with his own dataset. For $\Delta T_{TBL,b}$ ($R^2 = 0.9700$), our data fit the scaling law proposed by Moore (2008) much better than our theoretical prediction (fig. 2c, $R^2 = -0.9971$), but it is again poorer than the fit obtained with his own dataset. A careful examination of the results in fig. 1 indicates that the poor fit to our dataset is essentially due to the cases with $Ra \leqslant 10^5$ and $H \approx H_{crit}$. Excluding these cases improves the fit for $\Delta T_{TBL,t}$ from $R^2 = 0.8197$ to $R^2 = 0.9815$. Furthermore, these cases are almost absent in the set of numerical simulations performed by Moore (2008). Discrepancies between our scaling laws and that obtained by Moore (2008) may thus be explained by differences in the explored dataset. The failure of the scaling laws to reproduce the results for low Ra when internal heating is dominant was already noted in the purely volumetrically heating case (Deschamps *et al.* 2012; Vilella & Kaminski 2017). Furthermore, Vilella & Kaminski (2017) showed that scaling laws based on the hot temperature profile, similar to the theoretical scaling laws developed in this work, are more successful in predicting thermal structure of the TBL at low Rayleigh number. They further argued that scaling laws based on the horizontally averaged temperature profile do not capture the physics correctly, because this profile averages the TBL during different stages of its evolution.

Figure 1b,d,f plot the theoretical predictions from Moore (2008) against our measurements for cases where $H \geqslant H_{crit}$. The predictions are in clear disagreement with the measurements, and in that case our scaling laws are much more successful (fig. 4). This is not surprising, since the scaling laws of Moore (2008) was not established for cases where $H \geqslant H_{crit}$. It however provides an additional evidence that the convective regime for $H \geqslant H_{crit}$ is different from the one for $H \leqslant H_{crit}$.

The results presented here confirm the differences found by Vilella & Kaminski (2017) between scaling laws obtained using the hot and horizontally averaged temperature profiles. In particular, it is important to emphasize that the precise understanding of the convective system is required to establish scaling laws. For instance, our theoretical framework neglects important processes of the convective system for $H \leqslant H_{crit}$ and is therefore not accurate in predicting the thermal structure of the two TBLs. However, previous scaling laws based on the horizontally averaged temperature profile also neglects part of the physics of the system. In particular, this temperature profile averaged the TBL at different stage of its evolution, and lead to inconsistent results for some values of Ra and H. When one excludes these specific cases, however, our numerical results agree well with the scaling laws proposed by Moore (2008), further supporting the validity and consistency of our numerical results.

REFERENCES

Choblet, G. 2012 On the scaling of heat transfer for mixed heating convection in a spherical shell. *Physics of the Earth and Planetary Interiors* **206–207**, 31–42.

Choblet, G. & Parmentier, E. M. 2009 Thermal convection heated both volumetrically and from below: Implications for predictions of planetary evolution. *Physics of the Earth and Planetary Interiors* **173**, 290–296.

Deschamps, F., Tackley, P. J. & Nakagawa, T. 2010 Temperature and heat flux scalings for isoviscous thermal convection in spherical geometry. *Geophysical Journal International* **182**, 137–154.

Deschamps, F., Yao, C., Tackley, P. J. & Sanchez-Valle, C. 2012 High Rayleigh number thermal convection in volumetrically heated spherical shells. *Journal of Geophysical Research* **117**, E09006.

Jarvis, G. T. 1993 Effects of curvature on two-dimensional models of mantle convection: Cylindrical polar coordinates. *Journal of Geophysical Research: Solid Earth* **98**, 4477–4485.

Moore, W. B. 2008 Heat transport in a convecting layer heated from within and below. *Journal of Geophysical Research: Solid Earth* **113**, B11407.

Shahnas, M. H., Lowman, J. P., Jarvis, G. T. & Bunge, H. P. 2008 Convection in a spherical shell heated by an isothermal core and internal sources: Implications for the thermal state of planetary mantles. *Physics of the Earth and Planetary Interiors* **168**, 6–15.

Sotin, C. & Labrosse, S. 1999 Three-dimensional thermal convection in an iso-viscous, infinite Prandtl number fluid heated from within and from below: applications to the transfer of heat through planetary mantles. *Physics of the Earth and Planetary Interiors* **112**, 171–190.

Vilella, K. & Kaminski, E. 2017 Fully determined scaling laws for volumetrically heated convective systems, a tool for assessing habitability of exoplanets. *Physics of the Earth and Planetary Interiors* **266**, 18–28.

Weller, M. B., Lenardic, A. & Moore, W. B. 2016 Scaling relationships and physics for mixed heating convection in planetary interiors: Isoviscous spherical shells. *Journal of Geophysical Research: Solid Earth* **121**, 7598–7617.

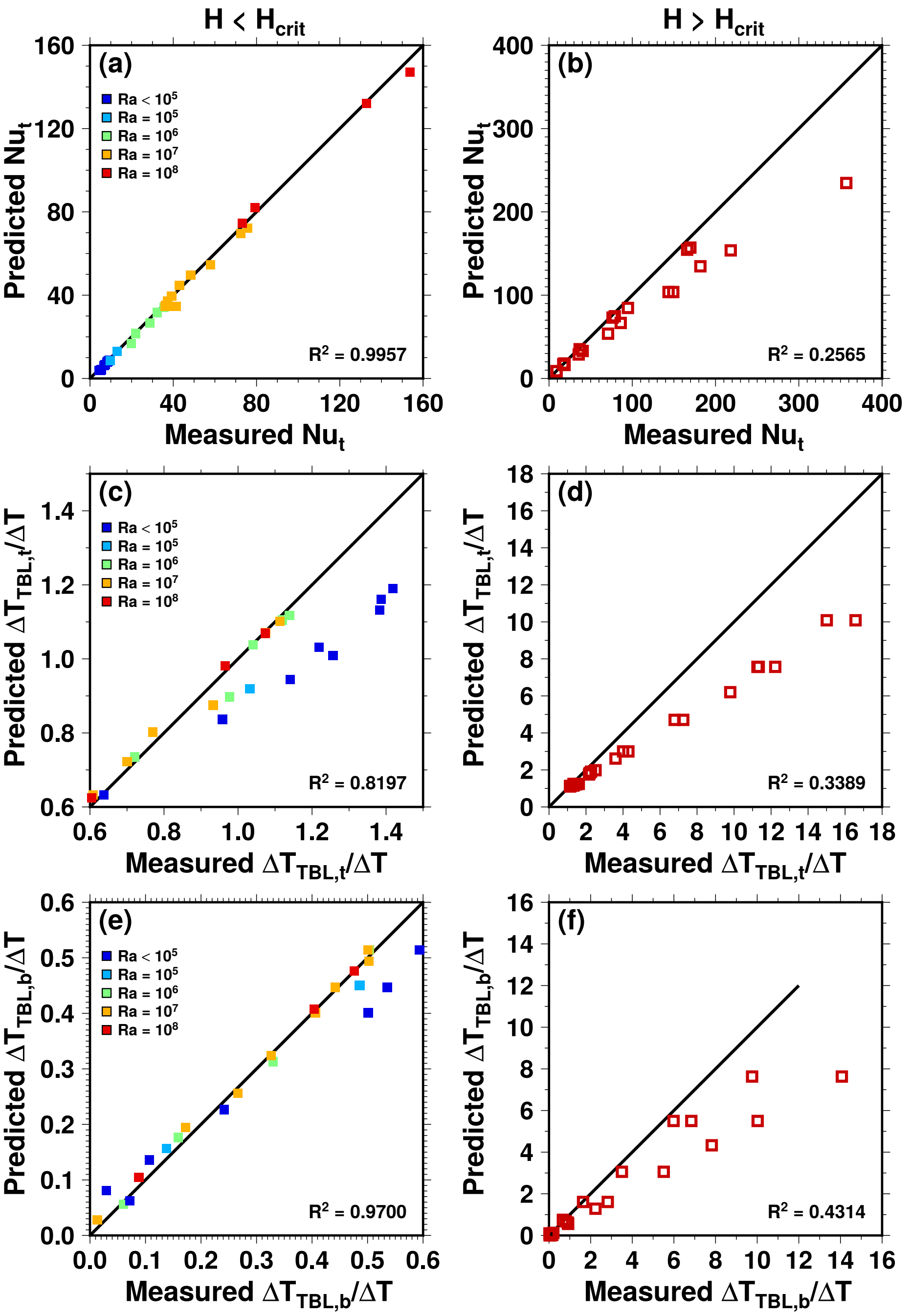

FIGURE 1. Comparison between predictions and measurements for (a), (b) the dimensionless surface heat flux (Nu_t) (c), (d) the dimensionless temperature jump across the top thermal boundary layer ($\Delta T_{TBL,t}/\Delta T$), and (e), (f) the temperature jump across the bottom thermal boundary layer ($\Delta T_{TBL,b}/\Delta T$), for the cases where $H \leqslant H_{crit}$ and $H \geqslant H_{crit}$, respectively. The scaling laws used for predictions are from Moore (2008). The solid lines correspond to a perfect agreement between predictions and measurements, while the R^2 values of the fits are indicated in each panel. The symbol colors in the left panels represents the corresponding value for Ra.

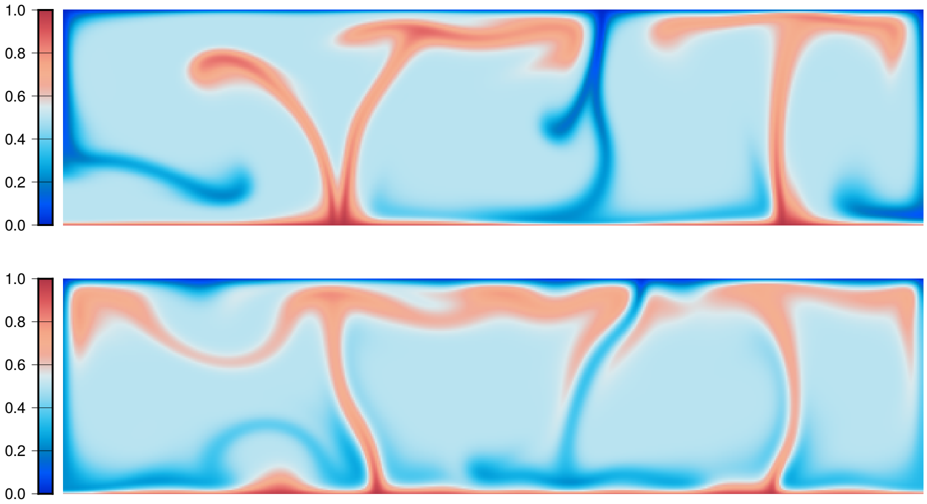

FIGURE 2. Dimensionless temperature fields of a 2D-Cartesian numerical simulation conducted for $Ra = 10^6$ and free slip top and bottom boundary conditions. The two temperature fields are separated by a short time, the top panel being earlier than the bottom one, and are used in figure 6.

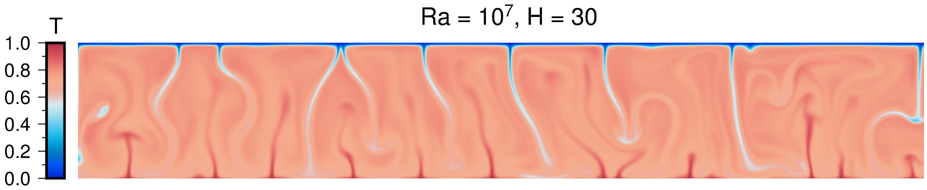

FIGURE 3. Dimensionless temperature fields of a 2D-Cartesian numerical simulation conducted for $Ra = 10^7$, $H = 30$ and free slip top and bottom boundary conditions. This temperature field is used in figure 7.