Celestial Mechanics: Homework II

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due in class on May 6th, 2009

1. J_2 **and oblateness:** This problem is a slightly modified version of Prob. 4.1 in Murray & Dermott. In general, the nth harmonic coefficient of a planet's gravity field is given by the integral

$$J_n = -\frac{2\pi}{MR^n}\int_0^R\int_{-1}^1 P_n(\mu)r^{n+2}\rho(r,\mu)d\mu dr, \tag{1}$$

where P_n is the Legendre polynomial of degree n, $\mu = \cos\theta$, M and R are the planet's mass and mean radius, and $\rho(r,\mu)$ is the interior density distribution. Consider a uniform density planet with oblateness $\epsilon = (a-b)/a$, where a and b are the equatorial and polar radii.

   1.1 Prove that $J_2 = (C-A)/Ma^2$, which has been mentioned in class.

   1.2 By dividing the planet into a sphere of radius b, surrounded by a thin shell of variable thickness $h(\theta) = \epsilon R\sin^2\theta$, and evaluating the integral separately for the two pieces, show that $J_2 \approx 2\epsilon/5$, in the limit that $\epsilon \ll 1$.

   1.3 Why does this procedure fail for a planet with a radial density gradient?

   1.4 For a planet in hydrostatic equilibrium, show that $\epsilon = (3/2)J_2 + q/2$, where the quantity q is related to the Love number k_2 by $k_2 = 3J_2/q$.

2. **Tidal Dissipation in Planets:** Solve Prob. 4.6 in Murray & Dermott.

3. **Orbital Precession of Saturn:** Solve Prob. 6.3 in Murray & Dermott.

4. **Secular Interactions of Two Planets:** Solve Prob. 7.1 in Murray & Dermott.

5. **GJ 876:** In 2000, two planets orbiting the M4V star GJ 876 were discovered[1]. The mass of GJ876 is estimated to be 0.32 solar masses. Deduced from the radial-velocity method and astrometry, the two planets are found to be locked in the 2:1 mean motion resonance and their masses are $m_1 = 0.597M_{Jup}$ and $m_2 = 1.90M_{Jup}$, where M_{Jup} denotes one Jupiter mass. Their orbital elements are as follows: $P_1 = 30.38$ d, $P_2 = 60.93$ d, $e_1 = 0.218$, $e_2 = 0.029$, $\varpi_1 = 154.4°$, and $\varpi_2 = 149.1°$. Write down the equations of motion for e and ϖ of these two planets due to the resonant perturbation.

[1] A third planet of $0.018M_{Jup}$ was discovered later in 2005.