Celestial Mechanics: Homework III

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due in class on May 27th, 2009

1. **Relaxation time:** Evaluate the relaxation time due to star-star collisions for the following star systems: disk galaxy ($v \approx 30$ km/s, $n \approx 0.1\text{pc}^{-3}$, size$\approx$ 30kpc), globular cluster ($v \approx 10$ km/s, $n \approx 10^4\text{pc}^{-3}$, size$\approx$ 5pc), open cluster ($v \approx 1$ km/s, $n \approx 10\text{pc}^{-3}$, size$\approx$ 5pc).

2. **Gravitational Focusing:** The phenomenon of gravitational focusing is ubiquitous in astronomical studies. For instance, you have seen in class that many x-ray binaries can form from two unrelated stars via this process in a crowded stellar environment such as a globular cluster. In the lecture on exoplanets, you have heard that planetesimals in a proto-planetary disk are massive enough to merge together through the same process even though their original separation is much larger than their sizes. Consider a massive body of physical radius R and mass M as well as a small particle of mass m moving at a dispersive speed v_{dis} relative to the massive body at infinity.

   2.1 Applying the conservation of energy and angular momentum to the motion of the test particle toward the massive body with an impact parameter b, show that the impact parameter required for the test particle to plunge to the surface of the massive body is given by

   $$b = R\sqrt{1 + \left(\frac{v_{esc}}{v_{dis}}\right)^2}, \tag{1}$$

   where v_{esc} is the escape velocity from the massive body.

   2.2 Comparing the above result to the minimum impact parameter b_{min} for the stellar close encounter derived in class, explain why they are different.

   2.3 If we assume that a group of massive and light bodies reach a state of energy equipartition during the process of dynamical friction, does a more massive or a less massive body grow more quickly via the merge process due to gravitational focusing? Explain your result.