Celestial Mechanics: Homework IV

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due in class on June 3rd, 2009

1. **Equations in cylindrical coordinates:** The natural coordinate system to work with for a disk galaxy is the cylindrical coordinates (r, ϕ, z) with the origin at the galactic center. z is the vertical coordinate parallel to the rotation axis and $z = 0$ represents the midplane of the disk. r is the radial coordinate perpendicular to z-axis and ϕ is the azimuthal coordinate.

   2.1 Show that the collisionless Boltzmann equation in cylindrical coordinates is given by eq(4-17) in the reference book "Galactic Dynamics" 1st edition by Binney & Tremaine (hereafter "BT").

   2.2 The moment equations in general coordinates were derived in class. Following the similar procedures, derive eqs(4-28), (4-29a), (4-29b), and (4-29c)[1] from eq(4-17) in BT under the axisymmetric assumption (i.e. Φ and all average quantities do not vary with ϕ).

   2.3 Fluid equations in general coordinates were described in class. Derive the continuity and momentum equations in cylindrical coordinates:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(\rho r u_r\right) + \frac{1}{r}\frac{\partial}{\partial \phi}\left(\rho u_\phi\right) = 0, \tag{1}$$

$$\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\phi}{r}\frac{\partial u_r}{\partial \phi} - \frac{u_\phi^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} - \frac{\partial \Phi}{\partial r}, \tag{2}$$

$$\frac{\partial u_\phi}{\partial t} + u_r\frac{\partial u_\phi}{\partial r} + \frac{u_\phi}{r}\frac{\partial u_\phi}{\partial \phi} + \frac{u_\phi u_r}{r} = -\frac{1}{\rho r}\frac{\partial P}{\partial \phi} - \frac{1}{r}\frac{\partial \Phi}{\partial \phi}. \tag{3}$$

[1] The page of "BT" describing these equations can be found online at http://books.google.com/books/princeton?id=01yNf7mipb0C&printsec=frontcover#PPA197,M1