Stellar Physics: Homework II

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due in class on Apr 2nd, 2014

1. **Radiative diffusion and Kelvin-Helmholtz timescale:** Use the radiative diffusion equation to show that the diffusion time of photons through the sun is $t_D \sim \rho c_p R_\odot^2/k$, where $k = 16\sigma T^3/3\kappa_R \rho$. Using representative values of the opacity, density, and specific heat, show that t_D is of the order of 10^7 years. Comment on the agreement between the radiative diffusion timescale and the Kelvin-Helmholtz timescale.

2. **Photospheric boundary conditions**

   2.1 Take a opacity equal to 1 cm^2/g at the solar photosphere. Estimate the pressure at the photosphere in terms of 1 atm. Roughly estimate the ratio of the photospheric pressure p_p to the pressure at the center of the Sun p_c (p_c is taken from the last homework assignment).

   2.2 In class, I derive the photospheric boundary condition in the absence of any external irradiation. Assume that there exists isotropic external irradiation with the flux given by F_{irr}.[1] Show that

   $$\sigma T^4 = \frac{1}{2} F \left(\frac{3}{2} \tau + 1 \right) + F_{irr}. \quad (1)$$

   Note that at the photosphere (i.e. $\tau = 2/3$), the above equation leads to $\sigma T^4 = F + F_{irr}$, meaning that the blackbody temperature at the photosphere is contributed from the "intrinsic" energy flux from the stellar interior F plus the external irradiation F_{irr}.

3. **What if no quantum tunneling?** This is an exercise from the book "Modern Astophysics" by Carroll & Ostlie.

   3.1 What temperature would be required for to protons to collide if quantum mechanical tunneling is neglected? Assume that nuclei having velocities ten times the root-mean-square (rms) value for the Maxwell-Boltzmann distribution can overcome the Coulumb barrier. Compare your answer with the estimated central temperature for the Sun $T \approx 10^7$ K.

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[1] F_{irr} can originate from a companion star in a close binary system.

3.2 Calculate the ratio of the number of protons having velocities ten times the rms value to those moving at the rms velocity.

3.3 Assuming (incorrectly) that the Sun is pure hydrogen, estimate the number of hydrogen nuclei in the Sun. Could there be enough proton moving with a speed times the rms value to account for the Sun's luminosity?

4. **Thermaonuclear reaction:**

4.1 In class, I show the nuclear energy liberated from pp chain to produce one α particle is about 26.7 MeV. Use this information to estimate the flux of pp chain solar neutrinos at the Earth given the solar luminosity $L_\odot$. Assume that the pp chain is entirely responsible for the solar energy production. If 2% of the energy is lost this way, what is the average neutrino energy?

4.2 Show that the energy generation rates per unit mass for pp chain ϵ_{pp} and for the CNO cycle ϵ_{CNO} are

$$\epsilon_{pp} = c_1 \rho X^2 T^{-2/3} \exp\left[-33.8\left(\frac{10^6}{T}\right)^{1/3}\right], \qquad (2)$$

$$\epsilon_{CNO} = c_2 \rho X X_{CNO} T^{-2/3} \exp\left[-155\left(\frac{10^6}{T}\right)^{1/3}\right], \qquad (3)$$

where X is the mass fraction of hydrogen, X_{CNO} is the total mass fraction of carbon, nitrogen, and oxygen, c_1 and c_2 are constants. Note that deriving approximate numerical values in the exponential terms are O.K.

4.3 Calculate their power-law indices ν_{pp} and ν_{CNO} and explain what causes the difference in the temperature dependence.

4.4 Consider the following model of a massive star. Its mass is $M = 10M_\odot$ and its radius $R = 3.6R_\odot$. Its central density is 10 g/cm^{-3} and the bulk of energy generation takes place within a radial distance $0.15R$. Assume constant density for this region and a temperature 2.7×10^7 K. Adopt $X_{CNO} = 0.005X$ and $c_2 = 8 \times 10^{31}$ in c.g.s. units. Determine the rate of energy generation by the star. Also, calculate how soon it will use up the hydrogen in its central core.

4.5 Repeat the same calculations for the Sun assuming burning in the central region out to $0.2R$. Take a central density of about 55 g/cm^{-3} and a central temperature 10^7 K. Adopt $c_1 = 2.5 \times 10^{10}$ in c.g.s. units.