Stellar Physics (Radiative Processes): Homework IV

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due in class on May 23rd, 2014

1. **Effective temperature of a geometrically thin disk:** When we derived the emergent flux from a blackbody $F = \sigma T^4$ in class, we have made use of the relation $F = \pi B$ for an isotropic emitter without proving it beforehand. But you can find the proof in the text around eq.(1.14) and Figure 1.6 in Rybicki & Lightman. Now make use of Figure 1.6 in the book again, but consider a star of radius R (as shown in the figure), surrounded by a geometrically thin, flat disk with its disk plane aligned with the dashed line in the figure. In other words, point P denotes any point that lies on the disk with a distance r from the stellar center. The flat disk is assumed to be infinitesimally thin in geometry but optically thick.

   1.1 You can employ spherical coordinates with the origin at point P such that the axis of the coordinate system points to the stellar center (see Figure 1.6). Show that the stellar flux passing through the disk surface is $F = \int I_* \sin\theta \cos\phi d\Omega$, where I_* is the constant brightness on the spherical photosphere of the star.

   1.2 Given the fact that the top surface of the optically thick disk can only be illuminated by the flux coming from the top half of the star, show that

   $$F = \frac{\sigma T_*^4}{\pi}\left[\sin^{-1}\left(\frac{R}{r}\right) - \left(\frac{R}{r}\right)\sqrt{1-\left(\frac{R}{r}\right)^2}\right], \tag{1}$$

   where T_* is the effective temperature of the star.

   1.3 Assume that the disk attains thermal equilibrium. In the limit that $(R/r) \ll 1$ (i.e. far from the star), show that $T_{disk} \propto r^{-3/4}$.

2. **radiation force:** Consider a plane-parallel stellar atmosphere with known extinction coefficient $\chi_\nu(z) = \alpha_\nu(z) + \sigma_\nu(z)$ and known radiation field $I_\nu(z, \mu)$. Find an expression for the force per unit volume on the stellar matter exerted by the radiation field due to extinction.

3. **Eddington approximation:** In class we assumed the intensity to be expanded to linear order $I(\mu) = a + b\mu$ to solve the radiative transfer equation. This procedure leads to the Eddington approximation: $f \equiv K/J = 1/3$. In this problem, we shall study various versions of the approximation.

3.1 Consider the intensity of the form $I(\mu) = I_0 + \sum_n I_n \mu^n$. If the sum includes only odd powers n, what should be the value of f?

3.2 Suppose that $I(\mu) = I_1$ for $0 \leq \mu \leq 1$ and $I(\mu) = I_2$ for $-1 \leq \mu \leq 0$. What is the value of f?

3.3 Consider $I(\mu) = a(1 - \mu^2)$, which is analogous to the emission from electron scattering. What is f in this case?

4. **Eddington limit of luminosity:**

4.1 Rybicki & Lightman: Problem 1.4

4.2 Follow up the preceding exercise. Assume that the luminosity of an accreting object reaches its maximum value, i.e. the Eddington luminosity L_{EDD}. The mass and radius of the central object is one solar mass and 10 km, respectively (i.e. typical parameters for a neutron star). What is the effective temperature of the accreting HII gas? Also estimate the viral temperature of the accreting HII gas, which is assumed to be in quasi-hydrostatic equilibrium? Which temperature is higher? Make an attempt to explain why.

5. **more from Ribicky & Lightman:**

5.1 Ribicky & Lightman: problem 1.7

5.2 Ribicky & Lightman: problem 1.8