Stellar Physics
lecture 6: stellar evolution

辜品高

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pre-main-sequence stars

Calvet 2004

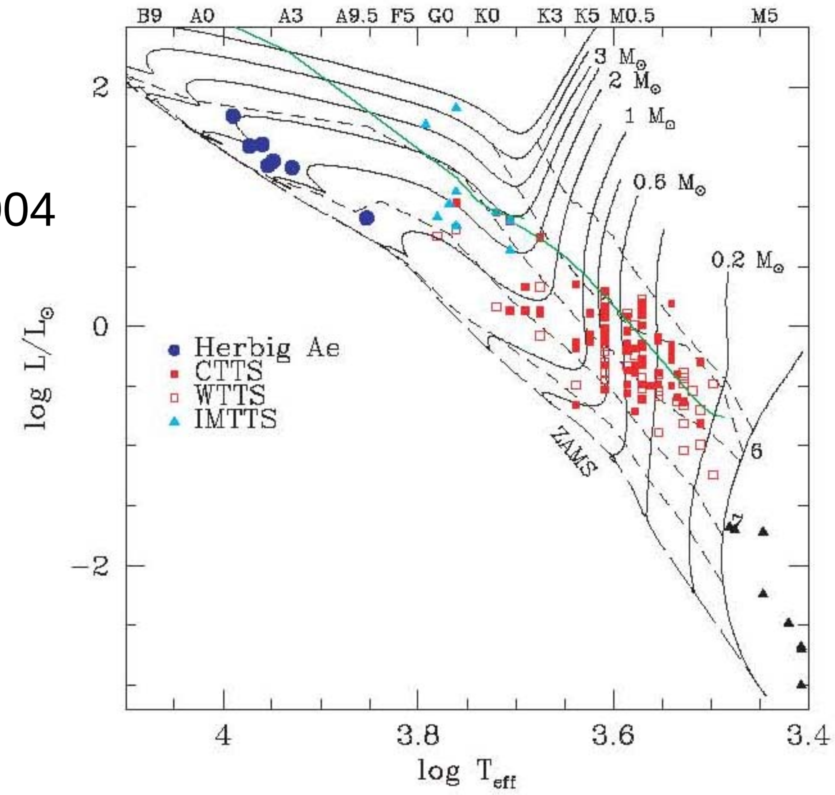


Fig. 1. HR diagram showing the location of visible YSO: Herbig Ae/Be stars, Classical T Tauri stars, Weak T Tauri stars, and intermediate mass T Tauri stars. The luminosity refers to the purely stellar luminosity. Evolutionary tracks for masses between 0.1 and 4 $\dot{M}$ (light solid lines) and isochrones between log $t = 5.5$ and 8 (dashed lines) are indicated. The heavy solid line is the birthline.

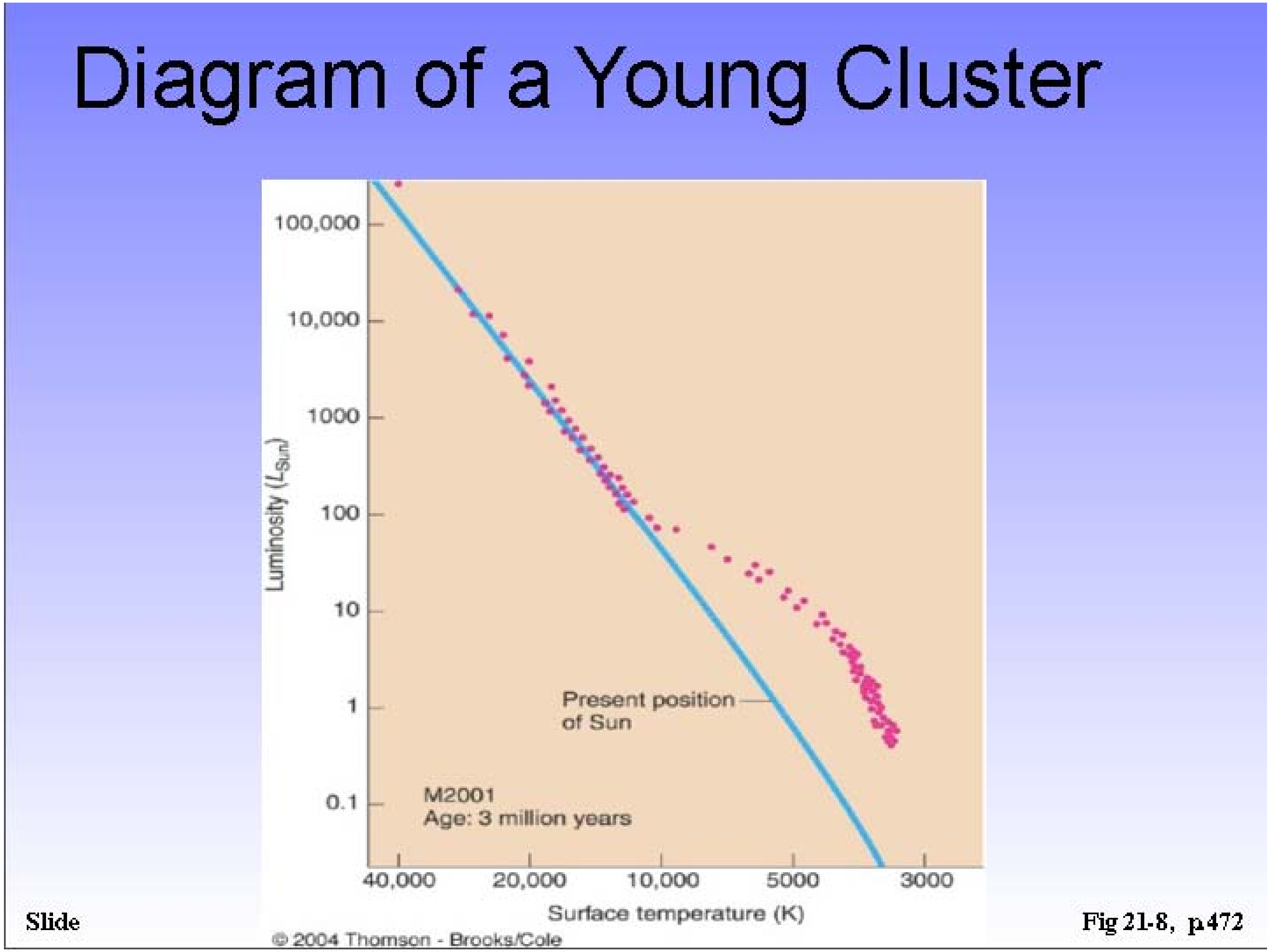
vertical: Hayashi track
H- opacity acts as a thermostat

horizontal: Henyey track, radiative core develops

Stellar birth line: corresponds to the upper envelope of pre main sequence stars on the HR diagram, which also coresonds to the D burning phase (Stahler 1988)

http://adsabs.harvard.edu/abs/2004adjh.conf..521C

Age of a young cluster



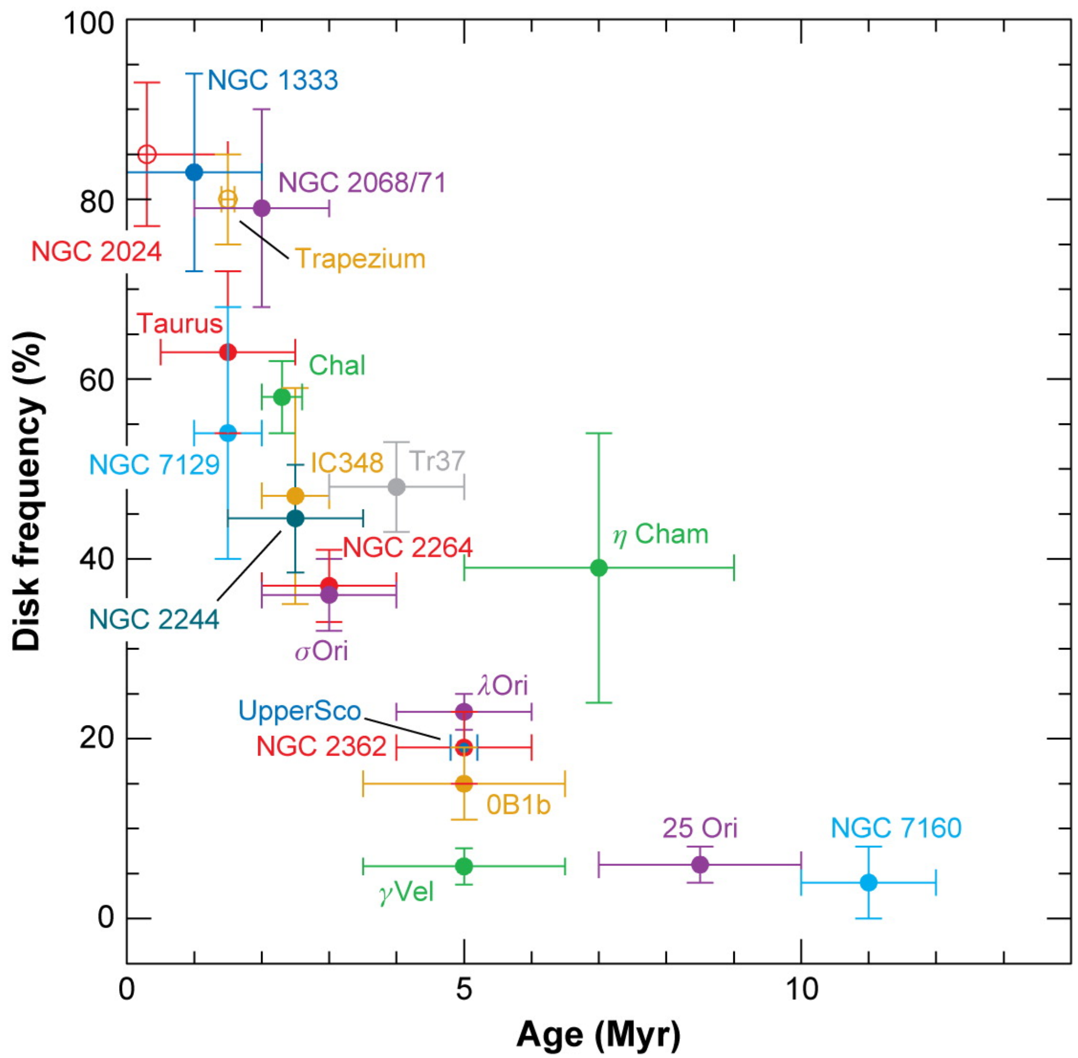
depletion of protostellar disks around sun-like proto-stars

Near-Infrared excess comes from micron-sized, hot (about 900K) dust grains.

This may imply that gas giant planets form in ~ a few million years.

cf. solar-type stars spend Gyrs on main sequence.

Causes: accretion, photoevaporation, disk winds?



Wyatt MC. 2008.
Annu. Rev. Astron. Astrophys. 46:339–83

radiative stars

Mass - Luminosity (M - L) relation for Radiative Stars

$$L(r) = -\frac{16\pi\sigma r^2}{3\kappa\rho}\frac{dT^4}{dr} \Rightarrow \frac{1}{\overline{\kappa\rho}}\frac{T^4}{R_*} \propto \frac{L_*}{R_*^2}$$

Hydrostatic equilibrium :

$$\frac{GM}{R} \sim \frac{k\overline{T}}{\mu m_u} \Rightarrow \overline{T} \propto \frac{M_*}{R_*},\ \overline{\rho} \sim \frac{M_*}{R_*^3}$$

so, $L \propto \frac{M_*^3}{\overline{\kappa}}$

High - mass star (electron scattering dominated) : $L \propto M_*^3$

Low - mass star (b - f, f - f transitions : $\kappa \propto \rho T^{-3.5}$) : $L \propto \frac{M_*^3}{\overline{\kappa}} \propto M_*^{11/2} R_*^{-1/2}$

If stars are so massive ($M >> 140 M_{sun}$) that radiation pressure ($P = aT^4/3, \kappa = \kappa_{es}$) dominates :

$$\frac{1}{\rho}\frac{dP}{dr} = \frac{4aT^3}{3\rho}\frac{dT}{dt} = -\frac{GM}{r^2} \Rightarrow \frac{L}{4\pi r^2} = F = -\frac{4acT^3}{3\rho\kappa}\frac{dT}{dr} = \frac{cGM}{\kappa r^2}$$

$\Rightarrow L = \frac{4\pi cGM}{\kappa_{es}} \propto M$ (Eddington Luminosity, independent of R

because both radiation pressure force and gravitational force $\propto 1/r^2$, so they cancel)

These M-L scaling laws, though oversimplified, are useful for main-sequence and horizontal tracks in the H-R disgram.

main sequence

$M > 140M_{sun}, aT_c^4 \propto \frac{GM^2}{R^4}$ (i.e. due to radiation pressure support)

$\varepsilon_{CNO} \propto \rho_c T_c^{16} \propto \frac{M^9}{R^{19}}$

total generation rate $\propto M\varepsilon_{CNO} \propto \frac{M^{10}}{R^{19}}$

Eddington luminosity $L \propto M$

In thermal equilibrium, the above two equal $\Rightarrow R \propto M^{9/19} \approx M^{0.5}$

$2M_{sun} < M < 20M_{sun}$ ($T_c \propto \frac{M}{R}$ due to pressure support)

$\varepsilon_{CNO} \propto \rho_c T_c^{16} \propto \frac{M}{R^3}\left(\frac{M}{R}\right)^{16} \propto \frac{M^{17}}{R^{19}}$

total generation rate $\propto M\varepsilon_{CNO} \propto \frac{M^{18}}{R^{19}}$

luminosity $L \propto M^3$

In thermal equilibrium, the above two equal $\Rightarrow R \propto M^{15/19}$

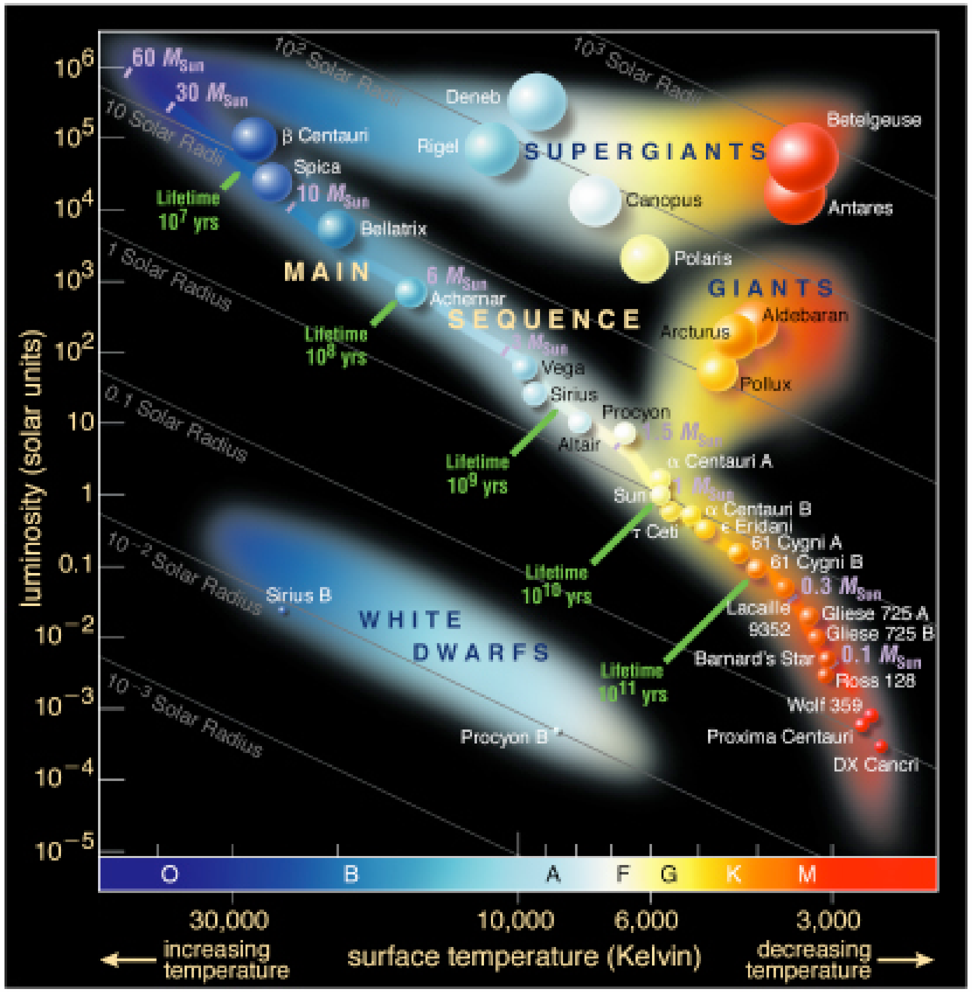
$0.5M_{sun} < M < 2M_{sun}$: appreciable outer convection zone

$M < 0.5M_{sun}$: completely convective

The above scaling approaches for radiative stars fail.

computer modeling $\Rightarrow R \propto M^{0.9}$

Hertzprung-Russell diagram



stellar types

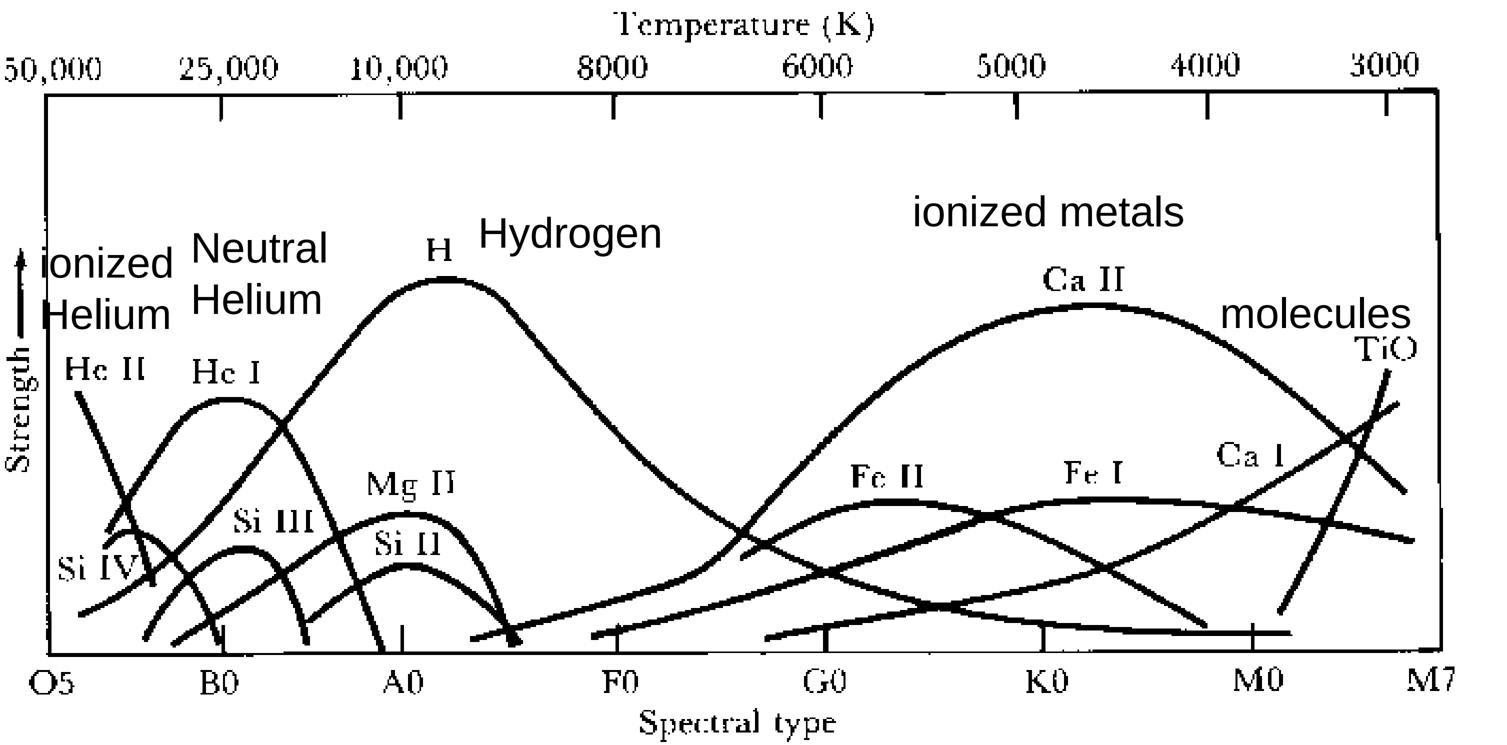
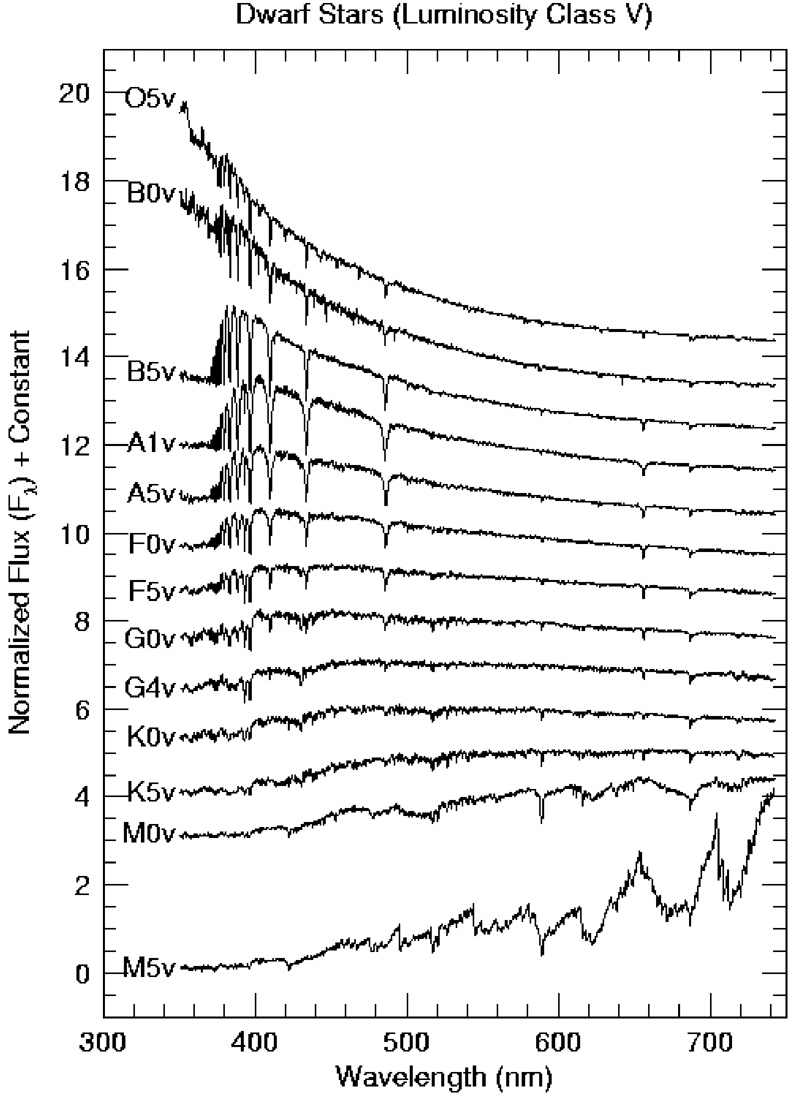
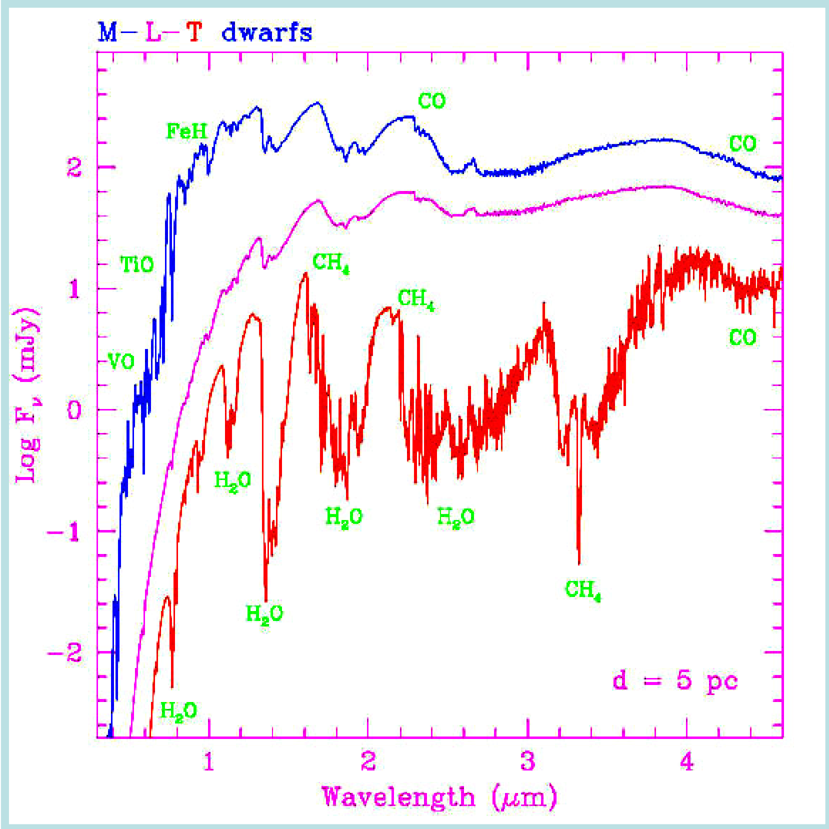


Figure 12-9 7-36
Kaufmann
DISCOVERING THE UNIVERSE
Second Edition

stellar spectra

http://frigg.physastro.mnsu.edu/~eskridge/astr101/dwarfs.gif





interior of man-sequence stars

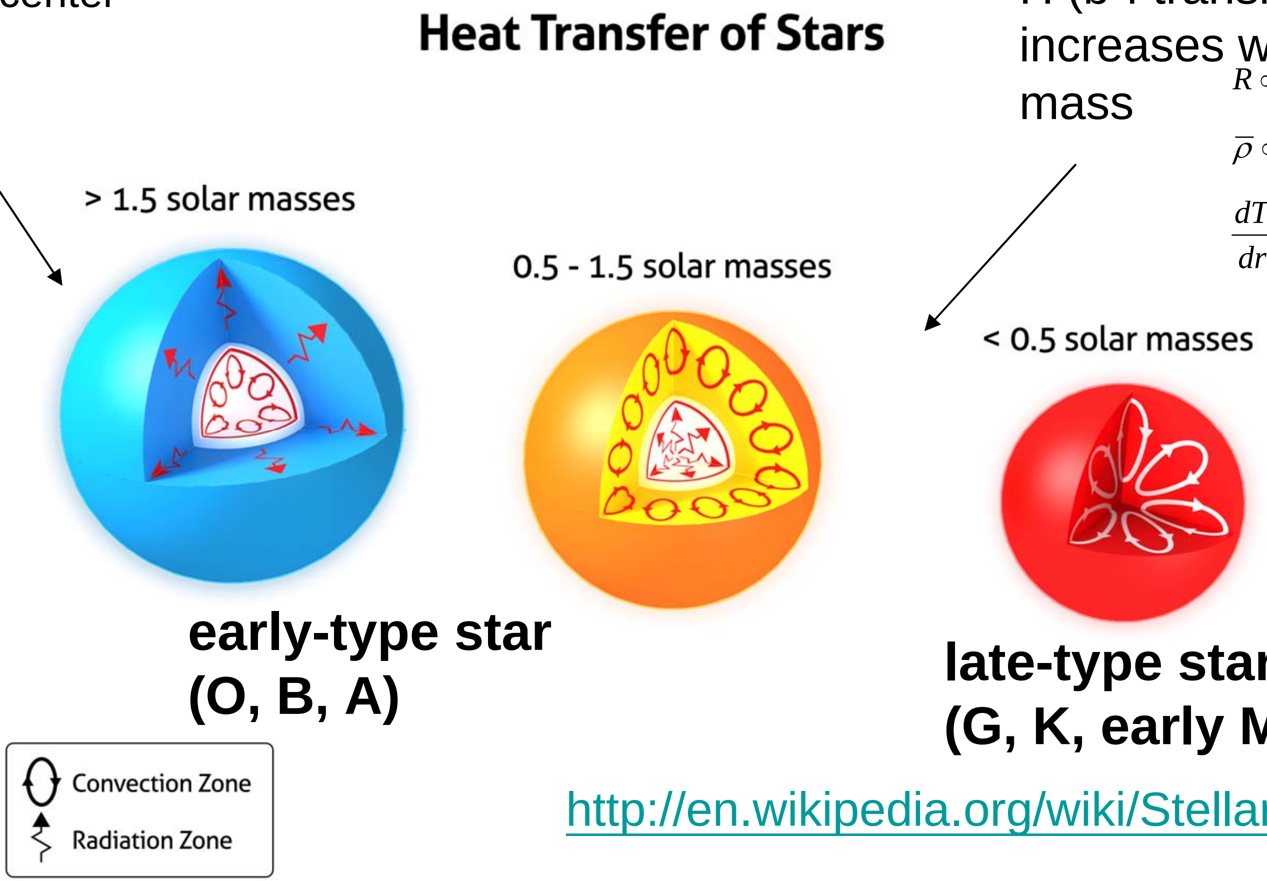
CNO cycles is extremely sensitive to temperature
→ energy generation concentrated toward the center
→ $\nabla > \nabla_{ad}$

large opacity κ
due to partially ionized H (b-f transition), also density increases with decreasing mass

$$R \propto M$$

$$\bar{\rho} \propto \frac{M}{R^3} \propto \frac{1}{M^2}$$

$$\frac{dT}{dr} \propto \rho\kappa$$

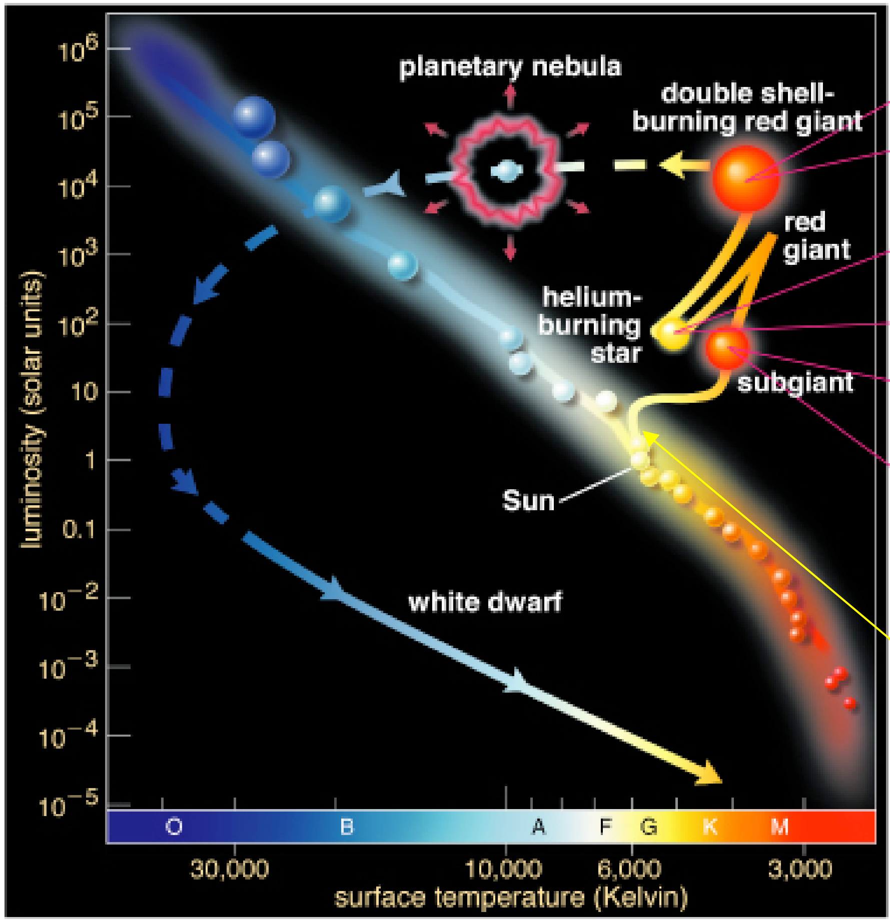


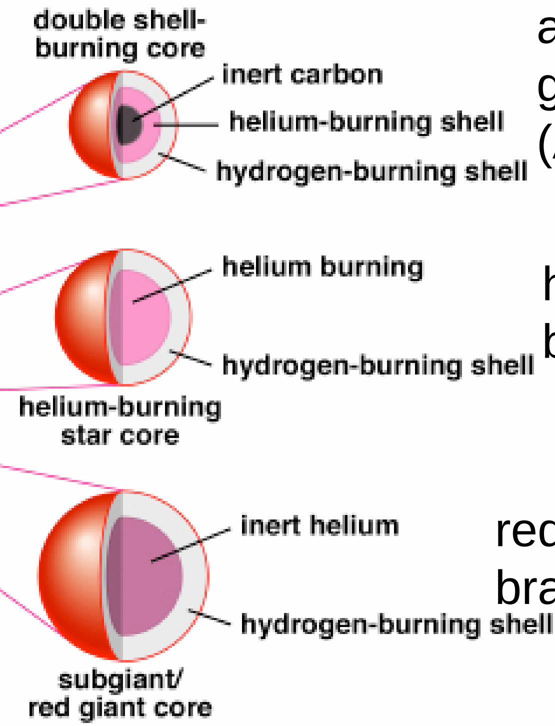
early-type star (O, B, A)

late-type star (G, K, early M)

http://en.wikipedia.org/wiki/Stellar_structure

evolutionary track of a solar-mass star





asymptotic giant branch (AGB)

horizontal branch (HB)

red giant branch (RGB)

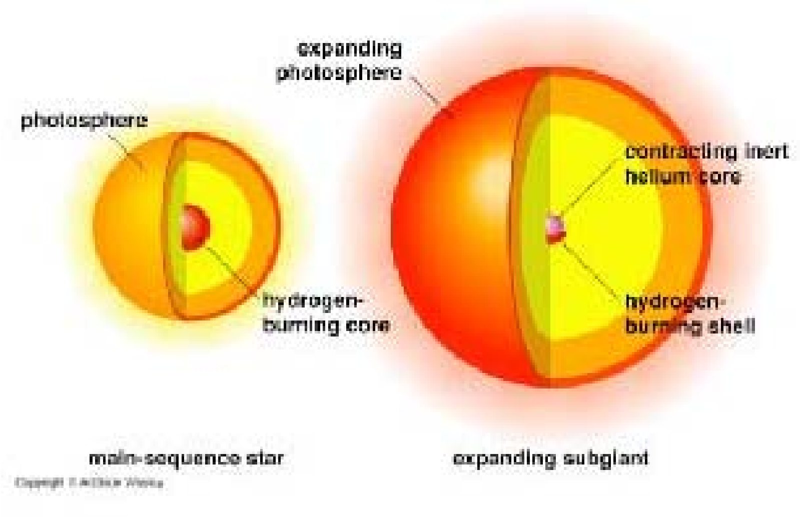
The Sun gets brighter on the main sequence

$$\varepsilon_{pp} \propto \rho X_H^2 T^4, \quad P = \frac{\rho R T}{\mu}$$

Although X_H decreases with time in the H-burning core, μ increases with time $\rightarrow$ the H-burning core is then compressed, increasing ρ and T, which in turn compensates the offset of the decrease in X_H and in fact enhances ε_{pp}!

Because of the large luminosities on the red giant and asymptotic giant branches, the exhaustion of the majority of the fuel takes only 10% of the time that the main-sequence required to exhaust a minority amount.

from main sequence to subgiant



Schönberg-Chandrasekhar limit

end of the main-sequence: core H exhaustion → a He core surrounded by a H-rich envelope
In what condition does the inner He core start to gravitationally contract? Ans: Schönberg & Chandrasekhar limit

Assume that the He core is an ideal gas, radiative and almost in thermal and hydrostatic equilibrium.

$$\frac{\partial L}{\partial m} = \varepsilon = 0 \;\; \& \;\; L(m=0) = 0 \Rightarrow L = 0 \text{ in the core}$$

$$\Rightarrow \frac{\partial T}{\partial m} = -\frac{3\kappa L}{64\pi^2 r^4 acT^3} = 0 \Rightarrow \text{isothermal core}$$

Applying the virial theorem to the core but with a surface pressure from its envelope:

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2} \Rightarrow 4\pi R_c^3 P(R_c) - 3\int_0^{R_c} P 4\pi r^2 dr = -\int_0^{M_c} \frac{Gmdm}{r}$$

$$\Rightarrow 4\pi R_c^3 P(R_c) = 3\frac{RT_c}{\mu_c} M_c - \alpha \frac{GM_c^2}{R_c} (= 2E_i + E_g)$$

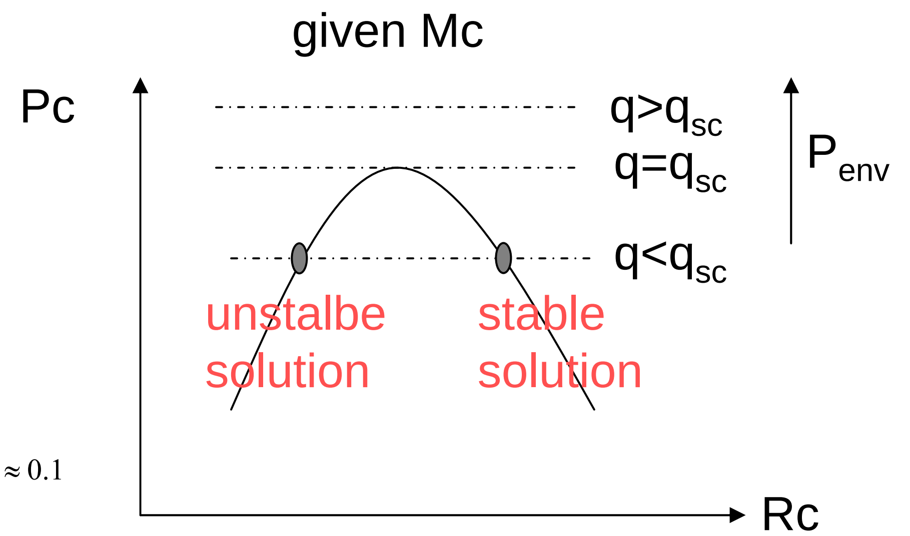
$$\Rightarrow P(R_c) = \frac{3M_c RT_c}{4\pi R_c^3 \mu_c} - \alpha \frac{GM_c^2}{4\pi R_c^4}$$

$$\Rightarrow P(R_c)_{\max} = \left(\frac{9^4 R^4 \alpha G}{12\pi(4\alpha G\mu_c)^4}\right)\frac{T_c^4}{M_c^2} \quad \text{at} \quad R_c = \frac{4\alpha GM_c\mu_c}{9RT_c}$$

Moreover, we may assume at the bottom of the envelope

$$P_{env}(R_c) \sim \frac{M_*^2}{R_*^4}, T_c \sim T_{env}(R_c) \sim \frac{M_*}{R_*} \Rightarrow P(R_c) \propto \frac{T_c^4}{M_*^2}$$

$$\text{Hence, } P_{env}(R_c) < P(R_c)_{\max} \Rightarrow q \equiv \frac{M_c}{M_*} \le q_{sc} = 0.37\left(\frac{\mu_{env}}{\mu_c}\right)^2 \approx 0.37\left(\frac{0.65}{4/3}\right)^2 \approx 0.1$$



Core contraction & H burning shell

When an isothermal He core is built up to the Schönberg & Chandrasekhar limit (i.e. $q > q_{sc} \approx 0.1M_*$), the core cannot maintain quasi-hydrostatic equilibrium but starts to contract. the star leaves the main-sequence

The He core contracts on the KH timescale. The weight of the envelope increases the pressure at the base of the envelope to maintain the quasi-hydrostatic equilibrium. The increase of the pressure and temperature to ignite H burning in the shell at the base of the envelope.

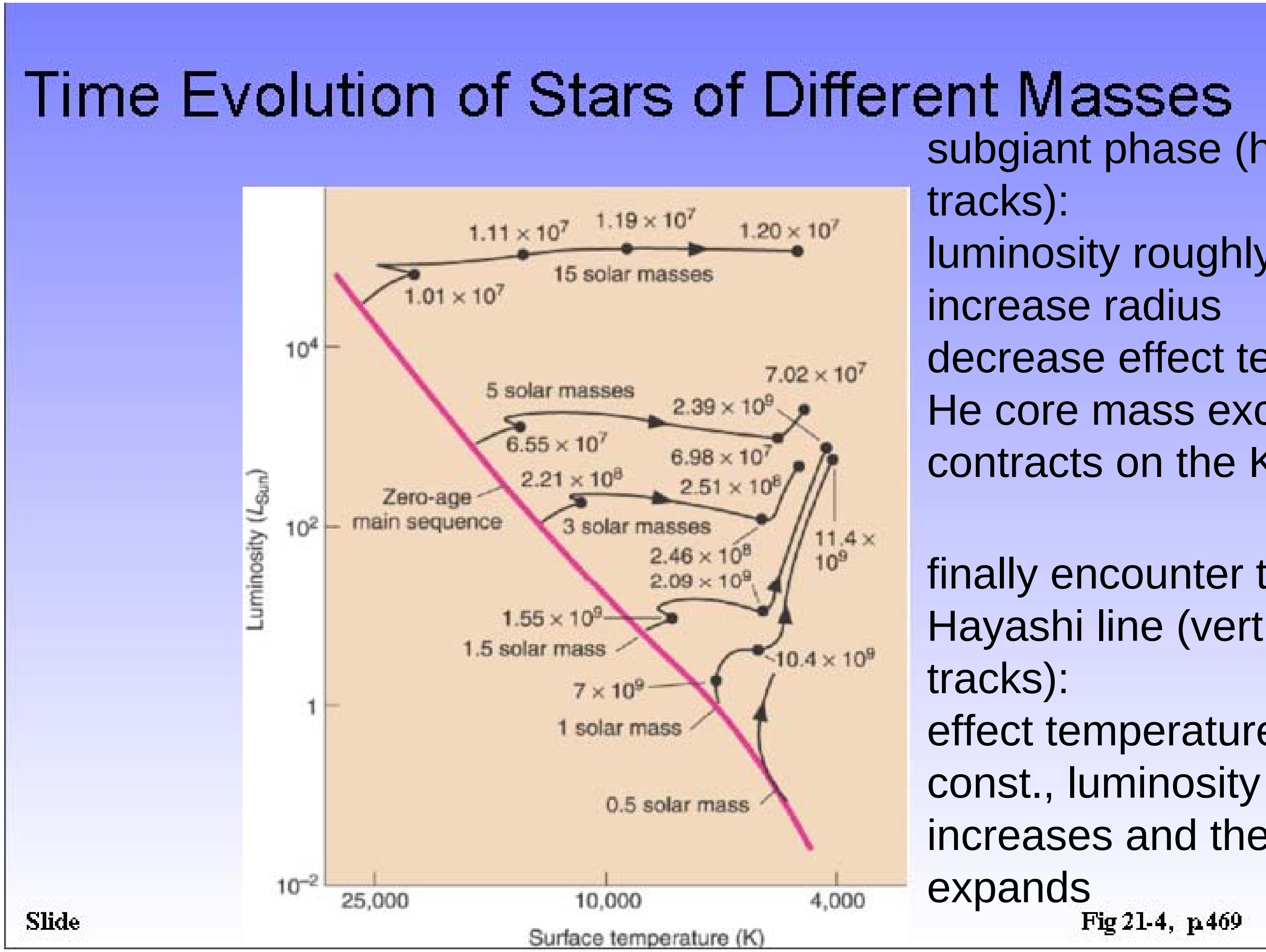
H burning shell drops He “ash” into core, which adds to its gravity and tendency to contract even more. Weight and therfore pressure of H burning shell becomes greater which makes H fusion occurs faster. The gain in heat by the envelope is larger than what is lost from the photosphere. The envelope expands (i.e. due to pdV work) even as the core contracts. The star evolves to a subgiant.

If this expansion occurs while the envelope is mostly radiative, then the star must follow a more-or-less horizontal track in the H-R diagram. It finally evolves to the Hayashi line. As stars ascend their Hayashi tracks to the red giant branch, their envelopes becomes increasing convective.

evolutionary tracks on H-R diagam

Roughly speaking, evolutionary tracks of stars in the H-R diagram either go horizontal (if the envelopes are radiative) or they do vertical (if the envelopes are convective). Evolutionary tracks in the H-R diagram therefore never go, for example, at a diagonal (in either branch of an X).

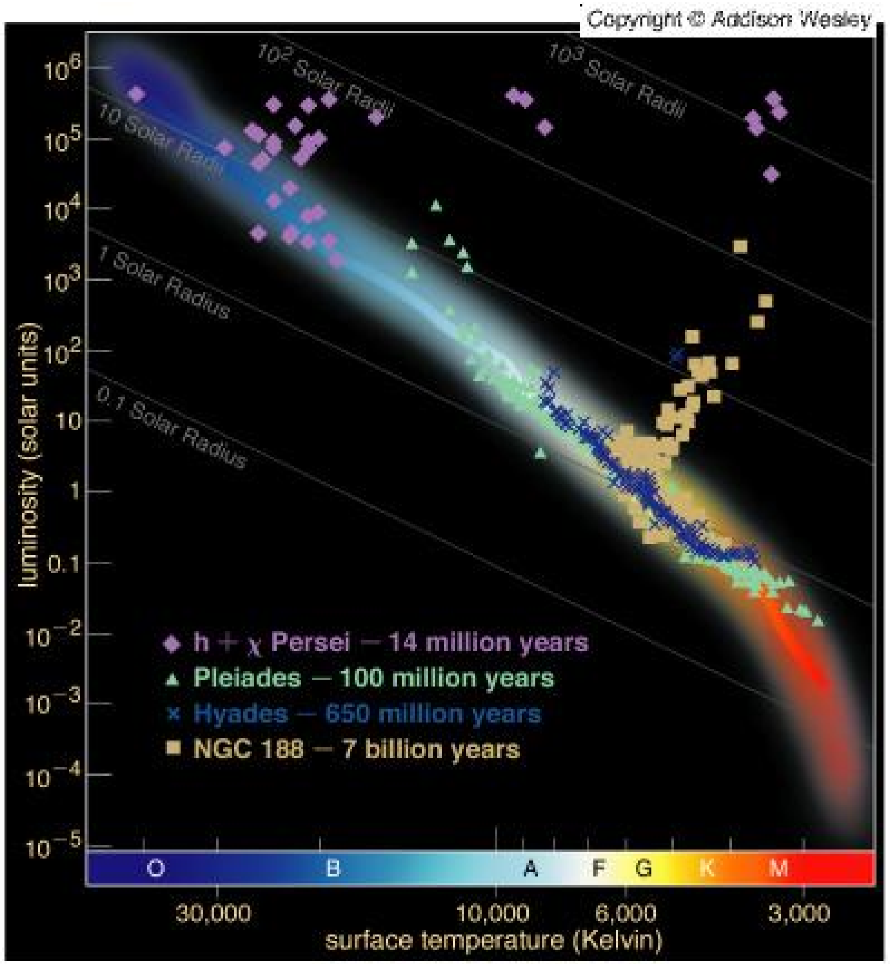
Ascent to red giant



subgiant phase (horizontal tracks):
luminosity roughly const.
increase radius
decrease effect temperature,
He core mass exceeds q_{sc},
contracts on the KH timescale

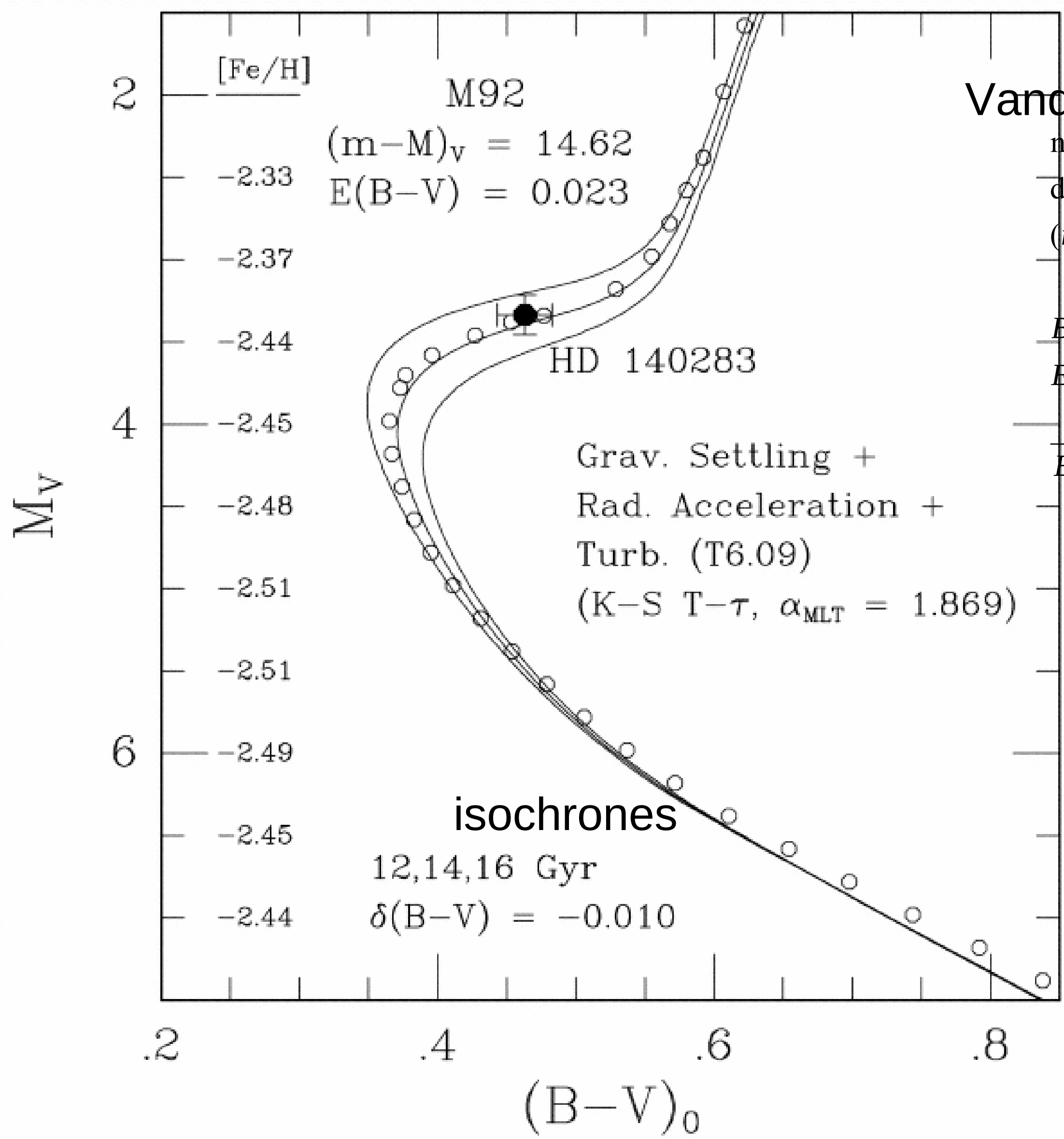
finally encounter the Hayashi line (vertical tracks):
effect temperature almost const., luminosity increases and the red giant expands

Age of a stellar cluster



turn-off point from the main sequence
→ cluster age

age of globular clusters



Vandenberg et al. 2002

need distance candles $\Rightarrow$

distance modulus:

$$(m-M)_V = 5\log d(\mathrm{pc}) - 5 + A_V$$

$$B-V = M_B - M_V$$

$$E(B-V) = (B-V)_{\mathrm{observed}} - (B-V)_{\mathrm{intrinsic}}$$

$$\frac{A_V}{E(B-V)} = 3.2$$

best fit:
age ~ 13.5 Gyrs
almost as old
as the universe

RGB & He flash at the tip

contracting radiative core becomes increasingly dense, eventually making electrons in it more degenerate. (He “white dwarf” inside a star)

→ the H burning shell deposits He “ash” into the core.

→ surface gravity of the core ($GM_c/R_c^2 \propto M_c^{5/3}$) increases with Mc, and therefore raises T and ρ of the H burning shell

→ enhance nuclear-energy generation in the shell, raise *T* of the shell and core

→ larger luminosity enters the base of the convective envelope

→ star expands tremendously and approaches the “tip” of the RGB, some mass is lost from the stellar convective envelope

→ core becomes hot enough ($\sim 10^8$ K) to ignite He fusion into C in a flash (triple alpha reactions)

→ He flash: the extra energy release raises the local temperature, but a slight increase in temperature adds little to the pressures that are already present in the form of the electron degenerate pressure. However, the rate of He burning near T $\sim 10^8$ K scales as T^{40}. Thus, even a modest increase in T creates a vastly enhanced rate of nuclear burning. More nuclear burning means higher temperatures, which means more nuclear burning.

→ The He flash occurs on time scales that are still appreciably longer than the sound-crossing time scale in the inner regions of the star; thus, the helium flash produces only relatively slow expansional motions. The expansion “lifts” the degeneracy of the core. In the H-R diagram, the star now descends to the horizontal branch.

Electron degeneracy of the core

a gas of free electrons begins to become noticeably degenerate when

$$n_e \lambda^3_{\text{de Broglie}} \geq 2, \text{ where } \lambda_{\text{de Broglie}} \equiv \left(\frac{h^2}{2\pi m_e kT} \right)^{1/2}. \leftarrow \text{also obtained from } \varepsilon_F \geq kT$$

$$\text{with } \varepsilon_F = m_e c^2 x_F^2 / 2 \text{ for an N.R. gas and } n = \frac{2}{h^3} \int_0^{\varepsilon_F} f(\varepsilon) 4\pi p^2 dp = \frac{8\pi}{h^3} \int_0^{\varepsilon_F} p^2 dp = \frac{8\pi}{3} \left(\frac{h}{mc} \right)^{-3} x_F^3.$$

The dividing line between non-degenerate and degenerate conditions:

$$\left(n_e T^{-3/2} \right)_{crit} = 2 \left(\frac{h^2}{2\pi m_e k} \right)^{-3/2} = 4.83 \times 10^{15} \ \text{cm}^{-3} \ \text{K}^{-3/2}.$$

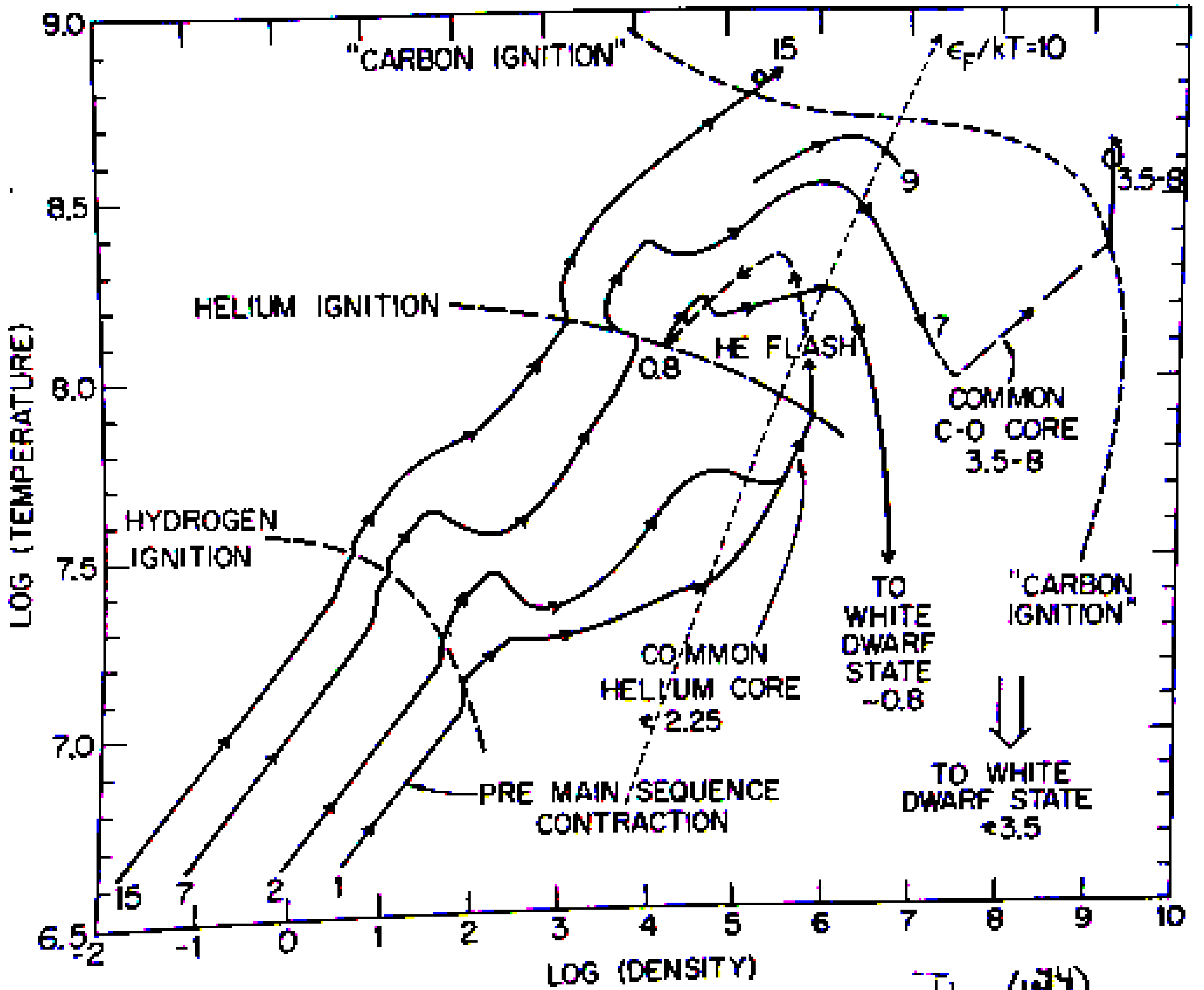
For a pure ^{4}He plama, $n_e = \dfrac{\rho}{2m_u}$. So the contracting He core becomes degenerate when

$$\left(\rho_c \right)_{crit} = 9.4 \times 10^2 \ \text{g cm}^{-3} \left(\frac{T_e}{1.5 \times 10^7 \text{K}} \right)^{3/2},$$

where $T_e = 1.5 \times 10^7$ K is the estimated temperature that H burning in the shell might be expected to hold the core at when the post-main sequence core begins its gravitational contraction. The core density at the end of the main-sequence of the Sun will not be very different from the central density at present ~100 g/cm^3. The core is not far even initially from being partially electron-degenerate conditions. Ascending to the giant branch is in a more-or-less continuous fashion.

In contrast, the central densities of high-mass post-main sequence stars are farther from the critical value for electron degeneracy, which means that the relatively rapid phase of core contraction occupies a larger span of effective temperature.

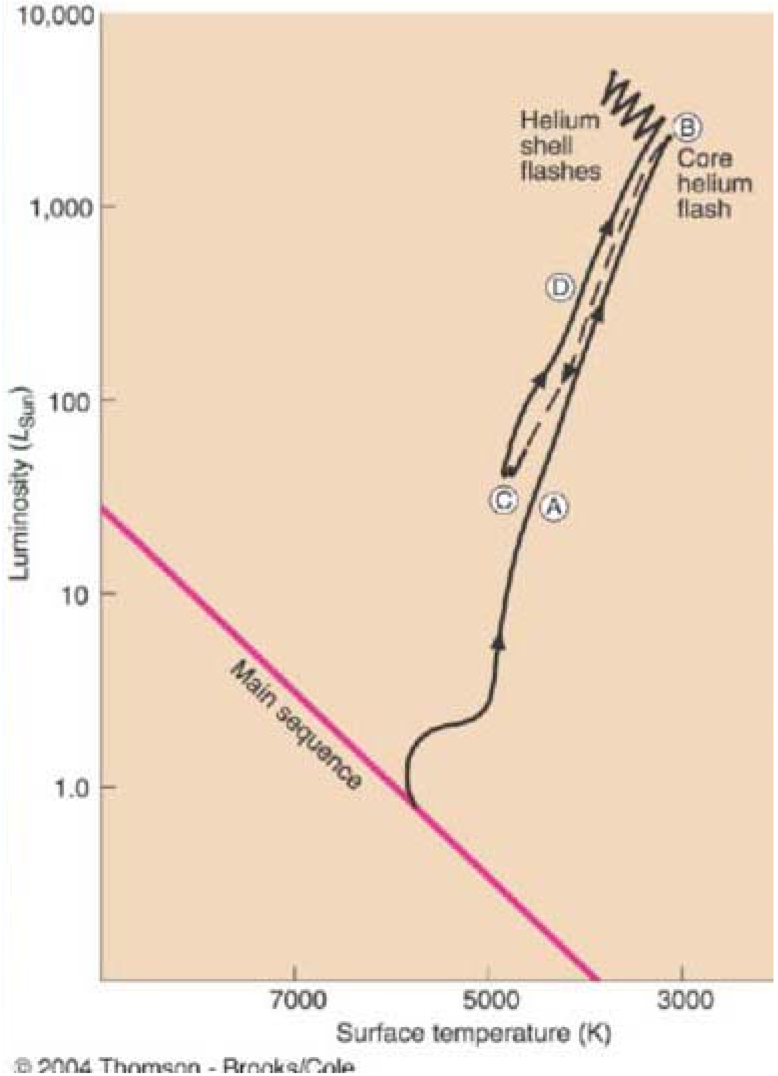
He core flash:
He ignites under the conditions of partial electron degeneracy for solar-mass stars, but not for stars 7-15 solar masses

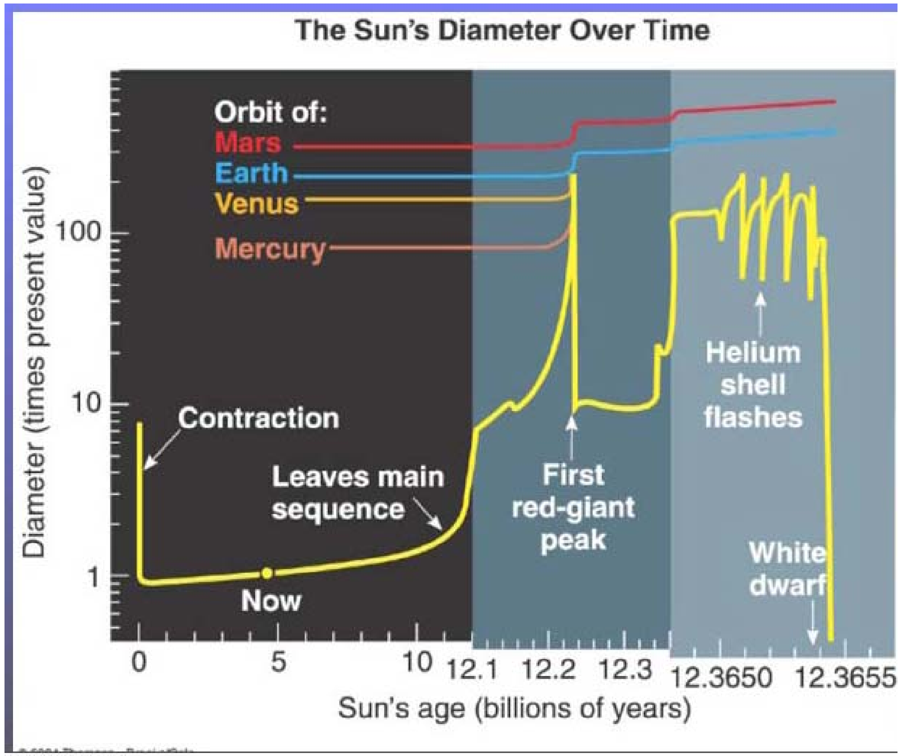


HB and AGB

- HB: He burning in the core, H burning in the shell around the core
- AGB: H and He burning shells above the inert C/O core.
- mass loss during AGB probably due to stellar pulsation and radiation pressure on condensed dust grains

He shell flashes





ay, August 8, 13

thermal instability of a shell burning

$m_{shell} \sim \rho r_{shell}^2 D$ with $dm_{shll} = 0$ & $r_{shell} \sim$ const. $\Rightarrow \dfrac{\dot\rho}{\rho} = -\dfrac{\dot D}{D} = -\dfrac{r}{D}\dfrac{\dot r}{r}$

Besides, $P = \displaystyle\int_{shell} \frac{GM}{4\pi r^4} dm \Rightarrow \dot P = -4\frac{\dot r}{r}\int_{shell}\frac{Gm}{4\pi r^4} dm \Rightarrow \frac{\dot P}{P} = -4\frac{\dot r}{r}$

The above two equations give $\dfrac{\dot\rho}{\rho} = \dfrac{r}{4D}\dfrac{\dot P}{P}$

We also have $\dfrac{\dot\rho}{\rho} = \alpha\dfrac{\dot P}{P} - \delta\dfrac{\dot T}{T}$, where $\alpha = \left(\dfrac{\partial \ln\rho}{\partial \ln P}\right)_T$ and $\delta = -\left(\dfrac{\partial \ln\rho}{\partial \ln T}\right)_P$.

Eliminating $\dfrac{\dot\rho}{\rho}$, we get $\dfrac{\dot P}{P} = \dfrac{\delta}{\alpha - r/4D}\dfrac{\dot T}{T}$

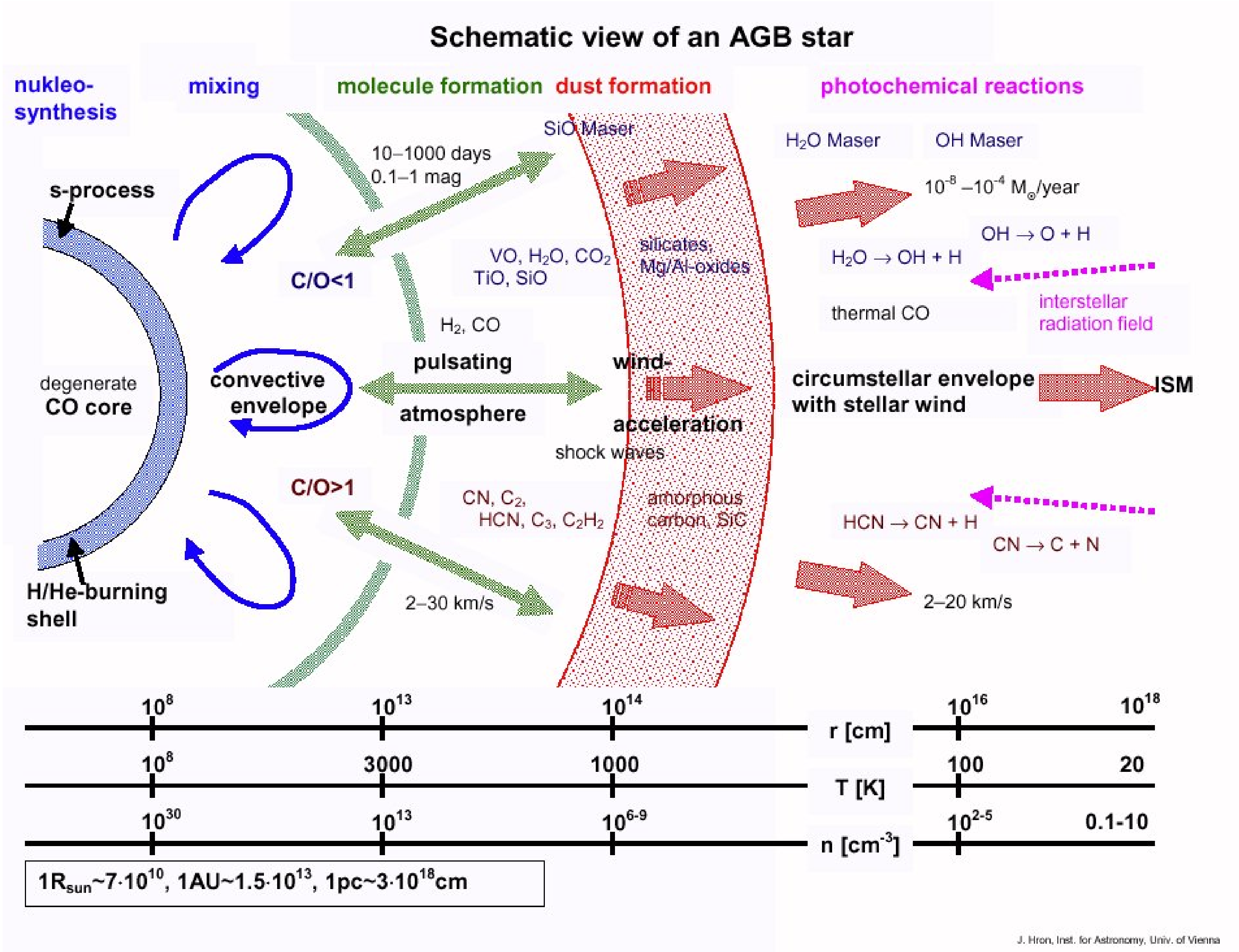
Now, study the 1st law of thermodynamics of the burning shell:

$\dot q = \dot u + P\dot v = c_p \rho \dot T - \dfrac{\delta}{\rho}\dot P = c^* \dot T$, where the gravothermal specific heat $c^* = c_p\left(1 - \nabla_{ad}\dfrac{4\delta}{4\alpha - r/D}\right)$

For an ideal monatomic gas ($\alpha = \delta = 1, \nabla_{ad} = 2/5$),

if D/r is small enough, c^* is positive and the shell burning is unstable.

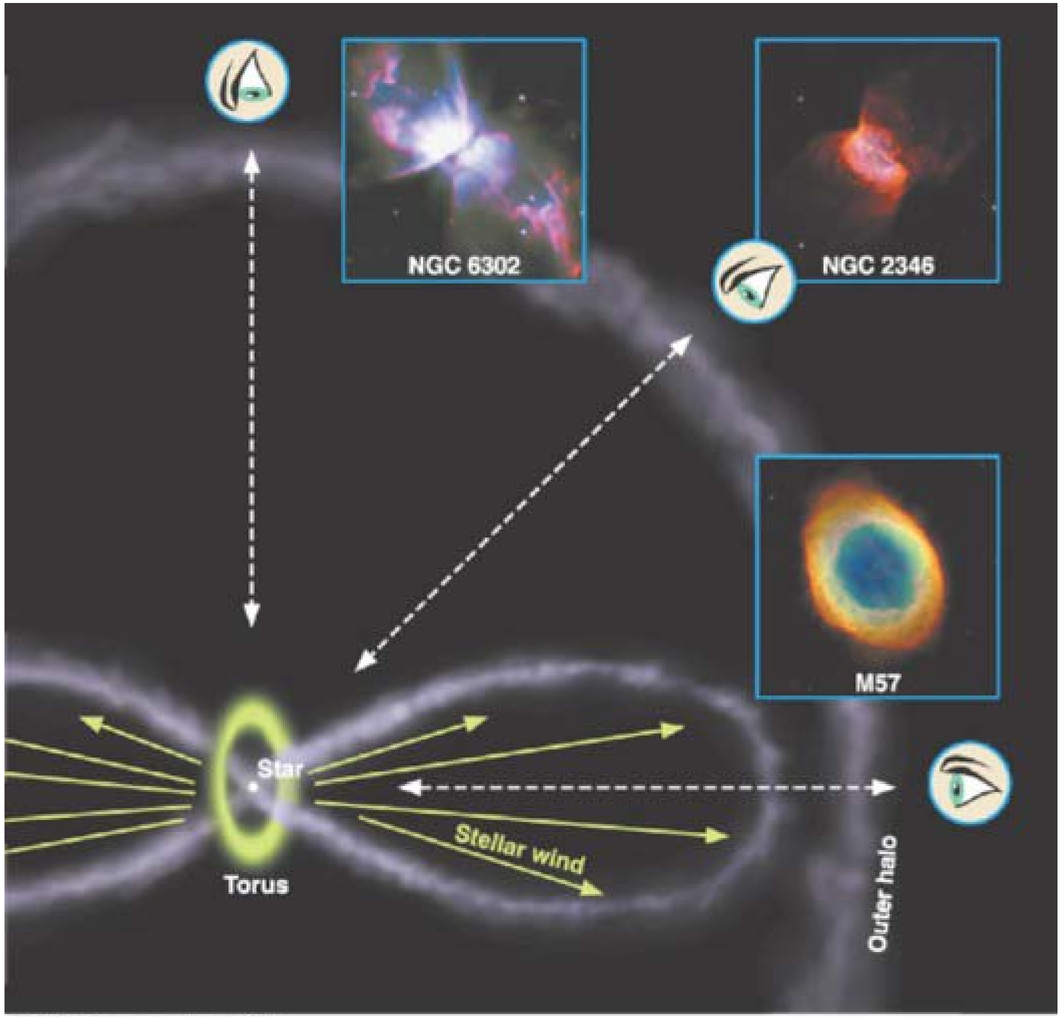
Positive feedback: $\dot q \to T \to \dot q$...... (recall $\dot q$ for 3α reactions is an extremely sensitive funtion of T)



http://www.srl.caltech.edu/personnel/mseibert/galex/protected/mira/about_mira.html

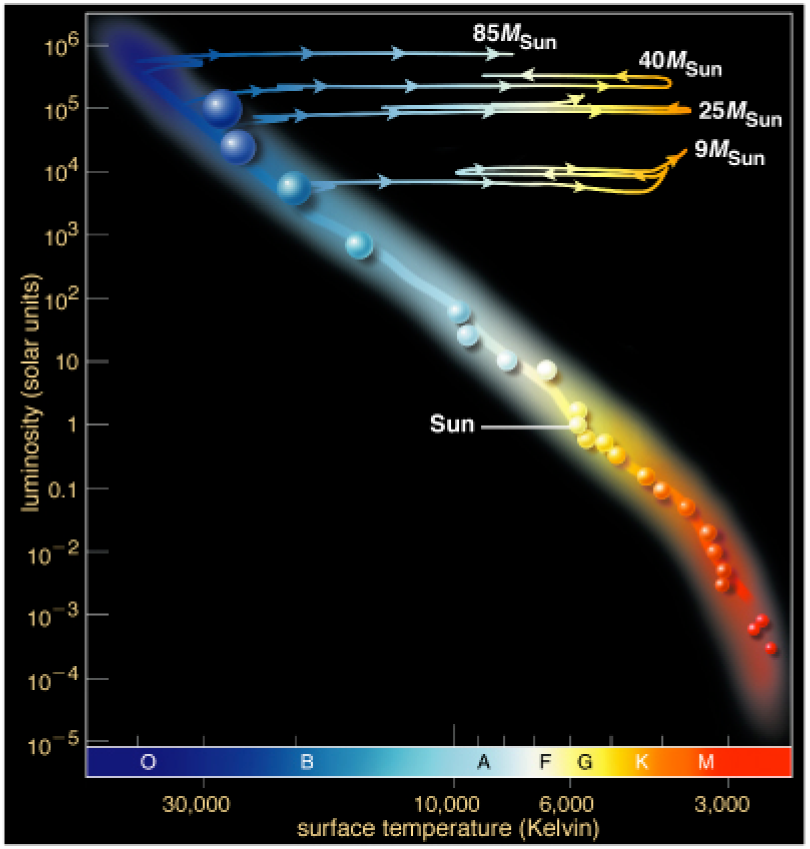
planetary nebulae and viewing angle

planetary nebula: C/O core contracts as the fusion has ceased. The stellar winds and radiation from the core sweep up the remnant ejecta ionizing it and leading to the formation of a planetary nebula



evolution of high-mass stars

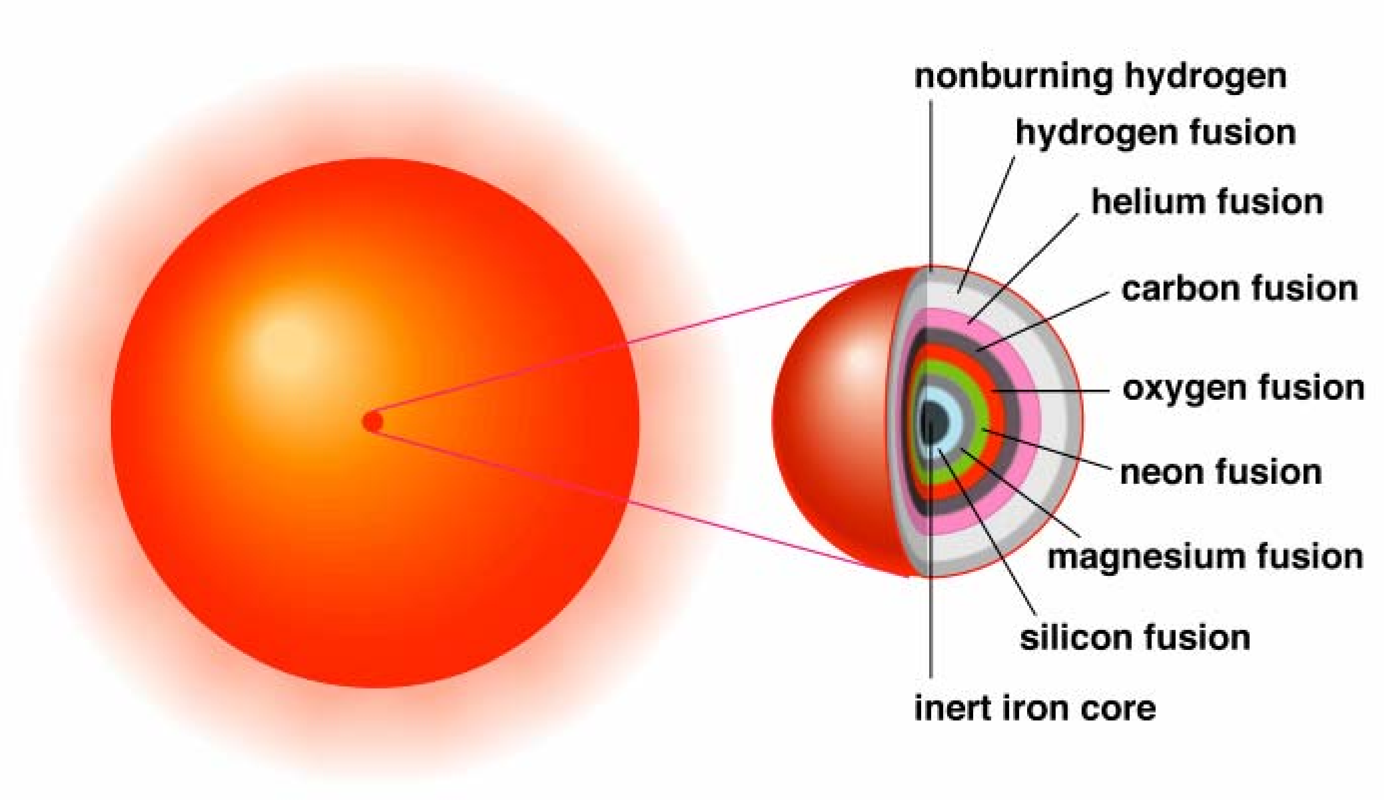
$M_* > 8M_\odot$ will not produce white dwarfs after mass loss



photospheres expand, nuclear core sources turn off and shell sources switch on

photospheres contract, new nuclear core sources turn on and previous shell sources weaken

a pre-supernova of high mass star



the inert iron core that approaches the Chandrasekhar limit as nuclear ash continues to drop in from the burning layers above.

sorry, I couldn’t finish the entire subject

- pulsating stars
- asteroseismology
- stellar atmospheres
- stellar rotation
- evolution of close binary systems
- and many many others!