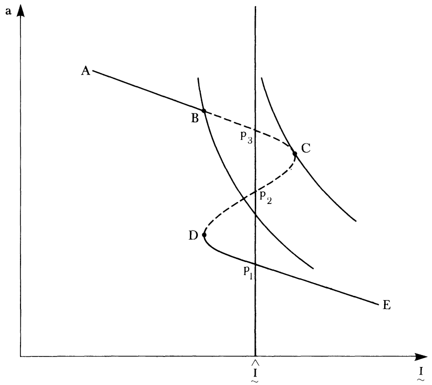
An Analysis of the Principal-Agent Problem (S. Grossman and O. Hart, 1983)

- Themes
  1. An alternative procedure to solve for the principal-agent problem without using the first-order condition approach.
  2. Characterization and properties of solution under this procedure.

Problem of the First-Order Condition Approach

- Unless strong assumptions are imposed, using the first-order condition for e in the IC usually results in the wrong solution.
- Usually the true solution is the "corner" solution.



For a given I the agent strictly prefers lower actions

FIGURE 1.

Model

- One risk-neutral principal, one risk-averse agent.
- n possible output, $y_1 < y_2 < \cdots < y_n$.
- A: Set of available efforts for agent; compact subset of a finite dimensional space. $e \in A$.
- Let $S \equiv \{(\pi_1, \ldots, \pi_n) | \sum_{i=1}^{n} \pi_i = 1, \pi_i \geq 0\}$ be the n-dimensional complex.
- $\pi_i(e)$: probability that y_i is realized, given e.
- Assume π_i is continuous, and $\pi_i(e) > 0$ for all e and i.
- $\pi(e) \equiv \big(\pi_1(e), \pi_2(e), \ldots, \pi_n(e)\big) \in S$.
- Utility of agent: $U(e, y) = u(w) - v(e)$; where w is wage.

First-Best Benchmark: Effort Level Verifiable

- $\underline{U}$: reservation utility of agent.
- When e is observable, wage depends on e.
- For any $e \in A$, let w be such that $u(w) - v(e) = \underline{U}$, i.e., $w = u^{-1}(\underline{U} + v(e))$
- $C_{FB}(e) \equiv h\big(\underline{U} + v(e)\big)$; where $h \equiv u^{-1}$.
- $C_{FB}(e)$ is the lowest cost for the principal to implement e.
- $B(e) \equiv \sum_i \pi_i(e) y_i$: The expected revenue to the principal when agent's effort is e.

First-Best Benchmark: Effort Level Verifiable

- The first-best effort is one that maximizes $B(e) - C_{FB}(e)$.
- Let solutions be e^{FB}.
- The optimal contract for the principal is therefore $(e^{FB}, C_{FB}(e^{FB}))$.

Second-Best: Two-Step Solution

- When e is non-obeservable, the solution procedure can be separated into two steps.
- Step 1: Cost Minimization
  Given any $e \in A$, if the principal wants to implement effort e, he solves

$$\min_{\{w_i\}_{i=1}^n} \sum_{i=1}^n \pi_i(e) w_i$$

$$s.t. \quad \sum_{i=1}^n \pi_i(e) U(e, w_i) \geq \underline{U}, \qquad \text{(IR)}$$

$$\sum_{i=1}^n \pi_i(e) U(e, w_i) \geq \sum_{i=1}^n \pi_i(e') U(e', w_i) \text{ for all } e' \in A, \qquad \text{(IC)}$$

Second-Best Two-Step Solution

- Let $u_i \equiv u(w_i)$.
- Step 1's optimization problem can be rewritten as

$$\min_{\{u_i\}_{i=1}^n} \sum_{i=1}^n \pi_i(e)h(u_i)$$

$$s.t. \sum_{i=1}^n \pi_i(e)u_i - v(e) \geq \sum_{i=1}^n \pi_i(e')u_i - v(e') \text{ for all } e' \in A;$$

$$\sum_{i=1}^n \pi_i(e)u_i - v(e) \geq \underline{U}.$$

- IR is binding: If $U(e, h(u_i(e))) > \underline{U}$, then replace $u_i(e)$ by $u_i(e) - \varepsilon$.
- The solution to step 1, $\big(u_1(e), u_2(e), \ldots, u_n(e)\big) \equiv u(e)$, will be said to implement e.

Second-Best Two-Step Solution

- $u_i(e)$ is agent's wage utility when output is y_i.
- $h(u_i(e))$ is the agent's wage when output is y_i.
- Let $C(e) = \sum_{i=1}^{n} \pi_i(e) h(u_i(e))$: The principal's lowest cost to implement e.
- Step 2: Profit maximization

$$\max_{e \in A} B(e) - C(e).$$

- Let the solution be e^*.

Efficiency Loss in Second-Best

- As in the case using first-order condition approach, there is efficiency loss in the second-best case.
- Let $L \equiv \left(B(e^{FB}) - C_{FB}(e^{FB})\right) - \left(B(e^*) - C(e^*)\right)$.
- Comparison of efficiency can be made according to the principal's profit, because IR is binding.

Proposition

(i) *$C(e) \geq C_{FB}(e)$ for all $e \in A$, implying $L \geq 0$.*

(ii) *If agent is risk-neutral, then $L = 0$.*

(iii) *Let $u'' < 0$ and $C_{FB}(e^{FB}) > \min_{e \in A} C_{FB}(e)$. Then $L > 0$.*

Proposition

Proof:

(i) *is obvious, since first-best must also be second-best.*

(ii) *Let the wage function be such that*
$w(y_i) = y_i - \big(B(e^{FB}) - C_{FB}(e^{FB})\big)$.
The principal's profit is then always $B(e^{FB}) - C_{FB}(e^{FB})$*, and the agent's optimal effort will be* e^{FB}*, which gives him a utility of* $\underline{U}$*.*

(iii)
$$C(e^*) = \sum_{i=1}^n \pi_i(e^*)h(u_i(e^*)) > h\big(\sum_{i=1}^n \pi_i(e^*)u_i(e^*)\big)$$
$$\geq h\big(\underline{U} + v(e^*)\big) = C_{FB}(e^*);$$

where the 1st inequality comes from Jensen's, and 2nd inequality comes from IR constraint. Therefore,
$B(e^{FB}) - C_{FB}(e^{FB}) \geq B(e^*) - C_{FB}(e^*) > B(e^*) - C(e^*)$.

Is Wage Increasing in Output?

- No: Even if we assume *MLRP*, the agent's wage might be lower when output is higher.
- However, wage cannot be decreasing in the whole range of outputs.