Holdup Problem

Kong-Pin Chen
Academia Sinica
National Taiwan University

Introduction

- In general, holdup problem arises because once a contract is signed and specific-relationship investment sunk, party who makes investment will subject to the other party's opportunistic behaviour ex post.
- Later it also describes the phenomenon that a party which makes investment that increases contracting value bears all it's cost, but reaps only a fraction of this value, resulting in underinvestment.

Model

- Two contracting parties, buyer (B) and seller (S).
- Benefit gained depends on B's valuation v and S's cost c.
- $v \in \{v_L, v_H\}$, with $v_H > v_L$ and $p_r(v_H) = j$.
- $c \in \{c_L, c_H\}$, with $c_H > c_L$ and $p_r(c_L) = i$.
- j and i can be seen to be the investment level of B and S, respectively.
- Costs of investment : $\phi(i)$ and $\psi(j)$.

- Buyer's payoff : $vq - p - \psi(j)$;

  where q is quantity traded, and p is price.

- Seller's payoff : $p - cq - \phi(i)$.
- Timing

  (1) Parties contract;

  (2) Simultaneously choose i and j;

  (3) Both learn values of v and c;

  (4) Execute contract.

- Assumption: $c_H > v_H > c_L > v_L$.
- Given assumption, ex post (after c and v realized) efficient trade is $q = 1$ if $v = v_H$ and $c = c_L$; and $q = 0$ otherwise.
- Ex ante, first-best is to solve for

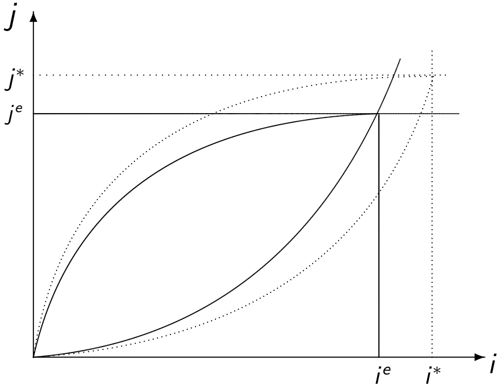
$$\max_{i,j} \; ij(v_H - c_L) - \psi(j) - \phi(i).$$

- FOC:

$$i^*(v_H - c_L) = \psi'(j^*)$$
$$j^*(v_H - c_L) = \phi'(i^*).$$

- When i and j are not contractible, the division of production surplus is subject to ex post negotiation.
- If gains are evenly divided, then i and j are determined by

$$\frac{1}{2}\, i(v_H - c_L) = \psi'(j^e);$$
$$\frac{1}{2}\, j(v_H - c_L) = \phi'(i^e).$$



- $i^e < e^*$ and $j^e < j^*$
- There is underinvestment for both parties.
- How to formulate long-term contract that can mitigate this problem?

- Contract specifies a default option that the parties can request whenever trade is possible.
- Let $\tilde{q}$ be such that

$$\tilde{q}(c_H - c_L) = \phi(i^*).$$

- Consider the following mechanism in which, after θ is realized, the following game is played :
  (1) B makes offer (p, q) to S;
  (2) S accepts (p, q), or rejects it, in which case $\tilde{q}$ is traded at prespecified price $\tilde{p}$, whose value reflects initial bargaining power.

- Since B has total bargaining power, he offers trade ex post only when efficient. Moreover, B's offer makes S indifferent to default option.
- S therefore solves

$$\max_i \tilde{p} - i\tilde{q}c_L - (1-i)\tilde{q}c_H - \phi(i).$$

- Resulting in FOC

$$\tilde{q}(c_H - c_L) = \phi'(i).$$

- By construction of $\tilde{q}$, optimal value of i is exactly i^*, the first-best level.

- B is residual claimer, and solves

$$\max_j \; i^* j(v_H - c_L) - [\, \tilde{p} - i^* \tilde{q} c_L - (1 - i^*)\tilde{q} c_H \,] - \psi(j).$$

- FOC:

$$i^*(v_H - c_L) = \psi'(j), \text{ implying } j = j^*.$$

- Specific performance contract solves the holdup problem.

Option Contracts

- The specific performance contract mentioned above is ex post inefficient: $\tilde{q}$ is default even when $c = c_H$ or $v = v_L$.
- Assume q is either 0 or 1.
- An option contract allows two price levels and renegotiation.
- An option contract consists of two prices to be paid to the seller: p_0 for when the good is not delivered (i.e., $q = 0$), and $p_1 = p_0 + K$ when it is ($q = 1$).
- Let $c_L < K < c_H$.
- Seller has the right to decide whether to deliver good.

- After $\theta = (v, c)$ is realized, buyer and seller renegotiate the contract terms.
- Without renegotiation, there will be ex post inefficiency when $c = c_L$ and $v = v_L$: Seller will deliver good when it should not.
- Buyer can, during renegotiation, raise p_0 to compensate seller for non-delivery.
- Seller's gain of delivery is $p_1 - c_L$.
- The price buyer has to pay, in order not to deliver (the new renegotiated P_0), is $p_1 - c_L$.

- Note that this mechanism is ex post efficient, as S will asks the good to be delivered iff $v > c$.
- Seller's expected payoff (since there is trade only under (v_H, c_L)):

$$\begin{aligned}(1-i)p_0 + i\big[j(p_1 - c_L) + (1-j)(p_1 - c_L)\big] - \phi(i) \\ = (1-i)p_0 + i(p_1 - c_L) - \phi(i) \\ = (1-i)p_0 + i(p_0 + K - c_L) - \phi(i) \\ = p_0 + i(K - c_L) - \phi(i)\end{aligned}$$

- FOC:

$$K - c_L = \phi'(i).$$

- Recall that the first-best investment, i^*, satisfies

$$j^*(v_H - c_L) = \phi'(i^*).$$

- Let $K - c_L = j^*(v_H - c_L)$, implying

$$K = j^* v_H + (1 - j^*) c_L.$$

- In this case seller's investment will equal the first-best level.
- Requiring that K a linear combination of v_H and c_L.
- Must be that $c_L < K < c_H$.

- Buyer's expected payoff:

$$(1-i)(-p_0) + i[j(v_H - p_1) - (1-j)(p_1 - c_L)]$$

- FOC:

$$i(v_H - c_L) = \psi'(j).$$

- Since $i = i^*$, j must also be the first-best level, i.e., $j = j^*$.
- Summary: An option contract can achieve
  (i) first-best investment, and
  (ii) efficient ex post renegotiation.