Optimal Contracting with Subjective Evalutaion (AER 2003)

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- Investigate the form of the optimal contract when it is based on the principal's and the agent's subjective signals.
- The standard model of subjective evaluation assumes that signals to performance are common knowledge (but not verifiable). This paper goes further by considering the case when principal and agent can observe private (thus subjective) signals.

- Results:

  1. Relative to the case of verifiable signals, the optimal contract exhibits more compressed pay.
  2. Agent's compensation does not depend on his own signals.
  3. There is agency cost in using subjective signals.
  4. The agent imposes a lost on the principal when he feels evaluation is unfair.
  5. When signals are only weakly correlated, principal will use more crude form of evaluation (e.g. only two levels of evaluation). When signals are highly correlated, there will be less pooling of evaluation. In particular, if two signals are perfectly correlated, there will be no agency cost for using subjective evaluation.

Model

- One period. One principal and one agent.
- Binary outcome: either output is $B > 0$ (outcome H), or is 0 (outcome L).
- Effort level of agent $\lambda \in [0, 1)$, which is also the probability that B is realized.
- Agent utility:

$$U(c, \lambda) = u(c) - V(\lambda);$$

  where $u' > \varepsilon > 0$, $u'' < 0$, $\lim_{c \to 0^+} u(c) = -\infty$, $V' > 0$, $V'' > 0$, $\lim_{\lambda \to 1^-} V(\lambda) = \infty$.

- B is not observable.
- The principal receives a signal $t \in \{1, \ldots, n\}$.
- The probability the principal receives t, when outcome is H, is r_t^H, and is r_t^L when outcome is L.
- $r^k \equiv [r_1^k, \ldots, r_n^k]$; where $k =$$H$, L.
- Assume higher value of t is better signal regarding output:

$$\frac{r_{t+1}^H}{r_{t+1}^L} > \frac{r_t^H}{r_t^L}$$

for all $t = 1, \ldots, n-1$ (MLRP). For no t is the case that $r_t^H / r_t^L = 1$.

- Given λ, the probability of observing t is thus $r_t(\lambda) = \lambda r_t^H + (1-\lambda) r_t^L$. Let $r(\lambda) = \lambda r^H + (1-\lambda) r^L$.

Benchmark: Principal's Signal Verifiable

- Contract can depend on the value of t.
- Contract: $\{C_t\}_{t=1}^n$; where C_t is the payment for the agent when output is t.
- When $t's$ are verifiable, the form of optimal contract can be derived using the standard Grossman-Hart algorithm:
  Step 1 (cost-minimization):
  Given λ, solve for

$$C^*(\lambda) \equiv \min_{c_1,\ldots,c_n} \sum c_t r_t(\lambda)$$

- subject to

$$\sum_t u(c_t)r_t(\lambda) - V(\lambda) \geq \underline{u},$$

$$\lambda \in \arg\max_{\tilde{\lambda}} \sum u(c_t)r_t(\tilde{\lambda}) - V(\tilde{\lambda});$$

- where $(c_1, \ldots, c_n)$ is the wage paid to agent when the principal observes signal t. $\underline{u}$ is agent's reservation utility. Call the solution $\{c_1^*, \ldots, c_n^*\}$.

- Step 2 (profit maximization):
  The principal choose λ to maximize profit

$$\max_{\lambda \in [0,1)} \lambda B - c^*(\lambda).$$

- Proposition 1:

$$\infty > c^*(\lambda) > 0 \text{ for } \lambda > 0, \text{ and}$$
$$c^*_{t+1} > c^*_t \text{ for all } t = 1, \ldots, n-1.$$

Optimal Contract with Subjective Evaluation

- Suppose that t is now unverifiable, and after the principal observes t, the agent also observes a signal $s \in \{1, \ldots, n\}$.
- The probability that agent receives s, when principal observes t, is p_{ts}.
- Given λ, the probability that ts occurs is $r_t(\lambda)p_{ts}$.

Optimal Contract with Subjective Evaluation

- Note that under this specification, s has no informational value, as t is sufficient statistics of s:

$$Prob(H|t) = \frac{prob(H\&t)}{prob(t)} = \frac{\lambda r_t^H(\lambda)}{\lambda r_t^H(\lambda) + (1-\lambda) r_t^L(\lambda)}.$$

$$prob(H|t,s) = \frac{prob(H\&t,s)}{prob(t,s)} = \frac{\lambda r_t^H(\lambda) p_{ts}}{(\lambda r_t^H(\lambda) + (1-\lambda) r_t^L(\lambda)) p_{ts}}$$

$$= \frac{\lambda r_t^H(\lambda)}{\lambda r_t^H(\lambda) + (1-\lambda) r_t^L(\lambda)} = prob(H|t).$$

The value of s does not provide information beyond t.

- Use revelation principal to characterize the optimal contract.
- The optimal contract is a function of principal's and agent's signals:

$$\Psi = \{c_{ts}, w_{ts}\}_{t,s};$$

where c_{ts} is consumption, and w_{ts} wage, of the agent when signal is ts.

- First Step (cost minimization):
  Given λ, the principal solves

$$C^s(\lambda) = \min_{\Psi} w_{ts} r_t(\lambda) p_{ts},$$

subject to

$$\sum_{t,s} u(c_{ts}) r_t(\lambda) p_{ts} - V(\lambda) \geq \underline{u}, \tag{1}$$

$$\lambda \in \arg\max_{\tilde{\lambda}} \sum_{t,s} u(c_{ts}) r_t(\tilde{\lambda}) p_{ts} - V(\tilde{\lambda}), \tag{2}$$

- $$\sum_s w_{ts} r_t(\lambda) p_{ts} \leq \sum_s w_{t's} r_t(\lambda) p_{ts}(\lambda) \tag{3}$$

  for all t', t,

  $$\sum_t u(c_{ts}) r_t(\lambda) p_{ts} \geq \sum_t u(c_{ts'}) r_t(\lambda) p_{ts} \tag{4}$$

  for all s, s',

  $$w_{ts} \geq c_{ts} \geq 0. \tag{5}$$

- (1) and (2) are the usual IR and IC constraints.
- (3) and (4) are self-revelation constraints for the principal and agent, respectively.
- (5) is the requirement that consumption of agent can't be higher than his wage: It can't be the case $W_{ts} = C_{ts} \, \forall t, s$, as this can only induce the minimum effort.

- The contract is subjective in the sense that neither the principal's nor the agent's signal is verifiable. Moreover, their signals are private so that they can disagree on agent's performance. As a result, in order that they reveal true value of signals, the self-revelation constraints must be satisfied.

- Proposition 2:
  Given any λ, for any a cost-minimizing contract $\Psi^s = \{c^s_{ts}, w^s_{ts}\}_{s,t}$ implementing λ, it must be that $c^s_{ts} = c^s_{ts'}$ for all t, s, s'.
- Proof:
  Suppose $c_{ts} \neq c_{ts'}$ for some s, s', and t:
  let $\hat{c}_t = \sum_s p_{ts} c_{ts}$. Then

$$\sum_t r_t(\lambda) \sum_s p_{ts} u(c_{ts}) < \sum_t r_t(\lambda) u(\sum_s p_{ts} c_{ts})$$
$$= \sum_t r_t(\lambda) u(\hat{c}_t).$$

  The LHS is the agent's expected utility under $\{c_{ts}\}_{t,s}$; while the RHS is that under $\{\hat{c}_t\}_t$.

- Let $w'_{ts} = w_{ts} - c_{ts} + \hat{c}_t$. Then $w'_{ts} - \hat{c}_t = w_{ts} - c_{ts} \geq 0$. Thus (5) holds. Also note that $\sum_s w'_{ts} p_{ts} = \sum_s w_{ts} p_{ts}$, which implies (3) holds. Equation (4) holds since $\hat{c}_t$ is independent of s.
- In summary, if $c_{ts} \neq c_{ts'}$ for some s, s', then we can find a feasible contract $\{\hat{c}_t, w'_{ts}\}$ under which the agent is made better all. Thus $\{c_{ts}, w_{ts}\}$ can't be an optimum.

Discussion

- The agent's information is not used in the optimal contract.
- The result actually depends on the fact that the principal's signal is more informative than the agent's.
- On the one hand, paying w_{ts}'s and w_{ts}'s does not make any difference to the principal; on the other hand, the agent prefers $\hat{c}_t$ to c_{ts} because he is risk-averse. On the third hand, there is no loss of information from c_{ts} to $\hat{c}_t$, as t is sufficient statistics of s.

- Second Step (profit maximization):

$$\max_{\lambda} \lambda B - C^s(\lambda).$$

- Since c_{ts} can't be always same as w_{ts}, certain output has to be dumped, which creates agency cost out of subjective signals.
- The expected deadweight loss from using subjective evaluation is

$$\sum_{t,s}(w^s_{ts} - c^s_t) r_t(\lambda) p_{ts},$$

which is strictly positive iff $C^s(\lambda) > C^*(\lambda)$.

Special case 1: Perfect correction

- Proposition 3:
  If the principle's and agent's signals are perfectly correlated ($p_{ts} = 1$ if $t = s$, and is 0 if not), then the optimal contract is the same as the case with verifiable information.
- Proof:
  Let $\{c_t^*\}_t$ be the optimal contract in Proposition 1. Set $w_{tt}^* = c_t^*$ and $w_{ts}^* = c_t^* + k$ ($k > 0$) if $t \neq s$. Easy to see that the contract $\{c_t^*, w_{ts}^*\}_{t,s}$ satisfies all constraints, and thus achieves the optimality for the case with verifiability.

Special Case 2: No correlation

- In this case $p_{ts} = p_{t's}$ for all t and t'.
- There is only one constraint for (3), therefore $w_{ts} = w_s$.
- However, since c_t does not depend on s.
- Therefore, w_s must be the maximum of c_t: $w_t = \bar{w} = \max_t c_t$.
- Proposition 3: $w_t = w + b$, and

$$C_t = \begin{cases} w + b, \text{ if } t > 1, \\ w, \text{ if } t = 1. \end{cases}$$

Proof:

The optimal contract is a solution to:

$$C^{NC}(\lambda) \equiv \min w,$$

$$\sum_t u(c_t) r_t(\lambda) - V(\lambda) \geq \bar{u}, \tag{6}$$

$$\sum_t u(c_t)(r_t^H - r_t^H) - V'(\lambda) \geq 0, \tag{7}$$

$$w - c_t \geq 0, \forall t. \tag{8}$$

The Lagrangin for the optimization problem is:

$$L = w - \mu_0\{\sum_t u(c_t)r_t(\lambda) - V(\lambda) - \mu_t\}$$
$$- \mu_1\{\sum_t u(c_t)(r_t^H - r_t^L) - V'(\lambda)\} - \sum_t \beta_t r_t(\lambda)(w - c_t)$$

Now consider a type t such that $w > c_t$, then $\beta_t = 0$, which combined with $\frac{\partial L}{\partial c_t} = 0$ implies

$$\mu_0 + \mu_1 \frac{r_t^H - r_t^L}{r_t(\lambda)} = 0. \tag{9}$$

From the MLRC, (9) cen be true for at most one performance level, say t'.

For the t such that $w = c_t$, there is a $\beta_t \geq 0$ satisfying:

$$\frac{\beta_t}{u'(w)} = \mu_0 + \mu_1 \frac{r_t^H - r_t^L}{r_t(\lambda)}.$$

This implies that

$$\mu_0 + \mu_1 \frac{r_t^H - r_t^L}{r_t(\lambda)} \geq 0 = \mu_0 + \mu_1 \frac{r_{t'}^H - r_{t'}^L}{r_{t'}(\lambda)},$$

which by the MLRC can only be satisfied if $t' = 1$, the lowest signal.

Therefore the optimal contract takes the form:

$$c_t = \begin{cases} w + b, & t > 1 \\ w, & t = 1. \end{cases}$$

Using $r_g^k (k = H, L)$ as defined in the statement of the proof of the proposition, (6) implies:

$$u(w+b)r_g(\lambda) + u(w)r_1(\lambda) = \bar{u} + V(\lambda).$$

(7) implies:

$$u(w+b)(r_g^H - r_g^L) + u(w)(r_1^H - r_1^L) = V'(\lambda).$$

■

Imperfect Correlation

- Assumption 3 (Parameterized Beliefs): Suppose P_{ts}^{p} is such that with probability $1 - p$ the A observes a "no information signal," denoted by $s = 0$, while with probability p she observes the signal $s = t$, where t is the signal observed by P.

Imperfect Correlation

		s 0	1	2	$\cdots$	$\cdots$	n
t	1	$1-p$	p	0	$\cdots$	$\cdots$	0
	2	$1-p$	0	p	$\cdots$	$\cdots$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$		$\vdots$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\ddots$	$\vdots$
	n	$1-p$	0	0	$\cdots$	$\cdots$	p

Continuity of the optimal contract

Optimal contract under parameterized beliefs

- $C(\hat{t}|t) = \begin{cases} w_{\hat{t}t} & \text{if } \hat{t} \neq t \neq 0, \\ c^*_{\hat{t}} & \text{otherwise.} \end{cases}$
  and let $w_{\hat{t}t} = c^*_t + \max\{0, (c^*_t - c^*_{\hat{t}})p\}$.
- notice that $c^*_t \leq (1-p)c^*_{\hat{t}} + p \cdot w_{\hat{t}t}$

Levin (2003)

- $\sup w(\varphi) - \inf w(\varphi) \leq \frac{\delta}{1-\delta}(s - \overline{s})$,
- $|w_{ts} - c_t| \leq S \ \forall\ t,\ s \in \mathcal{T}$.

Proposition (7)

Suppose Assumptions 1 and 2 are satisfied, and consider a sequence of beliefs $P_{ts}^{k} \to P_{ts}$*, where either (a)* $P_{ts} > 0$ *for all* $ts \in \mathcal{T} \times \mathcal{T}$ *or (b)* $|w_{ts} - c_t| \leq S$*. Then for* $\lambda \in [0, 1)$*, the optimal cost function converges,* $C^k(\lambda) \to C(\lambda)$*, and the limit points of the optimal contract,* c_t^k*, are optimal for the beliefs* P_{ts}*.*

Proposistion 8: *Suppose Assumption 1, 2 and 3 are satisfied and the amout of lose ex post is constrained by* S*. If* S *is sufficiently large (but finite) then for every* p *there is a type* $t(p)$*, such that*

$$\bar{c} = c_n = c_{n-1} \cdots$$
$$c_{t(p)+1} > c_{t(p)} > c_{t(p)-1} > \cdots > c_1,$$

with the property that for some $\bar{p}$ *sufficently close to zero* $t(p) = 1$*, for* $\bar{p} \geq p \geq 0$*. Moreover, when correlation is perfect the optimal contract is implemented (*$c_t^1 = c_t^*$*).*

Conclusion

- A's report has no effect on her compensation.
- The threat of conflict plays a role in ensuring that the principal has an incentive to threat the agent fairly.
- When there is np correlation between t_s, A is only punish when the worst signal of performance is observed.

Discussion

- Proof of Proposition (2) incomplete.
- Relationship between gap of the two parties' reports and their payoffs.

Some questions on the papers:

- Two possible (non-trivial) extensions:

1. Suppose the principal is also risk-averse. Will this imply $w_{ts} = w_{t's}\ \forall t, t'$?
2. Change the information structure so that the principal's information is not more precise. What happens then?

- Can it be shown that w_{ts} and c_{ts} decrease in $|t - s|$?