Lecture Notes on

Industrial Organization (I)

Chien-Fu CHOU

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Contents

1 Introduction

1.1 Classification of industries and products

中華民國商品標準分類, 中華民國行業標準分類;
台經院產經資料庫.

1.2 A model of industrial organization analysis:

(FS Ch1)

Structuralist:

1. The inclusion of conduct variables is not essential to the development of an operational theory of industrial organization.
2. a priori theory based upon structure-conduct and conduct-performance links yields ambiguous predictions.
3. Even if a priori stucture-conduct-performance hypotheses could be formulated, attempting to test those hypotheses would encounter serious obstacles.

Behaviorist: We can do still better with a richer model that includes intermediate behavioral links.

1.3 Law and Economics

Antitrust law, 公平交易法
Patent and Intellectual Property protection 專利與智慧財產權保護
Cyber law or Internet Law 網路法

1.4 Industrial Organization and International Trade

Basic Conditions

Supply	*Demand*
Raw materials	Price elasticity
Technology	Substitutes
Unionization	Rate of growth
Product durability	Cyclical and seasonal character
Value/weight	Purchase method
Business attitudes	Marketing type
Public polices	

↓

Market Structure

Number of sellers and buyers
Product differentiation
Barriers to entry
Cost structures
Vertical integration
Conglomerateness

↓

Conduct

Pricing behavior
Product strategy and advertising
Research and innovation
Plant investment
Legal tactics

↓

Performance

Production and allocative efficiency
Progress
Full employment
Equity

2 Two Sides of a Market

2.1 Comparative Static Analysis

Assume that there are n endogenous variables and m exogenous variables.
Endogenous variables: $x_1, x_2, \ldots, x_n$
Exogenous variables: $y_1, y_2, \ldots, y_m$.
There should be n equations so that the model can be solved.

$$\begin{aligned} F_1(x_1, x_2, \ldots, x_n; y_1, y_2, \ldots, y_m) &= 0 \\ F_2(x_1, x_2, \ldots, x_n; y_1, y_2, \ldots, y_m) &= 0 \\ &\vdots \\ F_n(x_1, x_2, \ldots, x_n; y_1, y_2, \ldots, y_m) &= 0. \end{aligned}$$

Some of the equations are behavioral, some are equilibrium conditions, and some are definitions.

In principle, given the values of the exogenous variables, we solve to find the endogenous variables as functions of the exogenous variables:

$$\begin{aligned} x_1 &= x_1(y_1, y_2, \ldots, y_m) \\ x_2 &= x_2(y_1, y_2, \ldots, y_m) \\ &\vdots \\ x_n &= x_n(y_1, y_2, \ldots, y_m). \end{aligned}$$

We use comparative statics method to find the differential relationships between x_i and y_j: $\partial x_i/\partial y_j$. Then we check the sign of $\partial x_i/\partial y_j$ to investigate the causality relationship between x_i and y_j.

2.2 Utility Maximization and Demand Function

2.2.1 Single product case

A consumer wants to maximize his/her utility function $U = u(Q) + M = u(Q) + (Y - PQ)$.
FOC: $\frac{\partial U}{\partial Q} = u'(Q) - P = 0$,
$\Rightarrow u'(Q_d) = P$ (inverse demand function)
$\Rightarrow Q_d = D(P)$ (demand function, a behavioral equation)
$\frac{\partial^2 U}{\partial Q \partial P} = U_{PQ} = -1 \Rightarrow \frac{dQ_d}{dP} = D'(P) < 0$, the demand function is a decreasing function of price.

2.2.2 Multi-product case

A consumer wants to maximize his utility function subject to his budget constraint:

$$\max U(x_1, \ldots, x_n) \qquad \text{subj. to } p_1 x_1 + \cdots + p_n x_n = I.$$

Endogenous variables: $x_1, \ldots, x_n$
Exogenous variables: $p_1, \ldots, p_n, I$ (the consumer is a price taker)
Solution is the demand functions $x_k = D_k(p_1, \ldots, p_n, I)$, $k = 1, \ldots, n$

Example: $\max U(x_1, x_2) = a \ln x_1 + b \ln x_2$ subject to $p_1 x_1 + p_2 x_2 = m$.
$\mathcal{L} = a \ln x_1 + b \ln x_2 + \lambda(m - p_1 x_1 - p_2 x_2)$.
FOC: $\mathcal{L}_1 = \frac{a}{x_1} - \lambda p_1 = 0$, $\mathcal{L}_2 = \frac{b}{x_2} - \lambda p_2 = 0$ and $\mathcal{L}_\lambda = m - p_1 x_1 - p_2 x_2 = 0$.
$\Rightarrow \frac{a}{b}\frac{x_2}{x_1} = \frac{p_1}{p_2} \Rightarrow x_1 = \frac{am}{(a+b)p_1}$, $x_2 = \frac{bm}{(a+b)p_2}$

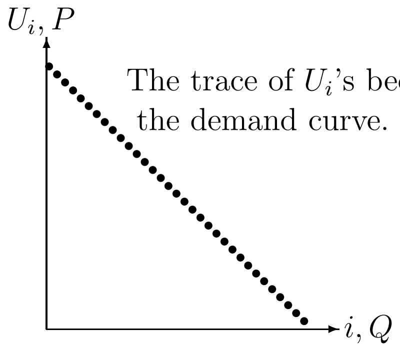
SOC: $\begin{vmatrix} 0 & -p_1 & -p_2 \\ -p_1 & \frac{-a}{x_1^2} & 0 \\ -p_2 & 0 & \frac{-b}{x_2^2} \end{vmatrix} = \frac{ap_2^2}{x_1^2} + \frac{bp_1^2}{x_2^2} > 0.$

$\Rightarrow x_1 = \frac{am}{(a+b)p_1}$, $x_2 = \frac{bm}{(a+b)p_2}$ is a local maximum.

2.3 Indivisibility, Reservation Price, and Demand Function

In many applications the product is indivisible and every consumer needs at most one unit.
Reservation price: the value of one unit to a consumer.
If we rank consumers according to their reservation prices, we can derive the market demand function.

Example: $U_i = 31 - i$, $i = 1, 2, \cdots, 30$.



2.4 Demand Function and Consumer surplus

Demand Function: $Q = D(p)$. Inverse demand function: $p = P(Q)$.
Demand elasticity: $\eta_D \equiv \frac{p}{Q}\frac{dQ}{dp} = \frac{pD'(p)}{D(p)} = \frac{P(Q)}{QP(Q)}$.
Total Revenue: $TR(Q) = QP(Q) = pD(p)$.
Average Revenue: $AR(Q) = \frac{TR(Q)}{Q} = P(Q) = \frac{pD(p)}{D(p)}$.
Marginal Revenue: $MR(Q) = \frac{dTR(Q)}{dQ}$ or

$$MR(Q) = P(Q) + QP'(Q) = P(Q)\left[1 + \frac{P'(Q)Q}{P(Q)}\right] = P(Q)\left[1 + \frac{1}{\eta}\right].$$

Consumer surplus: $CS(p) \equiv \int_p^\infty D(p)dp$.

2.4.1 Linear demand function: $Q = D(p) = \frac{A}{b} - \frac{1}{b}p$ or $P(Q) = A - bQ$

$TR = AQ - bQ^2$, $AR = A - bQ$, $MR = A - 2bQ$, $\eta = 1 - \frac{a}{bQ}$,
$CS(p) = \int_p^A D(p)dp = \frac{(A-p)p}{2b}$.

2.4.2 Const. elast. demand function: $Q = D(p) = ap^\eta$ or $P(Q) = AQ^{1/\eta}$

$TR = AQ^{1+\frac{1}{\eta}}$, $AR = AQ^{1/\eta}$,and $MR = \frac{1+\eta}{\eta}AQ^{1/\eta}$.

2.4.3 Quasi-linear utility function: $U(Q) = f(Q) + m \Rightarrow P(Q) = f'(Q)$

$TR = Qf'(Q)$, $AR = f'(Q)$, $MR = f'(Q) + Qf''(Q)$,
$CS(p) = f(Q) - pQ = f(Q) - Qf'(Q)$.

2.5 Profit maximization and supply function

2.5.1 From cost function to supply function

Consider first the profit maximization problem of a competitive producer:

$$\max_{Q} \Pi = PQ - C(Q), \quad \text{FOC } \Rightarrow \frac{\partial \Pi}{\partial Q} = P - C'(Q) = 0.$$

The FOC is the inverse supply function (a behavioral equation) of the producer: $P = C'(Q) = \text{MC}$. Remember that Q is endogenous and P is exogenous here. To find the comparative statics $\frac{dQ}{dP}$, we use the total differential method discussed in the last chapter:

$$dP = C''(Q)dQ, \ \Rightarrow \frac{dQ}{dP} = \frac{1}{C''(Q)}.$$

To determine the sign of $\frac{dQ}{dP}$, we need the SOC, which is $\frac{\partial^2 \Pi}{\partial Q^2} = -C''(Q) < 0$. Therefore, $\frac{dQ_s}{dP} > 0$.

2.5.2 From production function to cost function

A producer's production technology can be represented by a production function $q = f(x_1, \ldots, x_n)$. Given the prices, the producer maximizes his profits:

$$\max \Pi(x_1, \ldots, x_n; p, p_1, \ldots, p_n) = pf(x_1, \ldots, x_n) - p_1 x_1 - \cdots - p_n x_n$$

Exogenous variables: $p, p_1, \ldots, p_n$ (the producer is a price taker)
Solution is the supply function $q = S(p, p_1, \ldots, p_n)$ and the input demand functions, $x_k = X_k(p, p_1, \ldots, p_n)$ $k = 1, \ldots, n$

Example: $q = f(x_1, x_2) = 2\sqrt{x_1} + 2\sqrt{x_2}$ and $\Pi(x_1, x_2; p, p_1, p_2) = p(2\sqrt{x_1} + 2\sqrt{x_2}) - p_1 x_1 - p_2 x_2$,

$$\max_{x_1 . x_2} p(2\sqrt{x_1} + 2\sqrt{x_2}) - p_1 x_1 - p_2 x_2$$

FOC: $\frac{\partial \Pi}{\partial x_1} = \frac{p}{\sqrt{x_1}} - p_1 = 0$ and $\frac{\partial \Pi}{\partial x_2} = \frac{p}{\sqrt{x_2}} - p_2 = 0$.
$\Rightarrow x_1 = (p/p_1)^2, \ x_2 = (p/p_2)^2$ (input demand functions) and
$q = 2(p/p_1) + 2(p/p_2) = 2p(\frac{1}{p_1} + \frac{1}{p_2})$ (the supply function)
$\Pi = p^2(\frac{1}{p_1} + \frac{1}{p_2})$
SOC:

$$\begin{bmatrix} \frac{\partial^2 \Pi}{\partial x_1^2} & \frac{\partial^2 \Pi}{\partial x_1 \partial x_2} \\ \frac{\partial^2 \Pi}{\partial x_1 \partial x_2} & \frac{\partial^2 \Pi}{\partial x_1^2} \end{bmatrix} = \begin{bmatrix} \frac{-p}{2x_1^{-3/2}} & 0 \\ 0 & \frac{-p}{2x_2^{-3/2}} \end{bmatrix}$$

is negative definite.

2.5.3 Joint products, transformation function, and profit maximization

In more general cases, the technology of a producer is represented by a transformation function: $F^j(y_1^j, \dots, y_n^j) = 0$, where $(y_1^j, \dots, y_n^j)$ is called a production plan, if $y_k^j > 0$ (y_k^j) then k is an output (input) of j.

Example: a producer produces two outputs, y_1 and y_2, using one input y_3. Its technology is given by the transformation function $(y_1)^2 + (y_2)^2 + y_3 = 0$. Its profit is $\Pi = p_1y_1 + p_2y_2 + p_3y_3$. The maximization problem is

$$\max_{y_1,y_2,y_3} p_1y_1 + p_2y_2 + p_3y_3 \qquad \text{subject to } (y_1)^2 + (y_2)^2 + y_3 = 0.$$

To solve the maximization problem, we can eliminate y_3: $x = -y_3 = (y_1)^2 + (y_2)^2 > 0$ and

$$\max_{y_1,y_2} p_1y_1 + p_2y_2 - p_3[(y_1)^2 + (y_2)^2].$$

The solution is: $y_1 = p_1/(2p_3)$, $y_2 = p_2/(2p_3)$ (the supply functions of y_1 and y_2), and $x = -y_3 = [p_1/(2p_3)]^2 + [p_1/(2p_3)]^2$ (the input demand function for y_3).

2.6 Production function and returns to scale

Production function: $Q = f(L, K)$. $\qquad MP_K = \dfrac{\partial Q}{\partial L} \qquad MP_K = \dfrac{\partial Q}{\partial K}$

IRTS: $f(hL, hK) > hf(L, K)$. $\qquad$ CRTS: $f(hL, hK) = hf(L, K)$.

DRTS: $f(hL, hK) < hf(L, K)$.

Supporting factors: $\dfrac{\partial^2 Q}{\partial L \partial K} > 0$. $\qquad$ Substituting factors: $\dfrac{\partial^2 Q}{\partial L \partial K} < 0$.

Example 1: Cobb-Douglas case $F(L, K) = AL^aK^b$.

Example 2: CES case $F(L, K) = A[aL^\rho + (1-a)K^\rho]^{1/\rho}$.

2.7 Cost function: $C(Q)$

Total cost $TC = C(Q)$ $\qquad$ Average cost $AC = \dfrac{C(Q)}{Q}$ $\qquad$ Marginal cost $MC = C'(Q)$.

Example 1: $C(Q) = F + cQ$

Example 2: $C(Q) = F + cQ + bQ^2$

Example 3: $C(Q) = cQ^a$.

3 Competitive Market

Industry (Market) structure:
Short Run: Number of firms, distribution of market shares, competition decision variables, reactions to other firms.
Long Run: R&D, entry and exit barriers.

Competition: In the SR, firms and consumers are price takers.
In the LR, there is no barriers to entry and exit $\Rightarrow$ 0-profit.

3.1 SR market equilibrium

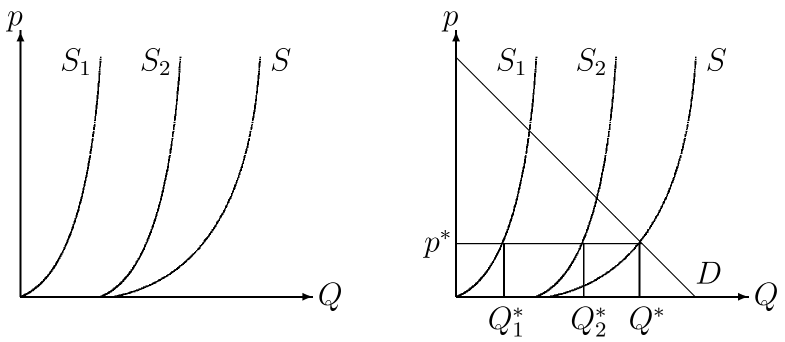
3.1.1 An individual firm's supply function

A producer i in a competitive market is a price taker. It chooses its quantity to maximize its profit:

$$\max_{Q_i} pQ_i - C_i(Q_i) \quad \Rightarrow p = C_i'(Q_i) \quad \Rightarrow Q_i = S_i(p).$$

3.1.2 Market supply function

Market supply is the sum of individual supply function $S(p) = \sum_i S_i(p)$.
On the Q-p diagram, it is the horizontal sum of individual supply curves.



3.2 Market equilibrium

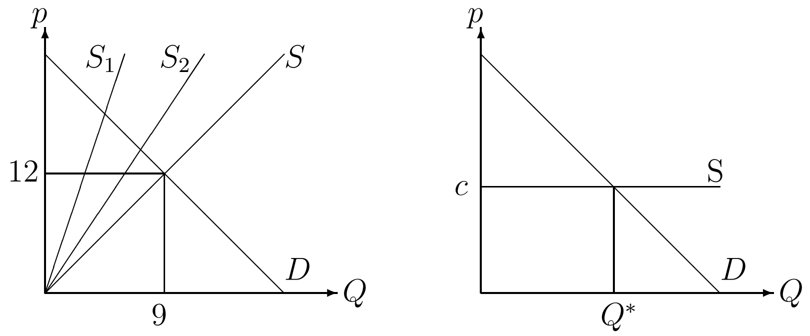
Market equilibrium is determined by the intersection of the supply and demand as in the diagram.

Formally, suppose there are n firms. A state of the market is a vector $(p, Q_1, Q_2, \ldots, Q_n)$.
An equilibrium is a state $(p^*, Q_1^*, Q_2^*, \ldots, Q_n^*)$ such that:
1. $D(p^*) = S(p^*)$.
2. Each Q_i^* maximizes $\Pi_i(Q_i) = p^*Q_i - C_i(Q_i)$, $i = 1, \ldots, n$.
3. $\Pi_i(Q_i^*) = p^*Q_i^* - c_i(Q_i^*) \geq 0$.

3.2.1 Example 1: $C_1(Q_1) = Q_1^2$, $C_2(Q_2) = 2Q_2^2$, $D = 12 - \frac{p}{4}$

$$p = C_1'(Q_1) = 2Q_1,\ p = C_2'(Q_2) = 4Q_2, \quad \Rightarrow S_1 = \frac{p}{2},\ S_2 = \frac{p}{4}, \quad S(P) = S_1 + S_2 = \frac{3p}{4}.$$

$$D(p^*) = S(p^*) \Rightarrow 12 - \frac{p^*}{4} = \frac{3p^*}{4} \Rightarrow p^* = 12,\ Q^* = S(p^*) = 9,\ Q_1^* = S_1(p^*) = 6,\ Q_2^* = S_2(p^*) = 3.$$



3.2.2 Example 2: $C(Q) = cQ$ (CRTS) and $D(p) = \max\{A - bp, 0\}$

If production technology is CRTS, then the equilibrium market price is determined by the AC and the equilibrium quantity is determined by the market demand.

$$p^* = c, \quad Q^* = D(p^*) = \max\{A - bp^*, 0\}.$$

If $\frac{A}{b} \leq c$ then $Q^* = 0$. If $\frac{A}{b} > c$ then $Q^* = A - bc > 0$.

3.2.3 Example 3: 2 firms, $C_1(Q_1) = c_1Q_1$, $C_2(Q_2) = c_2Q_2$, $c_1 < c_2$

$$p^* = c_1, \quad Q^* = Q_1^* = D(p^*) = D(c_1), \quad Q_2^* = 0.$$

3.2.4 Example 4: $C(Q) = F + cQ$ or $C''(Q) > 0$ (IRTS), no equilibrium

If $C''(Q) > 0$, then the profit maximization problem has no solution.
If $C(Q) = F + cQ$, then $p^* = c$ cannot be and equilibrium because
$\Pi(Q) = cQ - (F + cQ) = -F < 0$.

3.3 General competitive equilibrium

Commodity space: Assume that there are n commodities. The commodity space is $R^n_+ = \{(x_1, \ldots, x_n); x_k \geq 0\}$

Economy: There are I consumers, J producers, with initial endowments of commodities $\omega = (\omega_1, \ldots, \omega_n)$.
Consumer i has a utility function $U^i(x^i_1, \ldots, x^i_n)$, $i = 1, \ldots, I$.
Producer j has a production transformation function $F^j(y^j_1, \ldots, y^j_n) = 0$,

A price system: $(p_1, \ldots, p_n)$.

A private ownership economy: Endowments and firms (producers) are owned by consumers.
Consumer i's endowment is $\omega^i = (\omega^i_1, \ldots, \omega^i_n)$, $\sum_{i=1}^I \omega^i = \omega$.
Consumer i's share of firm j is $\theta^{ij} \geq 0$, $\sum_{i=1}^I \theta^{ij} = 1$.

An allocation: $x^i = (x^i_1, \ldots, x^i_n)$, $i = 1, \ldots, I$, and $y^j = (y^j_1, \ldots, y^j_n)$, $j = 1, \ldots, J$.

A competitive equilibrium:
A combination of a price system $\bar{p} = (\bar{p}_1, \ldots, \bar{p}_n)$ and an allocation $(\{\bar{x}^i\}_{i=1,\ldots,I}, \{\bar{y}^j\}_{j=1,\ldots,J})$ such that
1. $\sum_i \bar{x}^i = \omega + \sum_j \bar{y}^j$ (feasibility condition).
2. $\bar{y}^j$ maximizes Π^j, $j = 1, \ldots, J$ and $\bar{x}^i$ maximizes U^i, subject to i's budget constraint $p_1 x^i_1 + \ldots + p_n x^i_n = p_1 \omega^1_1 + \ldots + p_n \omega^i_n + \theta_{i1}\Pi^1 + \ldots + \theta_{iJ}\Pi^J$.

Existence Theorem:
Suppose that the utility functions are all quasi-concave and the production transformation functions satisfy some theoretic conditions, then a competitive equilibrium exists.

Welfare Theorems: A competitive equilibrium is efficient and an efficient allocation can be achieved as a competitive equilibrium through certain income transfers.

Constant returns to scale economies and non-substitution theorem:
Suppose there is only one nonproduced input, this input is indispensable to production, there is no joint production, and the production functions exhibits constant returns to scale. Then the competitive equilibrium price system is determined by the production side only.

4 Monopoly

A monopoly industry consists of one single producer who is a price setter (aware of its monopoly power to control market price).

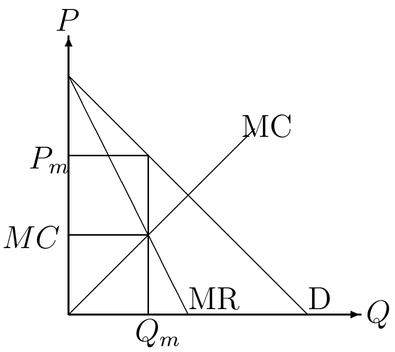
4.1 Monopoly profit maximization

Let the market demand of a monopoly be $Q = D(P)$ with inverse function $P = f(Q)$.
Its total cost is $\mathrm{TC} = C(Q)$. The profit maximization problem is

$$\max_{Q\geq 0} \pi(Q) = PQ - TC = f(Q)Q - C(Q) \Rightarrow f'(Q)Q + f(Q) = \mathrm{MR}(Q) = \mathrm{MC}(Q) = C'(Q) \Rightarrow Q_M.$$

The SOC is $\dfrac{d^2\pi}{dQ^2} = \mathrm{MR}'(Q) - \mathrm{MC}'(Q) < 0$.
Long-run existence condition: $\pi(Q_m) \geq 0$.

Example: $\mathrm{TC}(Q) = F + cQ^2$, $f(Q) = a - bQ$, $\Rightarrow$ $\mathrm{MC} = 2cQ$, $\mathrm{MR} = a - 2bQ$.
$\Rightarrow Q_m = \dfrac{a}{2(b+c)}$, $P_m = \dfrac{a(b+2c)}{2(b+c)}$, $\Rightarrow \pi(Q_m) = \dfrac{a^2}{4(b+c)} - F$.
When $\dfrac{a^2}{4(b+c)} < F$, the true solution is $Q_m = 0$ and the market does not exist.



4.1.1 Lerner index

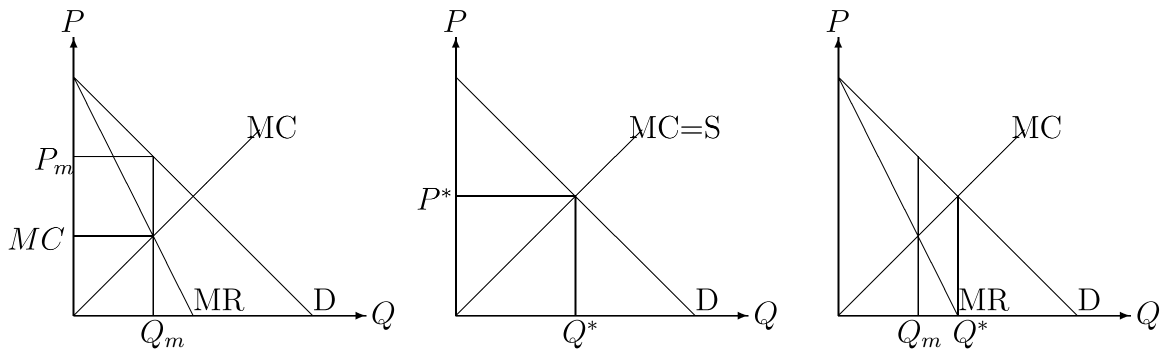
The maximization can be solved using P as independent variable:

$$\max_{P\geq 0} \pi(P) = PQ - TC = PD(P) - C(D(P))$$

$$\Rightarrow D(P) + PD'(P) = C'(D(P))D'(P) \Rightarrow \frac{P_m - C'}{P_m} = -\frac{D(P)}{D'(P)P} = \frac{1}{|\epsilon|}.$$

Lerner index: $\dfrac{P_m - C'}{P_m}$. It can be calculated from real data for a firm (not necessarily monopoly) or an industry. It measures the profit per dollar sale of a firm (or an industry).

4.1.2 Monopoly and social welfare



4.1.3 Rent seeking (競租) activities

R&D, Bribes, Persuasive advertising, Excess capacity to discourage entry, Lobby expense, Over doing R&D, etc are means taken by firms to secure and/or maintain their monopoly profits. They are called rent seeking activities because monopoly profit is similar to land rent. They are in many cases regarded as wastes because they don't contribute to improving productivities.

4.2 Monopoly price discrimination

Indiscriminate Pricing: The same price is charged for every unit of a product sold to any consumer.
Third degree price discrimination: Different prices are set for different consumers, but the same price is charged for every unit sold to the same consumer (linear pricing).
Second degree price discrimination: Different price is charged for different units sold to the same consumer (nonlinear pricing). But the same price schedule is set for different consumers.
First degree price discrimination: Different price is charged for different units sold to the same consumer (nonlinear pricing). In addition, different price schedules are set for different consumers.

4.2.1 Third degree price discrimination

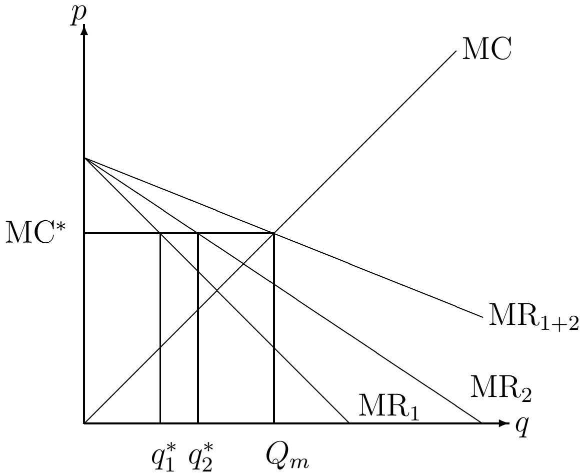
Assume that a monopoly sells its product in two separable markets.
Cost function: $C(Q) = C(q_1 + q_2)$
Inverse market demands: $p_1 = f_1(q_1)$ and $p_2 = f_2(q_2)$
Profit function: $\Pi(q_1, q_2) = p_1q_1 + p_2q_2 - C(q_1 + q_2) = q_1f_1(q_1) + q_2f_2(q_2) - C(q_1 + q_2)$
FOC: $\Pi_1 = f_1(q_1) + q_1f_1'(q_1) - C'(q_1 + q_2) = 0$, $\Pi_2 = f_2(q_2) + q_2f_2'(q_2) - C'(q_1 + q_2) = 0$; or $\mathrm{MR}_1 = \mathrm{MR}_2 = \mathrm{MC}$.
SOC: $\Pi_{11} = 2f_1' + q_1f_1'' - C'' < 0$, $\begin{vmatrix} 2f_1' + q_1f_1'' - C'' & -C'' \\ -C'' & 2f_2' + q_2f_2'' - C'' \end{vmatrix} \equiv \Delta > 0$.

Example: $f_1 = a - bq_1$, $f_2 = \alpha - \beta q_2$, and $C(Q) = 0.5Q^2 = 0.5(q_1 + q_2)^2$.
$f_1' = -b$, $f_2' = -\beta$, $f_1'' = f_2'' = 0$, $C' = Q = q_1 + q_2$, and $C'' = 1$.

FOC: $a - 2bq_1 = q_1 + q_2 = \alpha - 2\beta q_2 \Rightarrow \begin{pmatrix} 1+2b & 1 \\ 1 & 1+2\beta \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} a \\ \alpha \end{pmatrix}$

$\Rightarrow \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \frac{1}{(1+2b)(1+2\beta)-1}\begin{pmatrix} a(1+2\beta)-\alpha \\ \alpha(1+2b)-a \end{pmatrix}.$

SOC: $-2b - 1 < 0$ and $\Delta = (1+2b)(1+2\beta) - 1 > 0$.



$\text{MC} = \text{MR}_{1+2} \Rightarrow Q_m,\ \text{MC}^*$

$\text{MC}^* = \text{MR}_1 \Rightarrow q_1^*$

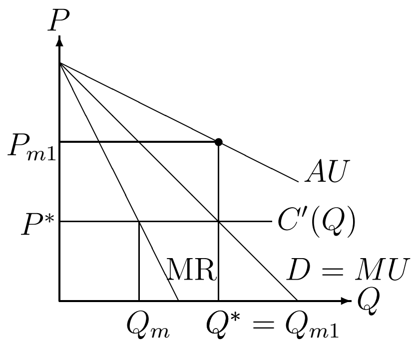
$\text{MC}^* = \text{MR}_2 \Rightarrow q_2^*$

4.2.2 First Degree

Each consumer is charged according to his total utility, i.e., $TR = PQ = U(Q)$. The total profit to the monopoly is $\Pi(Q) = U(Q) - C(Q)$. The FOC is $U'(Q) = C'(Q)$, i.e., the monopoly regards a consumer's MU ($U'(Q)$) curve as its MR curve and maximizes its profit.

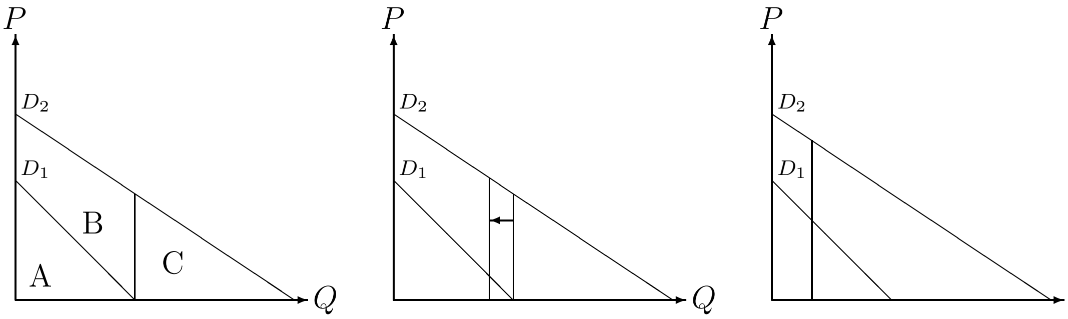
$$\max_Q \Pi = U(Q) - C(Q) \Rightarrow U'(Q) = C'(Q).$$

The profit maximizing quantity is the same as the competition case, $Q_{m1} = Q^*$. However, the price is much higher, $P_{m1} = \frac{U(Q^*)}{Q^*} = AU > P^* = U'(Q^*)$. There is no inefficiency. But there is social justice problem.



4.2.3 Second degree discrimination

See Varian Ch14 or Ch25.3 (under).



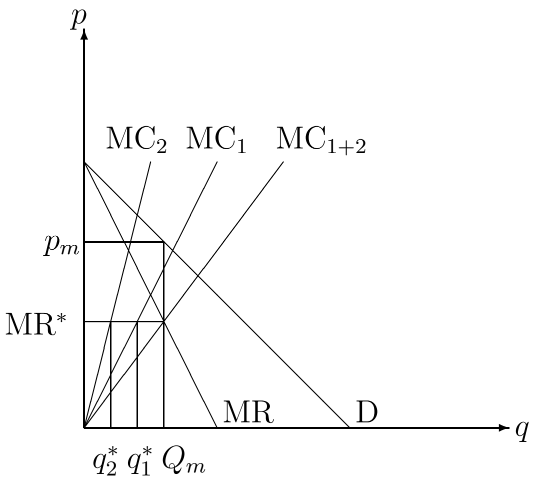
By self selection principle, $P_1Q_1 = A$, $P_2Q_2 = A + C$, $\Pi = 2A + C$ is maximized when Q_1 is such that the hight of D_2 is twice that of D_1.

4.3 Multiplant Monopoly and Cartel

Now consider the case that a monopoly has two plants.
Cost functions: $\mathrm{TC}_1 = C_1(q_1)$ and $\mathrm{TC}_2 = C_2(q_2)$
Inverse market demand: $P = D(Q) = D(q_1 + q_2)$
Profit function: $\Pi(q_1, q_2) = P(q_1 + q_2) - C_1(q_1) - C_2(q_2) = D(q_1 + q_2)(q_1 + q_2) - C_1(q_1) - C_2(q_2)$
FOC: $\Pi_1 = D'(Q)Q + D(Q) - C_1'(q_1) = 0$, $\Pi_2 = D'(Q)Q + D(Q) - C_2'(q_2) = 0$; or$\mathrm{MR} = \mathrm{MC}_1 = \mathrm{MC}_2$.
SOC: $\Pi_{11} = 2D'(Q) + D''(Q)Q - C_1'' < 0$,

$$\begin{vmatrix} 2D'(Q) + D''(Q)Q - C_1'' & 2D'(Q) + D''(Q)Q \\ 2D'(Q) + D''(Q)Q & 2D'(Q) + D''(Q)Q - C_1'' \end{vmatrix} \equiv \Delta > 0.$$

Example: $D(Q) = A - Q$, $C_1(q_1) = q_1^2$, and $C_2(q_2) = 2q_2^2$.
FOC: $MR = A - 2(q_1 + q_2) = MC_1 = 2q_1 = MC_2 = 4q_2$. $\begin{pmatrix} 4 & 2 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} A \\ A \end{pmatrix}$
$\Rightarrow q_1 = 0.2A,\ q_2 = 0.1A,\ P_m = 0.7A$



$\mathrm{MR} = \mathrm{MC}_{1+2} \Rightarrow Q_m$, MR^*

$\mathrm{MR}^* = \mathrm{MC}_1 \Rightarrow q_1^*$

$\mathrm{MR}^* = \mathrm{MC}_2 \Rightarrow q_2^*$

4.4 Multiproduct monopoly

Consider a producer who is monopoly (the only seller) in two joint products.

$$Q_1 = D_1(P_1, P_2), \quad Q_2 = D_2(P_1, P_2), \quad \mathrm{TC} = C(Q_1, Q_2).$$

The profit as a function of (P_1, P_2) is

$$\Pi(P_1, P_2) = P_1 D_1(P_1, P_2) + P_2 D_2(P_1, P_2) - C(D_1(P_1, P_2), D_2(P_1, P_2)).$$

Maximizing $\Pi(P_1, P_2)$ w. r. t. P_1, we have

$$D_1(P_1, P_2) + P_1 \frac{\partial D_1}{\partial P_1} + P_2 \frac{\partial D_2}{\partial P_1} - \frac{\partial C}{\partial Q_1}\frac{\partial D_1}{\partial P_1} - \frac{\partial C}{\partial Q_2}\frac{\partial D_2}{\partial P_1} = 0,$$

$$\Rightarrow \frac{P_1 - \mathrm{MC}_1}{P_1} = \frac{1}{|\epsilon_{11}|} + \frac{P_2 - \mathrm{MC}_2}{P_2}\frac{\mathrm{TR}_2}{\mathrm{TR}_1}\frac{\epsilon_{21}}{|\epsilon_{11}|}$$

Case 1: $\epsilon_{12} > 0$, goods 1 and 2 are substitutes, $\dfrac{P_1 - \mathrm{MC}_1}{P_1} > \dfrac{1}{|\epsilon_{11}|}$.

Case 2: $\epsilon_{12} < 0$, goods 1 and 2 are complements, $\dfrac{P_1 - \mathrm{MC}_1}{P_1} < \dfrac{1}{|\epsilon_{11}|}$.

Actually, both P_1 and P_2 are endogenous and have to be solved simultaneously.

$$\begin{pmatrix} 1 & -\dfrac{R_2\epsilon_{21}}{R_1|\epsilon_{11}|} \\ \dfrac{-R_1\epsilon_{12}}{R_2|\epsilon_{22}|} & 1 \end{pmatrix} \begin{pmatrix} \dfrac{P_1 - C_1'}{P_1} \\ \dfrac{P_2 - C_2'}{P_2} \end{pmatrix} = \begin{pmatrix} \dfrac{1}{|\epsilon_{11}|} \\ \dfrac{1}{|\epsilon_{22}|} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \dfrac{P_1 - C_1'}{P_1} \\ \dfrac{P_2 - C_2'}{P_2} \end{pmatrix} = \frac{1}{\epsilon_{11}\epsilon_{22} - \epsilon_{12}\epsilon_{21}} \begin{pmatrix} |\epsilon_{22}| + \dfrac{R_2}{R_1}\epsilon_{21} \\ |\epsilon_{11}| + \dfrac{R_1}{R_2}\epsilon_{12} \end{pmatrix}.$$

4.4.1 2-period model with goodwill (Tirole EX 1.5)

Assume that $Q_2 = D_2(p_2; p_1)$ and $\dfrac{\partial D_2}{\partial p_1} < 0$, ie., if p_1 is cheap, the monopoy gains goodwill in $t = 2$.

$$\max_{p_1, p_2} p_1 D_1(p_1) - C_1(D_1(p_1)) + \delta[p_2 D_2(p_2; p_1) - C_2(D_2(p_2; p_1))].$$

4.4.2 2-period model with learning by doing (Tirole EX 1.6)

Assume that $\mathrm{TC}_2 = C_2(Q_2; Q_1)$ and $\dfrac{\partial C_2}{\partial Q_1} < 0$, ie., if Q_1 is higher, the monopoy gains more experience in $t = 2$.

$$\max_{p_1, p_2} p_1 D_1(p_1) - C_1(D_1(p_1)) + \delta[p_2 D_2(p_2) - C_2(D_2(p_2), D_1(p_1))].$$

Continuous time (Tirole EX 1.7):

$$\max_{q_t, w_t} \int_0^\infty [R(q_t) - C(w_t)q_t]e^{-rt}dt, \quad w_t = \int_0^t q_\tau d\tau,$$

where $R(q_t)$ is the revenue at t,$R' > 0, R'' < 0$, r is the interest rate, $C_t = C(w_t)$ is the unit production cost at t, $C' < 0$, and w_t is the experience accumulated by t.
Example: $R(q) = \sqrt{q}$ and $C(w) = a + \frac{1}{w}$.

4.5 Durable good monopoly

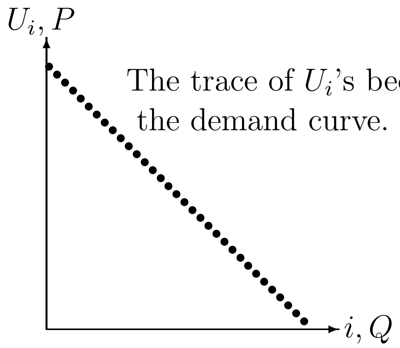
Flow (perishable) goods: 耗材
Durable goods: 耐久財

Coase (1972:) A durable good monopoly is essentially different from a perishable good monopoly.

Perishable goods: 不同時期市場獨立, 每期都要重新購買.
Durable goods: 不同時期市場不獨立, 這期購買下期就不必再買, 甚至可以轉賣.

4.5.1 A two-period model

There are 100 potential buyers of a durable good, say cars. The value of the service of a car to consumer i each period is $U_i = 101 - i$, $i = 1, \ldots, 100$.
Assume that MC = 0 and $0 < \delta < 1$ is the discount rate.



二種銷售方法: 1. 只租不賣. P_1^R, P_2^R are the rents in periods 1 and 2.
2. 賣斷. P_1^s, P_2^s are the prices of a car in periods 1 and 2.

4.5.2 只租不賣

The monopoly faces the same demand function $P = 100 - Q$ in each period. The monopoly profit maximization implies that MR = $100 - 2Q = 0$. Therefore,

$$P_1^R = P_2^R = 50, \quad \pi_1^R = \pi_2^R = 2500, \quad \Pi^R = \pi_1^R + \delta\pi_2^R = 2500(1 + \delta),$$

where $\delta < 1$ is the discounting factor.

4.5.3 賣斷

We use backward induction method to find the solution to the profit maximization problem. We first assume that those consumers who buy in period $t = 1$ do not resale their used cars to other consumers.

Suppose that $q_1^s = \bar{q}_1$, $\Rightarrow$ the demand in period $t = 2$ becomes
$q_2^s = 100 - \bar{q}_1 - P_2^s$, $\Rightarrow P_2^s = 100 - \bar{q}_1 - q_2$ and

$$\text{MR}_2 = 100 - \bar{q}_1 - 2q_2 = 0, \qquad q_2^s = 50 - 0.5\bar{q}_1 = P_2^s,\ \pi_2^s = (100 - \bar{q}_1)^2/4.$$

Now we are going to calculate the location of the marginal consumer $\bar{q}_1$ who is indifferent between buying in $t = 1$ and buying in $t = 2$.

$$(1+\delta)(100 - \bar{q}_1) - P_1^s = \delta[(100 - \bar{q}_1) - P_2^s] \Rightarrow (100 - \bar{q}_1) - P_1^s = -\delta P_2^s,$$

$$\Rightarrow P_1^s = 100 - \bar{q}_1 + \delta P_2^s = (1 + 0.5\delta)(100 - \bar{q}_1).$$

$\max_{q_1} \Pi^s = \pi_1^s + \delta\pi_2^s = q_1 P_1^s + \delta(50 - 0.5q_1)^2 = (1 + 0.5\delta)q_1(100 - q_1) + 0.25(100 - q_1)^2,$

FOC$\Rightarrow$ $q_1^s = 200/(4+\delta)$, $P_1^s = 50(2+\delta)^2/(4+\delta)$, $\Pi^s = 2500(2+\delta)^2/(4+\delta) < \Pi^R = (1+\delta)2500.$

When a monopoly firm sells a durable good in $t = 1$ instead of leasing it, the monopoly loses some of its monopoly power, that is why $\Pi^s < \Pi^R$.

4.5.4 Coase problem

Sales in t will reduce monopoly power in the future. Therefore, a rational expectation consumer will wait.

Coase conjecture (1972): In the ∞ horizon case, if $\delta \to 1$ or $\Delta t \to 0$, then the monopoly profit $\Pi^s \to 0$.

The conjecture was proved in different versions by Stokey (1981), Bulow (1982), Gul, Sonnenschein, and Wilson (1986).

Tirole EX 1.8.
1. A monopoly is the only producer of a durable good in $t = 1, 2, 3, \ldots$. If $(q_1, q_2, q_3, \ldots)$ and $(p_1, p_2, p_3, \ldots)$ are the quantity and price sequences for the monopoly product, the profit is

$$\Pi = \sum_{t=1}^{\infty} \delta^t p_t q_t.$$

2. There is a continuum of consumers indexed by $\alpha \in [0, 1]$, each needs 1 unit of the durable good.
$v_\alpha = \alpha + \delta\alpha + \delta^2\alpha + \ldots = \dfrac{\alpha}{1-\delta}$: The utility of the durable good to consumer α.
If consumer α purchases the good at t, his consumer surplus is

$$\delta^t(v_\alpha - p_t) = \delta^t\left(\frac{\alpha}{1-\delta} - p_t\right).$$

3. A linear stationary equilibrium is a pair (λ, μ), $0 < \lambda, \mu < 1$, such that
(a) If $v_\alpha > \lambda p_t$, then consumer α will buy in t if he does not buy before t.
(b) If at t, all consumers with $v_\alpha > v$ ($v_\alpha < v$) have purchased (not purchased) the durable good, then the monopoly charges $p_t = \mu v$.
(c) The purchasing strategy of (a) maximizes consumer α's consumer surplus, given the pricing strategy (b).
(d) The pricing strategy of (b) maximizes the monopoly profits, given the purchacing strategy (a).

The equilibrium is derived in Tirole as

$$\lambda = \frac{1}{\sqrt{1-\delta}}, \quad \mu = [\sqrt{1-\delta} - (1-\delta)]/\delta, \quad \lim_{\delta \to 1} \lambda = \infty, \quad \lim_{\delta \to 1} \mu = 0.$$

One way a monopoy of a durable good can avoid Coase problem is *price commitment.* By convincing the consumers that the price is not going to be reduced in the future, it can make the same amount of profit as in the rent case. However, the commitment equilibrium is not subgame perfect. Another way is to make the product less durable.

4.6 Product Selection, Quality, and Advertising

Tirole, CH2.
Product space, Vertical differentiation, Horizontal differentiation.
Goods-Characteristics Approach, Hedonic prices.
Traditional Consumer-Theory Approach.

4.6.1 Product quality selection, Tirole 2.2.1, pp.100-4.

Inverse Demand: $p = P(q, s)$, where s is the quality of the product.
Total cost: $\text{TC} = C(q, s)$, $C_q > 0$, $C_s > 0$.

Social planner's problem:

$$\max_{q,s} W(q, s) = \int_0^q P(x, s) ds - C(q, s),$$

$$\text{FOC:} \quad (1)\ W_q = P(q, s) - C_q = 0, \quad (2)\ W_s = \int_0^q P_s(x, s) dx - C_s = 0.$$

(1) $P = \text{MC}$,
(2) $\frac{1}{q} \int_0^q P_s dx = C_s / q$: Average marginal valuation of quality should be equal to the marginal cost of quality per unit.

Monopoly profit maximization:

$$\max_{q,s} \Pi(q, s) = qP(x, s) - C(q, s), \quad \text{FOC} \ \ \Pi_q = \text{MR} - C_q = 0, \ \Pi_s = qP_s(x, s) - C_s = 0.$$

$P_s = C_s/q$: Marginal consumer's marginal valuation of quality should be equal to the marginal cost of quality per unit.

Example 1: $P(q,s) = f(q) + s$, $C(q,s) = sq$, $\Rightarrow P_s = 1$, $C_s = q$, no distortion.

Example 2: There is one unit of consumers indexed by $x \in [0, \bar{x}]$. $U = xs - P$, $F(x)$ is the distribution function of x. $\Rightarrow P(q,s) = sF^{-1}(1-q) \Rightarrow \frac{1}{q}\int_0^q P_s dx \geq P_s(q,s)$, monopoly underprovides quality.

Example 3: $U = x + (\alpha - x)s - P$, $x \in [0, \alpha]$, $F(x)$ is the distribution function of $x \Rightarrow P(q,s) = \alpha s + (1-s)F^{-1}(1-q) \Rightarrow \frac{1}{q}\int_0^q P_s dx \leq P_s(q,s)$, monopoly overprovides quality.

5 Basis of Game Theory

In this part, we consider the situation when there are $n > 1$ persons with different objective (utility) functions; that is, different persons have different preferences over possible outcomes. There are two cases:
1. Game theory: The outcome depends on the behavior of all the persons involved. Each person has some control over the outcome; that is, each person controls certain strategic variables. Each one's utility depends on the decisions of all persons. We want to study how persons make decisions.

2. Public Choice: Persons have to make decision collectively, eg., by voting.
We consider only game theory here.

Game theory: the study of conflict and cooperation between persons with different objective functions.

Example (a 3-person game): The accuracy of shooting of A, B, C is 1/3, 2/3, 1, respectively. Each person wants to kill the other two to become the only survivor. They shoot in turn starting A.
Question: What is the best strategy for A?

5.1 Ingredients and classifications of games

A game is a collection of rules known to all players which determine what players may do and the outcomes and payoffs resulting from their choices.
The ingredients of a game:

1. Players: Persons having some influences upon possible income (decision makers).

2. Moves: decision points in the game at which players must make choices between alternatives (personal moves) and randomization points (called nature's moves).

3. A play: A complete record of the choices made at moves by the players and realizations of randomization.

4. Outcomes and payoffs: a play results in an outcome, which in turn determines the rewords to players.

Classifications of games:

1. according to number of players:
   2-person games – conflict and cooperation possibilities.
   n-person games – coalition formation (合縱連横) possibilities in addition.
   infinite-players' games – corresponding to perfect competition in economics.

2. according to number of strategies:
   finite – strategy (matrix) games, each person has a finite number of strategies,

payoff functions can be represented by matrices.
infinite – strategy (continuous or discontinuous payoff functions) games like duopoly games.

3. according to sum of payoffs:
   0-sum games – conflict is unavoidable.
   non-zero sum games – possibilities for cooperation.

4. according to preplay negotiation possibility:
   non-cooperative games – each person makes unilateral decisions.
   cooperative games – players form coalitions and decide the redistribution of aggregate payoffs.

5.2 The extensive form and normal form of a game

Extensive form: The rules of a game can be represented by a game tree.
The ingredients of a game tree are:
1. Players
2. Nodes: they are players' decision points (personal moves) and randomization points (nature's moves).
3. Information sets of player i: each player's decision points are partitioned into information sets. An information set consists of decision points that player i can not distinguish when making decisions.
4. Arcs (choices): Every point in an information set should have the same number of choices.
5. Randomization probabilities (of arcs following each randomization point).
6. Outcomes (end points)
7. Payoffs: The gains to players assigned to each outcome.
A pure strategy of player i: An instruction that assigns a choice for each information set of player i.
Total number of pure strategies of player i: the product of the numbers of choices of all information sets of player i.

Once we identify the pure strategy set of each player, we can represent the game in normal form (also called strategic form).

1. Strategy sets for each player: $S_1 = \{s_1, \ldots, s_m\}$, $S_2 = \{\sigma_1, \ldots, \sigma_n\}$.

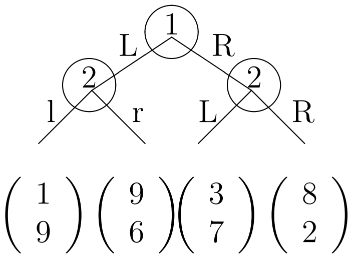
2. Payoff matrices: $\pi_1(s_i, \sigma_j) = a_{ij}$, $\pi_2(s_i, \sigma_j) = b_{ij}$. $A = [a_{ij}]$, $B = [b_{ij}]$.

Normal form:

I \ II	σ_1	$\ldots$	σ_n
s_1	(a_{11}, b_{11})	$\ldots$	(a_{1n}, b_{1n})
$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_m	(a_{m1}, b_{m1})	$\ldots$	(a_{mn}, b_{mn})

5.3 Examples

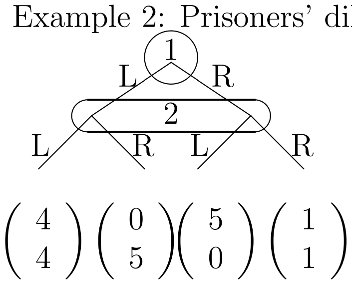
Example 1: A perfect information game



$S_1 = \{$ L, R $\}$, $S_2 = \{$ Ll, Lr, Rl, Rr $\}$.

I \ II	Ll	Rl	Lr	Rr
L	(1,9)	(1,9)	(9,6)	(9,6)
R	(3,7)*	(8,2)	(3,7)	(8,2)

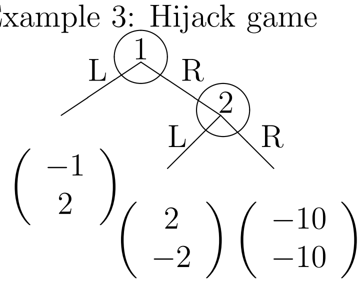
Example 2: Prisoners' dilemma game



$S_1 = \{$ L, R $\}$, $S_2 = \{$ L, R $\}$.

I \ II	L	R
L	(4,4)	(0,5)
R	(5,0)	(1,1)*

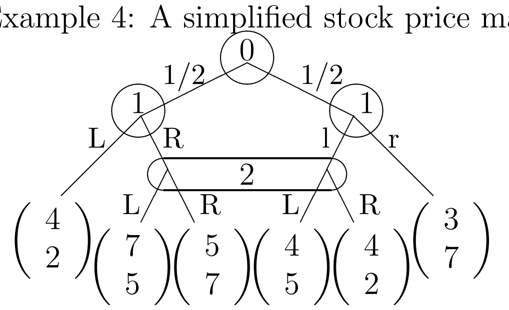
Example 3: Hijack game



$S_1 = \{$ L, R $\}$, $S_2 = \{$ L, R $\}$.

I \ II	L	R
L	(-1,2)	(-1,2)*
R	(2,-2)*	(-10,-10)

Example 4: A simplified stock price manipulation game



$S_1 = \{$ Ll, Lr, Rl, Rr $\}$, $S_2 = \{$ L, R $\}$.

I \ II	L	R
Ll	(4, 3.5)	(4, 2)
Lr	(3.5, 4.5)	(3.5, 4.5)
Rl	(5.5, 5)*	(4.5, 4.5)
Rr	(5,6)	(4,7)

Remark: Each extensive form game corresponds a normal form game. However, different extensive form games may have the same normal form.

5.4 Strategy pair and pure strategy Nash equilibrium

1. A Strategy Pair: (s_i, σ_j). Given a strategy pair, there corresponds a payoff pair (a_{ij}, b_{ij}).

2. A Nash equilibrium: A strategy pair (s_{i*}, σ_{j*}) such that $a_{i*j*} \geq a_{ij*}$ and $b_{i*j*} \geq b_{i*j}$ for all (i, j). Therefore, there is no incentives for each player to deviate from the equilibrium strategy. a_{i*j*} and b_{i*j*} are called the equilibrium payoff.

The equilibrium payoffs of the examples are marked each with a star in the normal form.

Remark 1: It is possible that a game does no have a pure strategy Nash equilibrium. Also, a game can have more than one Nash equilibria.
Remark 2: Notice that the concept of a Nash equilibrium is defined for a normal form game. For a game in extensive form (a game tree), we have to find the normal form before we can find the Nash equilibria.

5.5 Subgames and subgame perfect Nash equilibria

1. Subgame: A subgame in a game tree is a part of the tree consisting of all the nodes and arcs following a node that form a game by itself.

2. Within an extensive form game, we can identify some subgames.

3. Also, each pure strategy of a player induces a pure strategy for every subgame.

4. Subgame perfect Nash equilibrium: A Nash equilibrium is called **subgame perfect** if it induces a Nash equilibrium strategy pair for every subgame.

5. Backward induction: To find a subgame perfect equilibrium, usually we work backward. We find Nash equilibria for lowest level (smallest) subgames and replace the subgames by its Nash equilibrium payoffs. In this way, the size of the game is reduced step by step until we end up with the equilibrium payoffs.

All the equilibria, except the equilibrium strategy pair (L,R) in the hijack game, are subgame perfect.
Remark: The concept of a subgame perfect Nash equilibrium is defined only for an extensive form game.

5.5.1 Perfect information game and Zemelo's Theorem

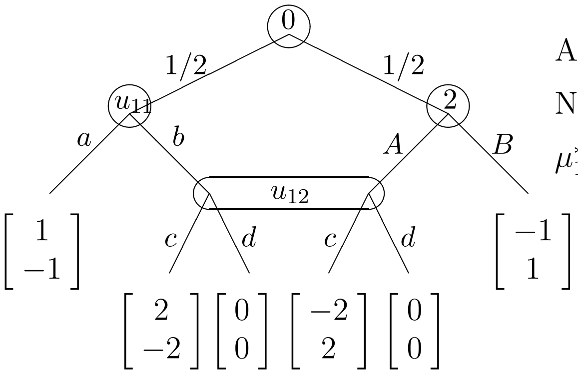
An extensive form game is called perfect information if every information set consists only one node. Every perfect information game has a pure strategy subgame perfect Nash Equilibrium.

5.5.2 Perfect recall game and Kuhn's Theorem

A local strategy at an information set $u \in U_i$: A probability distribution over the choice set at U_{ij}.
A behavior strategy: A function which assigns a local strategy for each $u \in U_i$.
The set of behavior strategies is a subset of the set of mixed strategies.

Kuhn's Theorem: In every extensive game with perfect recall, a strategically equivalent behavior strategy can be found for every mixed strategy.

However, in a non-perfect recall game, a mixed strategy may do better than behavior strategies because in a behavior strategy the local strategies are independent whereas they can be correlated in a mixed strategy.

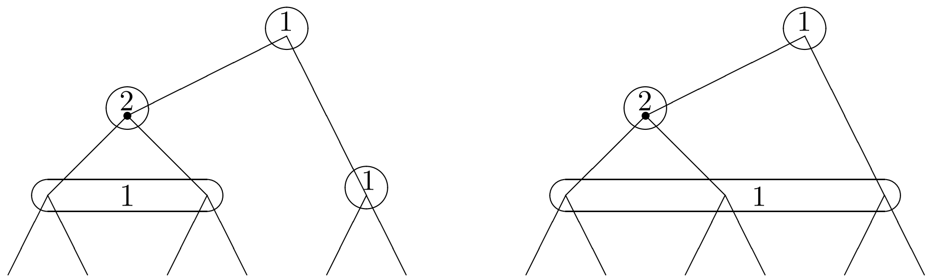


A 2-person 0-sum non-perfect recall game.
NE is $(\mu_1^*, \mu_2^*) = (\frac{1}{2}ac \oplus \frac{1}{2}bd, \frac{1}{2}A \oplus \frac{1}{2}B)$.
μ_1^* is not a behavioral strategy.

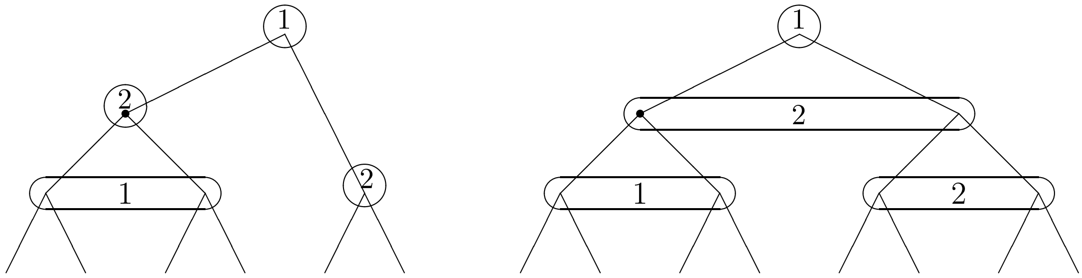
5.5.3 Reduction of a game

Redundant strategy: A pure strategy is redundant if it is strategically identical to another strategy.
Reduced normal form: The normal form without redundant strategies.
Equivalent normal form: Two normal forms are equivalent if they have the same reduced normal form.
Equivalent extensive form: Two extensive forms are equivalent if their normal forms are equivalent.

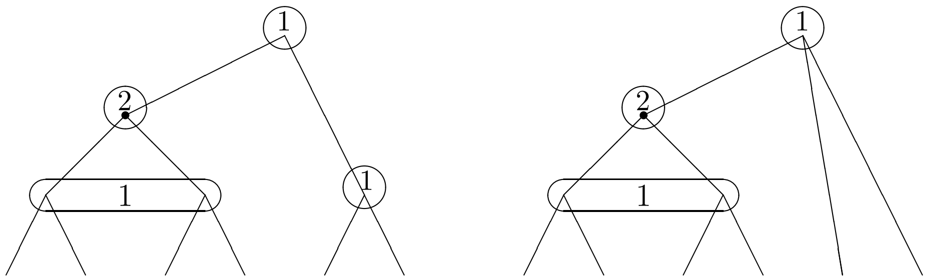
Equivalent transformation:
(1) Inflation-Deflation;



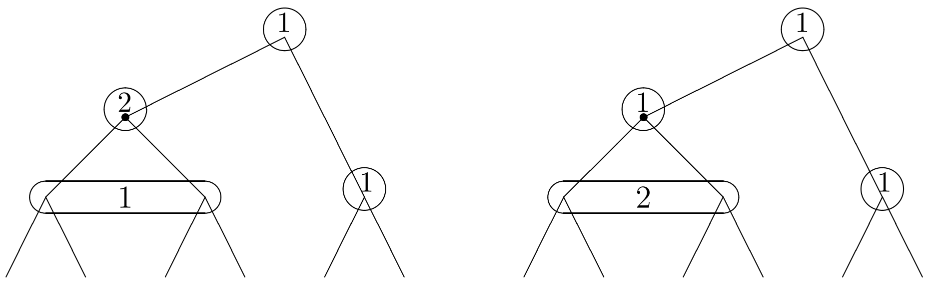
(2) Addition of superfluous move;



(3) Coalesing of moves;



(4) Interchange of moves.



5.6 Continuous games and the duopoly game

In many applications, S_1 and S_2 are infinite subsets of R^m and R^n Player 1 controls m variables and player 2 controls n variables (however, each player has infinite many strtategies). The normal form of a game is represented by two functions

$$\Pi^1 = \Pi^1(x; y) \quad \text{and} \quad \Pi^2 = \Pi^2(x; y), \quad \text{where} \quad x \in S_1 \subset R^m \quad \text{and} \quad y \in S_2 \subset R^n.$$

To simplify the presentation, assume that $m = n = 1$. A strategic pair is $(x, y) \in S_1 \times S_2$. A Nash equilibrium is a pair (x^*, y^*) such that

$$\Pi^1(x^*, y^*) \geq \Pi^1(x, y^*) \quad \text{and} \quad \Pi^2(x^*, y^*) \geq \Pi^2(x^*, y) \qquad \text{for all} \quad x \in S_1 \quad y \in S_2.$$

Consider the case when Π^i are continuously differentiable and Π^1 is strictly concave in x and Π^2 strictly concave in y (so that we do not have to worry about the SOC's).

Reaction functions and Nash equilibrium:
To player 1, x is his endogenous variable and y is his exogenous variable. For each y chosen by player 2, player 1 will choose a $x \in S_1$ to maximize his objective function Π^1. This relationship defines a behavioral equation $x = R^1(y)$ which can be obtained by solving the FOC for player 1, $\Pi^1_x(x; y) = 0$. Similarly, player 2 regards y as endogenous and x exogenous and wants to maximize Π^2 for a given x chosen by player 1. Player 2's reaction function (behavioral equation) $y = R^2(x)$ is obtained by solving $\Pi^2_y(x; y) = 0$. A Nash equilibrium is an intersection of the two reaction functions. The FOC for a Nash equilibrium is given by $\Pi^1_x(x^*; y^*) = 0$ and $\Pi^2_y(x^*; y^*) = 0$.

Duopoly game:
There are two sellers (firm 1 and firm 2) of a product.
The (inverse) market demand function is $P = a - Q$.
The marginal production costs are c_1 and c_2, respectively.
Assume that each firm regards the other firm's output as given (not affected by his output quantity).
The situation defines a 2-person game as follows: Each firm i controls his own output quantity q_i. (q_1, q_2) together determine the market price $P = a - (q_1 + q_2)$ which in turn determines the profit of each firm:

$$\Pi^1(q_1, q_2) = (P - c_1)q_1 = (a - c_1 - q_1 - q_2)q_1 \quad \text{and} \quad \Pi^2(q_1, q_2) = (P - c_2)q_2 = (a - c_2 - q_1 - q_2)q_2$$

The FOC are $\partial\Pi^1/\partial q_1 = a - c_1 - q_2 - 2q_1 = 0$ and $\partial\Pi^2/\partial q_2 = a - c_2 - q_1 - 2q_2 = 0$.
The reaction functions are $q_1 = 0.5(a - c_1 - q_2)$ and $q_2 = 0.5(a - c_2 - q_1)$.
The Cournot Nash equilibrium is $(q_1^*, q_2^*) = ((a - 2c_1 + c_2)/3, (a - 2c_2 + c_1)/3)$ with $P^* = (a + c_1 + c_2)/3$. (We have to assume that $a - 2c_1 + c_2,\ a - 2c_2 + c_1 \geq 0$.)

5.7 2-person 0-sum game

1. $B = -A$ so that $a_{ij} + b_{ij} = 0$.

2. Maxmin strategy: If player 1 plays s_i, then the minimum he will have is $\min_j a_{ij}$, called the security level of strategy s_i. A possible guideline for player 1 is to choose a strategy such that the security level is maximized: Player 1 chooses s_{i*} so that $\min_j a_{i*j} \geq \min_j a_{ij}$ for all i. Similarly, since $b_{ij} = -a_{ij}$, Player 2 chooses σ_j* so that $\max_i a_{ij*} \leq \max_i a_{ij}$ for all j.

3. Saddle point: If $a_{i*j*} = \max_i \min_j a_{ij} = \min_j \max_i a_{ij}$, then (s_{i*}, σ_{j*}) is called a saddle point. If a saddle point exists, then it is a Nash equilibrium.

$$A_1 = \begin{pmatrix} 2 & 1 & 4 \\ -1 & 0 & 6 \end{pmatrix} \quad A_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In example A_1, $\max_i \min_j a_{ij} = \min_j \max_i a_{ij} = 1$ (s_1, σ_2) is a saddle point and hence a Nash equilibrium. In A_2, $\max_i \min_j a_{ij} = 0 \neq \min_j \max_i a_{ij} = 1$ and no saddle point exists. If there is no saddle points, then there is no pure strategy equilibrium.

4. Mixed strategy for player i: A probability distribution over S_i. $p = (p_1, \ldots, p_m)$, $q = (q_1, \ldots, q_n)'$. (p, q) is a mixed strategy pair. Given (p, q), the expected payoff of player 1 is pAq. A mixed strategy Nash equilibrium (p^*, q^*) is such that $p^*Aq^* \geq pAq^*$ and $p^*Aq^* \leq p^*Aq$ for all p and all q.

5. Security level of a mixed strategy: Given player 1's strategy p, there is a pure strategy of player 2 so that the expected payoff to player 1 is minimized, just as in the case of a pure strategy of player 1.

$$t(p) \equiv \min_j \{\sum_i p_i a_{i1}, \ldots, \sum_i p_i a_{in}\}.$$

The problem of finding the maxmin mixed strategy (to find p^* to maximize $t(p)$) can be stated as

$$\max_p t \quad \text{subj. to} \quad \sum_i p_i a_{i1} \geq t, \ldots, \sum_i p_i a_{in} \geq t, \quad \sum_i p_i = 1.$$

6. Linear programming problem: The above problem can be transformed into a linear programming problem as follows: (a) Add a positive constant to each element of A to insure that $t(p) > 0$ for all p. (b) Define $y_i \equiv p_i/t(p)$ and replace the problem of $\max t(p)$ with the problem of $\min 1/t(p) = \sum_i y_i$. The constraints become $\sum_i y_i a_{i1} \geq 1, \ldots, \sum_i y_i a_{in} \geq 1$.

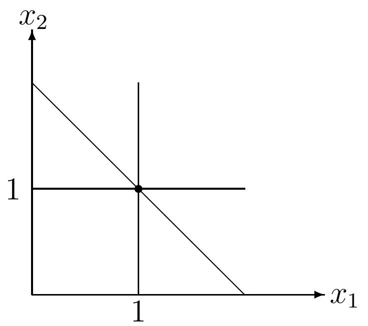
$$\min_{y_1,\ldots,y_m \geq 0} y_1 + \ldots + y_m \quad \text{subj. to} \quad \sum_i y_i a_{i1} \geq 1, \ldots, \sum_i y_i a_{in} \geq 1$$

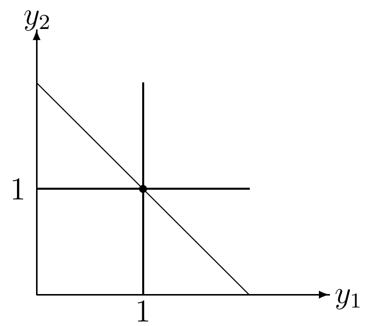
7. Duality: It turns out that player 2's minmax problem can be transformed similarly and becomes the dual of player 1's linear programming problem. The existence of a mixed strategy Nash equilibrium is then proved by using the duality theorem in linear programming.

Example (tossing coin game): $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
To find player 2's equilibrium mixed strategy, we solve the linear programming problem:

$$\max_{x_1,x_2 \geq 0} x_1 + x_2 \quad \text{subj. to} \quad x_1 \leq 1 \;\; x_2 \leq 1.$$

The solution is $x_1 = x_2 = 1$ and therefore the equilibrium strategy for player 2 is $q_1^* = q_2^* = 0.5$.





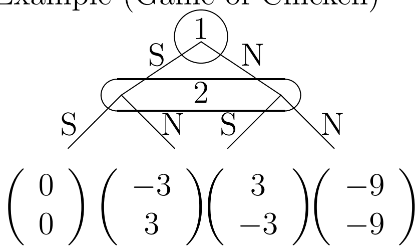
Player 1's equilibrium mixed strategy is obtained by solving the dual to the linear programming problem:

$$\min_{y_1, y_2 \geq 0} y_1 + y_2 \quad \text{subj. to} \quad y_1 \geq 1 \;\; y_2 \geq 1.$$

The solution is $p_1^* = p_2^* = 0.5$.

mixed strategy equilibria for non-zero sum games
The idea of a mixed strategy equilibrium is also applicable to a non-zero sum game. Similar to the simplex algorism for the 0-sum games, there is a Lemke algorism.

Example (Game of Chicken)



I \ II	Swerve	Don't
Swerve	(0,0)	(-3,3)*
Don't	(3,-3)*	(-9,-9)

$S_1 = \{$ S, N $\}$, $S_2 = \{$ S, N $\}$.

There are two pure strategy NE: (S, N) and (N, S).
There is also a mixed strategy NE. Suppose player 2 plays a mixed strategy $(q, 1-q)$. If player 1 plays S, his expected payoff is $\Pi^1(S) = 0q + (-3)(1-q)$. If he plays N, his expected payoff is $\Pi^1(N) = 3q + (-9)(1-q)$. For a mixed strategy NE, $\Pi^1(S) = \Pi^1(N)$, therefore, $q = \frac{2}{3}$.
The mixed strategy is symmetrical: $(p_1^*, p_2^*) = (q_1^*, q_2^*) = (\frac{2}{3}, \frac{1}{3})$.

5.8 Cooperative Game and Characteristic form

2-person 0-sum games are strictly competitive. If player 1 gains \$ 1, player 2 will loss \$ 1 and therefore no cooperation is possible. For other games, usually some cooperation is possible. The concept of a Nash equilibrium is defined for the situation when no explicit cooperation is allowed. In general, a Nash equilibrium is not efficient (not Pareto optimal). When binding agreements on strategies chosen can be contracted before the play of the game and transfers of payoffs among players after a play of the game is possible, players will negotiate to coordinate their strategies and redistribute the payoffs to achieve better results. In such a situation, the determination of strategies is not the key issue. The problem becomes the formation of coalitions and the distribution of payoffs.

Characteristic form of a game:
The player set: $N = \{1, 2, \ldots, n\}$.
A coalition is a subset of N: $S \subset N$.
A characteristic function v specifies the maximum total payoff of each coalition.

Consider the case of a 3-person game. There are 8 subsets of $N = \{1, 2, 3\}$, namely, $\phi, (1), (2), (3), (12), (13), (23), (123)$. Therefore, a characteristic form game is determined by 8 values $v(\phi), v(1), v(2), v(3), v(12), v(13), v(23), v(123)$.
Super-additivity: If $A \cap B = \phi$, then $v(A \cup B) \geq v(A) + v(B)$.
An imputation is a payoff distribution (x_1, x_2, x_3).
Individual rationality: $x_i \geq v(i)$.
Group rationality: $\sum_{i \in S} x_i \geq v(S)$.
Core C: the set of imputations that satisfy individual rationality and group rationality for all S.

Marginal contribution of player i in a coalition $S \cup i$: $v(S \cup i) - v(S)$
Shapley value of player i is an weighted average of all marginal contributions

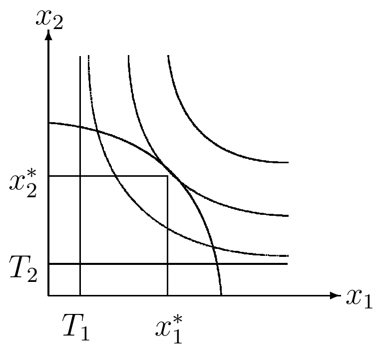
$$\pi_i = \sum_{S \subset N} \frac{|S|!(n - |S| - 1)!}{n!} [v(S \cup i) - v(S)].$$

Example: $v(\phi) = v(1) = v(2) = v(3) = 0, v(12) = v(13) = v(23) = 0.5, v(123) = 1$.
$C = \{(x_1, x_2, x_3),\ x_i \geq 0, x_i + x_j \geq 0.5, x_1 + x_2 + x_3 = 1\}$. Both (0.3, 0.3, 0.4) and (0.2, 0.4, 0.4) are in C.
The Shapley values are $(\pi_1, \pi_2, \pi_3) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.

Remark 1: The core of a game can be empty. However, the Shapley values are uniquely determined.
Remark 2: Another related concept is the von-Neumann Morgenstern solution. See CH 6 of Intriligator's *Mathematical Optimization and Economic Theory* for the motivations of these concepts.

5.9 The Nash bargaining solution for a nontransferable 2-person cooperative game

In a nontransferable cooperative game, after-play redistributions of payoffs are impossible and therefore the concepts of core and Shapley values are not suitable. For the case of 2-person games, the concept of Nash bargaining solutions are useful.
Let $F \subset R^2$ be the feasible set of payoffs if the two players can reach an agreement and T_i the payoff of player i if the negotiation breaks down. T_i is called the threat point of player i. The Nash bargaining solution (x_1^*, x_2^*) is defined to be the solution to the following problem:



$$\max_{(x_1, x_2) \in F} (x_1 - T_1)(x_2 - T_2)$$

See CH 6 of Intriligator's book for the motivations of the solution concept.

5.10 Problems

1. Consider the following two-person 0-sum game:

I \ II	σ_1	σ_2	σ_3
s_1	4	3	-2
s_2	3	4	10
s_3	7	6	8

   (a) Find the max min strategy of player I $s_{\max\min}$ and the min max strategy of player II $\sigma_{\min\max}$.

   (b) Is the strategy pair $(s_{\max\min}, \sigma_{\min\max})$ a Nash equilibrium of the game?

   (c) What are the equilibrium payoffs?

2. Find the maxmin strategy ($s_{\max\min}$) and the minmax strategy ($\sigma_{\min\max}$) of the following two-person 0-sum game:

I \ II	σ_1	σ_2
s_1	-3	6
s_2	8	-2
s_3	6	3

   Is the strategy pair $(s_{\max\min}, \sigma_{\min\max})$ a Nash equilibrium? If not, use simplex method to find the mixed strategy Nash equilibrium.

3. Find the (mixed strategy) Nash Equilibrium of the following two-person game:

I \ II	H	T
H	(-2, 2)	(2, -1)
T	(2, -2)	(-1,2)

4. Suppose that two firms producing a homogenous product face a linear demand curve $P = a - bQ = a - b(q_1 + q_2)$ and that both have the same constant marginal costs c. For a given quantity pair (q_1, q_2), the profits are $\Pi_i = q_i(P - c) = q_i(a - bq_1 - bq_2 - c)$, $i = 1, 2$. Find the Cournot Nash equilibrium output of each firm.

5. Suppose that in a two-person cooperative game without side payments, if the two players reach an agreement, they can get (Π_1, Π_2) such that $\Pi_1^2 + \Pi_2 = 47$ and if no agreement is reached, player 1 will get $T_1 = 3$ and player 2 will get $T_2 = 2$.

   (a) Find the Nash solution of the game.

   (b) Do the same for the case when side payments are possible. Also answer how the side payments should be done?

6. A singer (player 1), a pianist (player 2), and a drummer (player 3) are offered \$ 1,000 to play together by a night club owner. The owner would alternatively pay \$ 800 the singer-piano duo, \$ 650 the piano drums duo, and \$ 300 the piano alone. The night club is not interested in any other combination. Howeover, the singer-drums duo makes \$ 500 and the singer alone gets \$ 200 a night in a restaurant. The drums alone can make no profit.

   (a) Write down the characteristic form of the cooperative game with side payments.

   (b) Find the Shapley values of the game.

   (c) Characterize the core.

6 Duopoly and Oligopoly–Homogeneous products

6.1 Cournot Market Structure

2 Sellers producing a homogenous product.
$\mathrm{TC}_i(q_i) = c_i q_i$, $i = 1, 2$.
$P(Q) = a - bQ$, $a, b > 0$, $a > \max_i c_i$, $Q = q_1 + q_2$.

Simultaneous move: both firms choose (q_1, q_2) simultaneously.
$\pi_1(q_1, q_2) = P(Q)q_1 - c_1 q_1 = (a - bq_1 - bq_2)q_1 - c_1 q_1$,
$\pi_2(q_1, q_2) = P(Q)q_2 - c_2 q_2 = (a - bq_1 - bq_2)q_2 - c_2 q_2$.

Definition of a Cournot equilibrium: $\{P^c, q_1^c, q_2^c\}$ such that $P^c = P(Q^c) = a - b(q_1^c + q_2^c)$ and

$$\pi_1(q_1^c, q_2^c) \geq \pi_1(q_1, q_2^c), \quad \pi_2(q_1^c, q_2^c) \geq \pi_2(q_1^c, q_2), \quad \forall (q_1, q_2).$$

The first order conditions (FOC) are

$$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - c_1 = 0, \quad \frac{\partial \pi_2}{\partial q_2} = a - bq_1 - 2bq_2 - c_2 = 0.$$

In matrix form,

$$\begin{pmatrix} 2b & b \\ b & 2b \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} a - c_1 \\ a - c_2 \end{pmatrix}, \Rightarrow \begin{pmatrix} q_1^c \\ q_2^c \end{pmatrix} = \frac{1}{3b} \begin{pmatrix} a - 2c_1 + c_2 \\ a - 2c_2 + c_1 \end{pmatrix}.$$

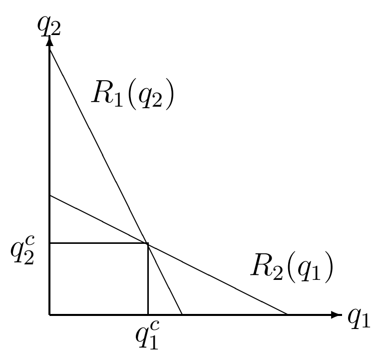
$$Q^c = \frac{2a - c_1 - c_2}{3b}, \; P^c = \frac{a + c_1 + c_2}{3}, \quad \begin{pmatrix} \pi_1^c \\ \pi_2^c \end{pmatrix} = \frac{1}{9b} \begin{pmatrix} (a - 2c_1 + c_2)^2 \\ (a - 2c_2 + c_1)^2 \end{pmatrix}.$$

If $c_1 \downarrow$ (say, due to R&D), then $q_1^c \uparrow$, $q_2^c \downarrow$, $Q^c \uparrow$, $P^c \downarrow$, $\pi_1^c \uparrow$, $\pi_2^c \downarrow$.

6.1.1 Reaction function and diagrammatic solution

From FOC, we can derive the reaction functions:

$$q_1 = \frac{a - c_1}{2b} - 0.5q_2 \equiv R_1(q_2), \quad q_2 = \frac{a - c_2}{2b} - 0.5q_1 \equiv R_2(q_1).$$



6.1.2 N-seller case

N sellers, $\mathrm{MC}_i = c_i$, $i = 1, \ldots, N$, $P = P(Q) = a - bQ = a - b\sum_{j=1}^{N} q_j$.

$$\pi_i(q_1, \ldots, q_N) = P(Q)q_i - c_i q_i = \left(a - b\sum_{j=1}^{N} q_j\right) q_i - c_i q_i.$$

FOC is

$$\frac{\partial \pi_i}{\partial q_i} = a - b\sum_j q_j - bq_i - c_i = P - bq_i - c_i = 0, \Rightarrow Na - (N+1)b\sum_j q_j - \sum_j c_j = 0,$$

$$\Rightarrow Q^c = \sum_j q_j^c = \frac{Na - \sum_j c_j}{(N+1)b}, \ P^c = \frac{a + \sum_j c_j}{N+1}, \ q_i^c = \frac{P - c_i}{b} = \frac{a + \sum_j c_j - (N+1)c_i}{b(N+1)}.$$

Symmetric case $c_i = c$:

$$q_i^c = \frac{a-c}{(N+1)b}, \ P^c = \frac{a + Nc}{N+1}, \ Q^c = \frac{N}{N+1}\frac{a-c}{b}.$$

When $N = 1$, it is the monopoly case.
As $N \to \infty$, $(P^c, Q^c) \to (c, \frac{a-c}{b})$, the competition case.

6.1.3 Welfare analysis for the symmetric case

Consumer surplus $\mathrm{CS} = \frac{(a-P)Q}{2}$, Social welfare $W = \mathrm{CS} + \sum_j \pi_j$.

For the symmetric case, CS as functions of N are

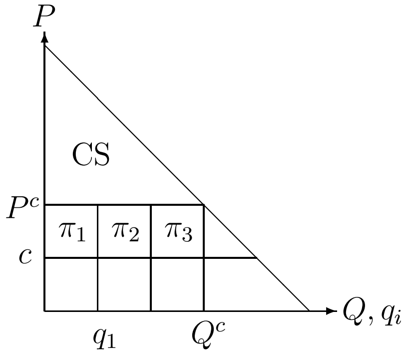
$$\mathrm{CS}^c(N) = \frac{1}{2}\left(a - \frac{a+Nc}{N+1}\right)\left(\frac{N}{N+1}\right)\left(\frac{a-c}{b}\right) = \frac{1}{2b}\left(\frac{N}{N+1}\right)^2 (a-c)^2.$$

The sum of profits is

$$\sum_j \pi_j = \sum_j (P^c - c)q_j = \frac{a-c}{N+1}\frac{N}{N+1}\frac{a-c}{b} = \frac{N}{(N+1)^2}\frac{(a-c)^2}{b}.$$

$$W^c(N) = \mathrm{CS}^c(N) + \sum_j \pi_j = \frac{(a-c)^2}{b}\frac{N + 0.5N^2}{(N+1)^2}.$$

$$\lim_{N\to\infty} W^c(N) = \lim_{N\to\infty} \mathrm{CS}^c(N) = \frac{(a-c)^2}{2b}, \quad \lim_{N\to\infty} \sum_j \pi_j = 0.$$



6.2 Sequential moves – Stackelberg equilibrium

Consider now that the two firms move sequentially. At $t = 1$ firm 1 chooses q_1. At $t = 2$ firm 2 chooses q_2.

Firm 1 – leader, Firm 2 – follower.

The consequence is that when choosing q_2, firm 2 already knows what q_1 is. On the other hand, in deciding the quantity q_1, firm 1 takes into consideration firm 2's possible reaction, i.e., firm 1 assumes that $q_2 = R_2(q_1)$. This is the idea of backward induction and the equilibrium derived is a **subgame perfect** Nash equilibrium.
At $t = 1$, firm 1 chooses q_1 to maximize the (expected) profit $\pi_1 = \pi_1(q_1, R_2(q_1))$:

$$\max_{q_1}[a-b(q_1+R_2(q_1))]q_1-c_1q_1 = [a-b(q_1+\frac{a-c_2}{2b}-0.5q_1)]q_1-c_1q_1 = 0.5(a+c_2-bq_1)q_1-c_1q_1.$$

The FOC (interior solution) is

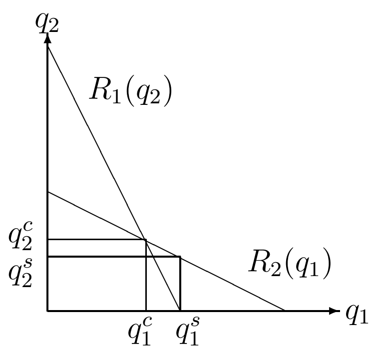
$$\frac{d\pi_1}{dq_1} = \frac{\partial \pi_1}{\partial q_1} + \frac{\partial \pi_1}{\partial q_2}\frac{dR_2}{dq_1} = 0.5(a - 2c_1 + c_2 - 2bq_1) = 0,$$

$$\Rightarrow q_1^s = \frac{a - 2c_1 + c_2}{2b} > q_1^c,\ q_2^s = \frac{a + 2c_1 - 3c_2}{4b} < q_2^c.$$

$$Q^s = \frac{3a - 2c_1 - c_2}{4b} > Q^c, \quad P^s = \frac{a + 2c_1 + c_2}{4} < P^c, \quad ((a + c_1 + c_2)/3 = P^c > c_1).$$

$$\pi_1^s = \frac{(a - 2c_1 + c_2)}{8b}, \quad \pi_2^s = \frac{(a + 2c_1 - 3c_2)}{16b}.$$

1. $\pi_1^s + \pi_2^s \gtrless \pi_1^c + \pi_2^c$ depending on c_1, c_2.
2. $\pi_1^s > \pi_1^c$ because $\pi_1^s = \max_{q_1} \pi_1(q_1, R_2(q_1)) > \pi_1(q_1^c, R_2(q_1^c)) = \pi_1^c$.
3. $\pi_2^s < \pi_2^c$ since $q_2^s < q_2^c$ and $P^s < P^c$.



6.2.1 Subgame non-perfect equilibrium

In the above, we derived a subgame perfect equilibrium. On the other hand, the game has many non-perfect equilibria. Firm 2 can threaten firm 1 that if q_1 is too large, he will chooses a large enough q_2 to make market price zero. This is an incredible threat because firm 2 will hurt himself too. However, if firm 1 believes that the threat will be executed, there can be all kind of equilibria.

6.2.2 Extension

The model can be extended in many ways. For example, when there are three firms choosing output quantities sequentially. Or firms 1 and 2 move simultaneously and then firm 3 follows, etc.

6.3 Conjecture Variation

In Cournot equilibrium, firms move simultaneously and, when making decision, expect that other firms will not change their quantities. In more general case, firms will form conjectures about other firms behaviors.

$$\pi_1^e(q_1, q_2^e) = (a - bq_1 - bq_2^e)q_1 - c_1 q_1, \quad \pi_2^e(q_1^e, q_2) = (a - bq_1^e - bq_2)q_2 - c_2 q_2.$$

The FOC are

$$\frac{d\pi_1^e}{dq_1} = \frac{\partial \pi_1^e}{\partial q_1} + \frac{\partial \pi_1^e}{\partial q_2^e}\frac{dq_2^e}{dq_1} = 0, \quad \frac{d\pi_2^e}{dq_2} = \frac{\partial \pi_2^e}{\partial q_2} + \frac{\partial \pi_2^e}{\partial q_1^e}\frac{dq_1^e}{dq_2} = 0.$$

Assume that the conjectures are

$$\frac{dq_2^e}{dq_1} = \lambda_1, \quad \frac{dq_1^e}{dq_2} = \lambda_2.$$

In equilibrium, $q_1^e = q_1$ and $q_2^e = q_2$. The FOCs become

$$a - 2bq_1 - bq_2 - b\lambda_1 q_1 - c_1 = 0, \quad a - 2bq_2 - bq_1 - b\lambda_2 q_2 - c_2 = 0.$$

In matrix form,

$$\begin{pmatrix} (2+\lambda_1)b & b \\ b & (2+\lambda_2)b \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} a - c_1 \\ a - c_2 \end{pmatrix},$$

$$\Rightarrow \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \frac{1}{[3 + 2(\lambda_1 + \lambda_2) + \lambda_1\lambda_2]b} \begin{pmatrix} (a - c_1)(2+\lambda_2) - (a - c_2) \\ (a - c_2)(2+\lambda_1) - (a - c_1) \end{pmatrix}.$$

If $\lambda_1 = \lambda_2 = 0$, then it becomes the Cournot equilibrium.

6.3.1 Stackelberg Case: $\lambda_2 = 0$, $\lambda_1 = R_2'(q_1) = 0.5$

Assuming that $c_1 = c_2 = 0$.

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \frac{1}{2b} \begin{pmatrix} a \\ a/2 \end{pmatrix}.$$

It is the Stackelberg leadership equilibrium with firm 1 as the leader.
Similarly, if $\lambda_1 = 0$, $\lambda_2 = R_1'(q_2) = 0.5$, then firm 2 becomes the leader.

6.3.2 Collusion case: $\lambda_1 = q_2/q_1, \lambda_2 = q_1/q_2$

Assume that $\text{MC}_1 = c_1 q_1$ and $MC_2 = c_2 q_2$. The FOCs become

$$a - 2b(q_1 + q_2) - c_1 q_1 = 0, \quad a - 2b(q_1 + q_2) - c_2 q_2 = 0.$$

That is, $\text{MR} = \text{MC}_1 = MC_2$, the collusion solution.

The idea can be generalized to N-firm case.

6.4 Bertrand Price Competition

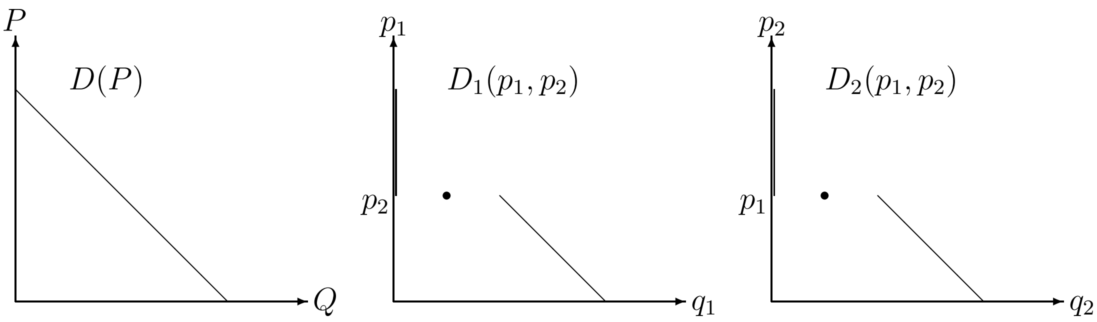
It is easier to change prices than to change quantities. Therefore, firms' strategic variables are more likely to be prices.

Bertrand model: firms determine prices simultaneously.

Question: In a homogeneous product market, given (p_1, p_2), how market demand is going to divided between firm 1 and firm 2?

Assumption: Consumers always choose to buy from the firm charging lower price. When two firms charge the same price, the market demand divided equally between them. Let $Q = D(P)$ be the market demand.

$$q_1 = D_1(p_1, p_2) = \begin{cases} 0 & p_1 > p_2 \\ 0.5D(p_1) & p_1 = p_2 \\ D(p_1) & p_1 < p_2, \end{cases} \qquad q_2 = D_2(p_1, p_2) = \begin{cases} D(p_2) & p_1 > p_2 \\ 0.5D(p_2) & p_1 = p_2 \\ 0 & p_1 < p_2. \end{cases}$$



Notice that individual firms' demand functions are discontinuous.

Bertrand game:

$$\pi_1(p_1, p_2) = (p_1 - c_1)D_1(p_1, p_2), \quad \pi_2(p_1, p_2) = (p_2 - c_2)D_2(p_1, p_2)$$

Bertrand equilibrium: $\{p_1^b, p_2^b, q_1^b, q_2^b\}$ such that $q_1^b = D_1(p_1^b, p_2^b)$, $q_2^b = D_2(p_1^b, p_2^b)$, and

$$\pi_1(p_1^b, p_2^b) \geq \pi_1(p_1, p_2^b), \quad \pi_2(p_1^b, p_2^b) \geq \pi_2(p_1^b, p_2) \;\; \forall \; (p_1, p_2).$$

We cannot use FOCs to find the reaction functions and the equilibrium as in the Cournot quantity competition case because the profit functions are not continuous.

6.4.1 If $c_1 = c_2 = c$, then $p_1^b = p_2^b = c$, $q_1^b = q_2^b = 0.5D(c)$

Proof: 1. $p_i^b \geq c$.
2. Both $p_1 > p_2 > c$ and $p_2 > p_1 > c$ cannot be equilibrium since the firm with a higher price will reduce its price.
3. $p_1 = p_2 > c$ cannot be equilibrium since every firm will reduce its price to gain the whole market.
4. $p_1 > p_2 = c$ and $p_2 > p_1 = c$ cannot be equilibrium because the firm with $p = c$ will raise its price to earn positive profit.
5. What left is $p_1 = p_2 = c$, where none has an incentive to change.

Bertrand paradox: 1. $\pi_1^b = \pi_2^b = 0$.
2. Why firms bother to enter the market if they know that $\pi_1^b = \pi_2^b = 0$.

6.4.2 If $c_i < c_j$ and $c_j \leq P_m(c_i)$, then no equilibrium exists

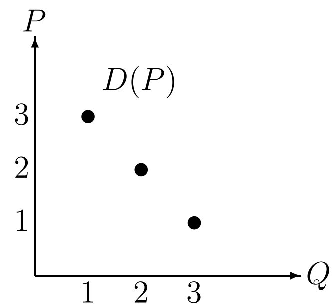
However, if $c_j > P_m(c_i)$, where $P_m(c_i)$ is the monopoly price corresponding to marginal cost c_i, then firm i becomes a monopoly.
Or if there is a smallest money unit e (say, e is one cent), $c_j - c_i > e$, then $p_i^b = c_2 - e$, $p_j^b = c_j$, $q_i = D(c_2 - e)$, $q_2^b = 0$ is an equilibrium because none has an incentive to change. Approximately, we say that $p_i^b = p_j^b = c_j$ and $q_i^b = D(c_j)$, $q_j^b = 0$.

6.5 Price competition, capacity constraint, and Edgeworth Cycle

One way to resolve Bertrand paradox is to consider DRTS or increasing MC. In Bertrand price competition, if the marginal cost is increasing ($\text{TC}_i''(q_i) > 0$), then we have to consider the possibility of mixed strategy equilibria. See Tirole's Supplement to Chapter 5. Here we discuss capacity constraint and Edgeworth cycle.

6.5.1 Edgeworth model

Assume that both firms has a capacity constraint $q_i \leq \bar{q}_i = 1$ and that the product is indivisible. $c_1 = c_2 = 0$. There are 3 consumers. Consumer i is willing to pay $4 - i$ dollars for one unit of the product, $i = 1, 2, 3$.



Edgeworth Cycle of (p_1, p_2): $(2,2) \rightarrow (3,2) \rightarrow (3,2.9) \rightarrow (2.8,2.9) \rightarrow (2.8,2.7) \rightarrow \cdots \rightarrow (2,2)$
Therefore, the is no equilibrium but repetitions of similar cycles.

6.6 A 2-period model

At $t = 1$, both firms determine quantities q_1, q_2.
At $t = 2$, both firms determine prices p_1, p_2 after seeing q_1, q_2.
Demand function: $P = 10 - Q$, MCs: $c_1 = c_2 = 1$.

Proposition: If $2q_i + q_j \leq 9$, then $p_1 = p_2 = 10 - (q_1 + q_2) = P(Q)$ is the equilibrium at $t = 2$.
Proof: Suppose $p_2 = 10 - (q_1 + q_2)$, then $p_1 = p_2$ maximizes firm 1's profit. The reasons are as follows.
1. If $p_1 < p_2$, $\pi_1(p_1) = q_1(p_1 - 1) < \pi_1(p_2) = q_1(p_2 - 1)$.
2. If $p_1 > p_2$, $\pi_1(p_1) = (10-q_2-p_1)(p_1-1)$, $\pi_1'(p_1) = 10-q_2-2p_1+1 = 2q_2+q_1-9 \leq 0$.

It follows that at $t = 1$, firms expect that $P = 10 - q_1 - q_2$, the reduced profit functions are

$$\pi_1(q_1, q_2) = q_1(P(Q)-c_1) = q_1(9-q_1-q_2), \quad \pi_2(q_1, q_2) = q_2(P(Q)-c_2) = q_2(9-q_1-q_2).$$

Therefore, it becomes an authentic Cournot quantity competition game.

6.7 Infinite Repeated Game and Self-enforcing Collusion

Another way to resolve Bertrand paradox is to consider the infinite repeated version of the Bertrand game. To simplify the issue, assume that $c_1 = c_2 = 0$ and $P = \min\{1 - Q, 0\} = \min\{1 - q_1 - q_2, 0\}$ in each period t. The profit function of firm i at time t is

$$\pi_i(t) = \pi_i(q_1(t), q_2(t)) = q_i(t)[1 - q_1(t) - q_2(t)].$$

A pure strategy of the repeated game of firm i is a sequence of functions $\sigma_{i,t}$ of outcome history H_{t-1}:

$$\sigma_i \equiv (\sigma_{i,0}, \sigma_{i,1}(H_0), \ldots, \sigma_{i,t}(H_{t-1}), \ldots).$$

where H_{t-1} is the history of a play of the repeated game up to time $t - 1$:

$$H_{t-1} = ((q_1(0), q_2(0)), (q_1(1), q_2(1)), \ldots, (q_1(t-1), q_2(t-1))),$$

and $\sigma_{i,t}$ maps from the space of histories H_{t-1} to the space of quantities $\{q_i : 0 \leq q_i < \infty\}$. Given a pair (σ_1, σ_2), the payoff function of firm i is

$$\Pi_i(\sigma_1, \sigma_2) = \sum_{t=0}^{\infty} \delta^t \pi_i(\sigma_{1,t}(H_{t-1}), \sigma_{2,t}(H_{t-1})) = \sum_{t=0}^{\infty} \delta^t \pi_i(q_1(t), q_2(t)).$$

Given the Cournot equilibrium $(q_1^c, q_2^c) = (\frac{1}{3}, \frac{1}{3})$, we can define a Cournot strategy for the repeated game as follows:

$$\sigma_{i,t}^c(H_{t-1}) = q_i^c = \frac{1}{3} \; \forall H_{t-1}, \quad \sigma_i^c \equiv (\sigma_{i,0}^c, \sigma_{i,1}^c, \ldots, \sigma_{i,t}^c, \ldots).$$

It is straightforward to show that the pair (σ_1^c, σ_2^c) is a Nash equilibrium for the repeated game.

6.7.1 Trigger strategy and tacit collusive equilibrium

A trigger strategy σ^T for firm i has a cooperative phase and a non-cooperative phase:
Cooperative phase 合作局面– If both firms cooperate (choose collusion quantity $q_i = 0.5Q_m = 0.25$) up to period $t-1$, then firm i will cooperate at period t.
Non-cooperative phase 不合作局面– Once the cooperation phase breaks down (someone has chosen a different quantity), then firm i will choose the Cournot equilibrium quantity q_i^c.
If both firms choose the same strigger strategy, then they will cooperate forever. If neither one could benefit from changing to a different strategy, then (σ^T, σ^T) is a subgame-perfect Nash equilbrium.

Formally, a trigger strategy for firm i is

$$\sigma^T = (\sigma_0^T, \sigma_1^T, \ldots, \sigma_t^T, \ldots), \quad \sigma_0^T = 0.5Q_m,$$

$$\sigma_t^T = \begin{cases} 0.5Q_m & \text{if } q_j(\tau) = 0.5Q_m \ \forall\, 0 \le \tau < t,\ j = 1,2 \\ q_i^c & \text{otherwise.} \end{cases}$$

6.7.2 (σ^T, σ^T) is a SPNE for $\delta > \dfrac{9}{17}$

Proof:
1. At every period t in the cooperative phase, if the opponent does not violate the cooperation, then firm i's gain to continue cooperation is

$$\Pi^* = 0.5\pi_m + \delta 0.5\pi_m + \delta^2 0.5\pi_m + \cdots = 0.5\pi_m(1 + \delta + \delta^2 + \cdots) = \frac{1}{8(1-\delta)}.$$

If firm i chooses to stop the cooperative phase, he will set $q_t = \dfrac{3}{8}$ (the profit maximization output when $q_j = 0.25$) and then trigger the non-cooperative phase and gains the Cournot profit of 1/9 per period. Firm i's gain will be

$$\Pi^v = \frac{9}{64} + \delta\pi_i^c + \delta^2 0.5\pi_i^c + \cdots = \frac{9}{64} + \pi_i^c(\delta + \delta^2 + \cdots) = \frac{9}{64} + \frac{\delta}{9(1-\delta)}.$$

$$\Pi^* - \Pi^v = \frac{1}{576}(17\delta - 9) > 0.$$

Therefore, during the cooperative phase, the best strategy is to continue cooperation.
2. In the non-cooperative phase, the Cournot quantity is the Nash equilibrium quantity in each period.

The cooperative phase is the equilibrium realization path. The non-cooperative phase is called off-equilibrium subgames.

6.7.3 Retaliation trigger strategy

A retaliation trigger strategy σ^{RT} for firm i has a cooperative phase and a retaliation phase:
Cooperative phase– the same as a trigger strategy.
Retaliation phase– Once the cooperation phase breaks down, then firm i will choose the retaliation quantity $q_i^r = 1$ to make sure $P = 0$.
If both firms choose the same trigger strategy, then they will cooperate forever. If neither one could benefit from changing to a different strategy, then $(\sigma^{RT}, \sigma^{RT})$ is a subgame-non-perfect Nash equilbrium. It is not a perfect equilibrium because retaliation will hurt oneself and is not a credible threat.

Formally, a retaliation trigger strategy for firm i is

$$\sigma^{RT} = (\sigma_0^{RT}, \sigma_1^{RT}, \ldots, \sigma_t^{RT}, \ldots), \quad \sigma_0^{RT} = 0.5Q_m,$$

$$\sigma_t^{RT} = \begin{cases} 0.5Q_m & \text{if } q_j(\tau) = 0.5Q_m \; \forall \, 0 \le \tau < t, \; j = 1, 2 \\ 1 & \text{otherwise.} \end{cases}$$

6.7.4 $(\sigma^{RT}, \sigma^{RT})$ is a NE for $\delta > \dfrac{1}{9}$

Proof:
At every period t in the cooperative phase, if the opponent does not violate the cooperation, then firm i's gain to continue cooperation is (same as the trigger strategy case)

$$\Pi^* = \frac{1}{8(1-\delta)}.$$

If firm i chooses to stop the cooperative phase, he will set $q_t = \dfrac{3}{8}$ (same as the trigger strategy case) and then trigger the retaliation phase, making 0 profit per period. Firm i's gain will be

$$\Pi^v = \frac{9}{64} \Rightarrow \Pi^* - \Pi^v = \frac{1}{8(1-\delta)} - \frac{9}{64} = \frac{9\delta - 1}{64(1-\delta)} > 0.$$

Therefore, during the cooperative phase, the best strategy is to continue cooperation.

The retaliation phase is off-equilibrium subgames and never reached. Since the retaliation strategy is not optimal, the Nash equilibrium is not subgame-perfect.

In summary, if $1 > \delta > \dfrac{9}{17}$, then the duopoly firms will collude in a SPNE; if $\dfrac{9}{17} > \delta > \dfrac{1}{9}$, then the duopoly firms will collusion in a non-perfect NE. If $\delta < \dfrac{1}{9}$, then collusion is impossible.

6.7.5 Folk Theorem of the infinite repeated game

In the above, we consider only the cooperation to divide the monopoly profit evenly. The same argument works for other kinds of distributions of monopoly profit or even aggregate profits less than the monopoly profit.

Folk Theorem: When $\delta \rightarrow 1$, every distributions of profits such that the average payoff per period $\pi_i \geq \pi_i^c$ can be implemented as a SPNE.

For subgame-non-perfect NEs, the individual profits can be even lower then the Cournot profit.

6.7.6 Finitely repeated game

If the duopoly game is only repeated finite time, $t = 1, 2, \ldots, T$, we can use backward induction to find Subgame-perfect Nash equilibria. Since the last period T is the same as a 1-period duopoly game, both firms will play Cournot equilibrium quantity. Then, since the last period strategy is sure to be the Cournot quantity, it does not affect the choice at $t = T - 1$, therefore, at $t = T - 1$ both firms also play Cournot strategy. Similar argument is applied to $t = T - 2$, $t + T - 3$, etc., etc.
Therefore, the only possible SPNE is that both firms choose Cournot equilibrium quantity from the beginning to the end.

However, there may exist subgame-non-perfect NE.

6.7.7 Infinite repeated Bertrand price competition game

In the above, we have considered a repeated game of quantity compeition. We can define an infinitely repeated price competition game and a trigger strategy similarily:
(1) In the cooperative phase, a firm sets monopoly price and gains one half of the monopoly profit $0.5\pi_m = 1/8$.
(2) In the non-cooperative phase, a firm sets Bertrand competition price $p_b = 0$ and gains 0 profit.
To deviate from the cooperative phase, a firm obtains the whole monopoly profit $\pi_m = 1/4$ instantly. If $1/[8(1-\delta)] > 1/4$ ($\delta > 0.5$), the trigger strategy is a SPNE.

6.8 Duopoly in International Trade

6.8.1 Reciprocal Dumping in International Trade

2 countries, $i = 1, 2$ each has a firm (also indexed by i) producing the same product. Assume that MC = 0, but the unit transportation cost is τ.
q_i^h, q_i^f: quantities produced by country i's firm and sold in domestic market and foreign market, respectively.
$Q_1 = q_1^h + q_2^f$, $Q_2 = q_2^h + q_1^f$: aggregate quantities sold in countries 1 and 2's market, respectively.

$P_i = a - bQ_i$: market demand in country i's market.

The profits of the international duopoly firms are

$$\Pi_1 = P_1 q_1^h + (P_2 - \tau) q_1^f = [a - b(q_1^h + q_2^f)] q_1^h + [a - (q_1^f + q_2^h) - \tau] q_1^f,$$

$$\Pi_2 = P_2 q_2^h + (P_1 - \tau) q_2^f = [a - b(q_2^h + q_1^f)] q_2^h + [a - (q_2^f + q_1^h) - \tau] q_2^f.$$

Firm i will choose q_i^h and q_i^f to maximize Π_i. The FOC's are

$$\frac{\partial \Pi_i}{\partial q_i^h} = a - 2bq_i^h - bq_j^f = 0, \quad \frac{\partial \Pi_i}{\partial q_i^f} = a - 2bq_i^f - bq_j^h - \tau = 0, \quad i = 1, 2.$$

In a symmetric equilibrium $q_1^h = q_2^h = q^h$ and $q_1^f = q_2^f = q^f$. In matrix form, the FOC's become:

$$\begin{pmatrix} 2b & b \\ b & 2b \end{pmatrix} \begin{pmatrix} q^h \\ q^f \end{pmatrix} = \begin{pmatrix} a \\ a - \tau \end{pmatrix}, \quad \Rightarrow \begin{pmatrix} q^h \\ q^f \end{pmatrix} = \begin{pmatrix} \dfrac{a+\tau}{3b} \\ \dfrac{a-2\tau}{3b} \end{pmatrix},$$

$$Q = q^h + q^f = \frac{2a - \tau}{3b} \; P = \frac{a + \tau}{3}.$$

It seems that there is reciprocal dumping: the FOB price of exports P^{FOB} is lower than the domestic price P.

$$P^{\mathrm{CIF}} = P = \frac{a+\tau}{3}, \quad P^{\mathrm{FOB}} = P^{\mathrm{CIF}} - \tau = \frac{a - 2\tau}{3} < P.$$

However, since $P^{\mathrm{FOB}} > \mathrm{MC} = 0$, there is no dumping in the MC definition of dumping.

The comparative statics with respect to τ is

$$\frac{\partial q^h}{\partial \tau} > 0, \; \frac{\partial q^f}{\partial \tau} < 0, \; \frac{\partial Q}{\partial \tau} < 0, \; \frac{\partial P}{\partial \tau} > 0.$$

In this model, it seems that international trade is a waste of transportation costs and is unnecessary. However, if there is no international competition, each country's market would become a monopoly.

Extensions: 1. 2-stage game. 2. comparison with monopoly.

6.8.2 Preferential Trade Agreement, Trade Creation, Trade Diversion

Free Trade Agreement FTA: Free trade among participants.
Customs Union CU: FTA plus uniform tariff rates towards non-participants.
Common Market CM: CU plus free factor mobility.

Consider the apple market in Taiwan.

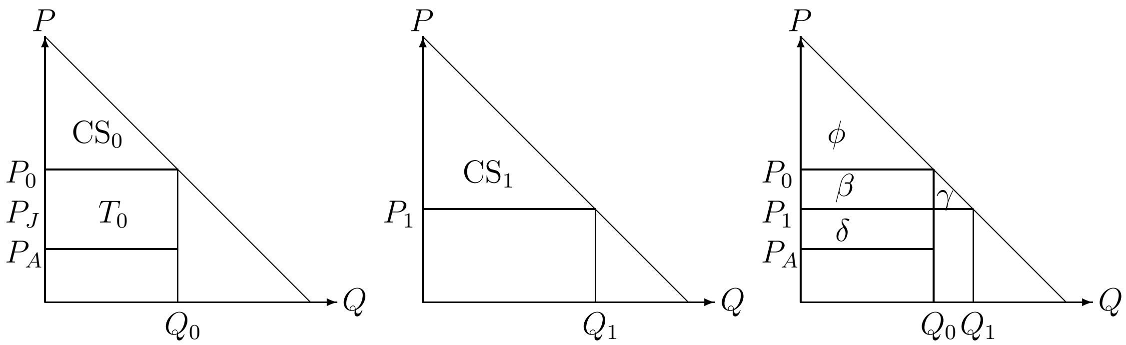
Demand: $P = a - Q$.
2 export countries: America and Japan, $P_A < P_J$.

At $t = 0$, Taiwan imposes uniform tariff of \$ t per unit.
$P_0 = P_A + t$, $Q_0 = a - P_A - t$, $W_0 = CS_0 + T_0 = \dfrac{Q_0^2}{2} + tQ_0 = \dfrac{(a - P_A)^2 - t^2}{2}$.

At $t = 1$, Taiwan and Japan form FTA, assume $P_J < P_A + t$.
$P_1 = P_J$, $Q_1 = a - P_J$, $W_1 = CS_1 + T_1 = \dfrac{Q_1^2}{2} = \dfrac{(a - P_J)^2}{2}$.

$$W_1 - W_0 = 0.5[(a - P_J)^2 + t^2 - (a - P_A)^2] = 0.5[t^2 - (2a - P_A - P_J)(P_J - P_A)].$$

Given a and P_A, FTA is more advantageous the higher a and the lower P_J.



$$W_1 - W_0 = (\phi + \beta + \gamma) - (\phi + \beta + \delta) = \gamma - \delta.$$

γ: trade creation effect.
δ: trade diversion effect.
$W_1 - W_0 > 0$ if and only if $\gamma > \delta$.

6.9 Duopoly under Asymmetric Information

6.9.1 Incomplete information game

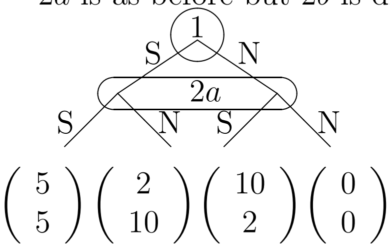
Imperfect information game: Some **information sets** contain more than one nodes, i.e., at some stages of the game, a player may be uncertain about the consequences of his choices.

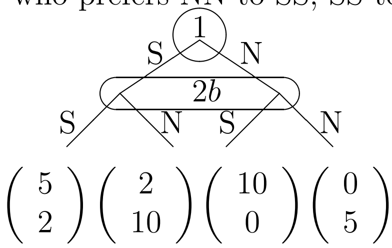
Incomplete information game: Some players do not completely know the **rule** of the game. In particular, a player does not know the payoff functions of other players. There are more than one **type** of a player, whose payoff function depends on his type. The type is known to the player himself but not to other players. There is a prior probability distribution of the type of a player.

Bayesian equilibrium (of a static game): Each type of a player is regarded as an independent player.
Bayesian perfect equilibrium (of a dynamic game): In a Bayesian equilibrium, players will use Bayes' law to estimate the posterior distribution of the types of other players.

6.9.2 Modified chicken game

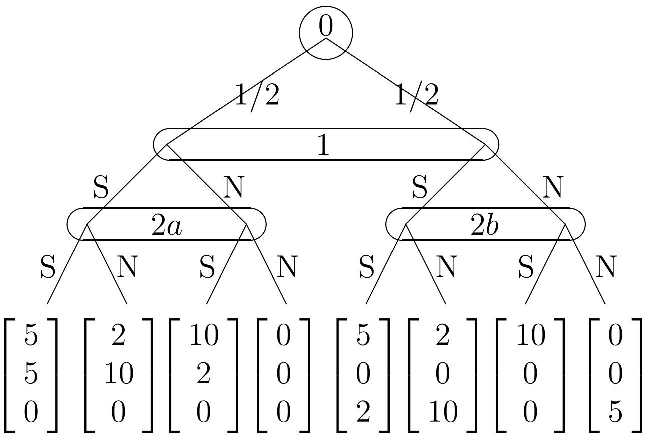
In the chicken dilemma game, assume that there are 2 types of player 2, $2a$ and $2b$, $2a$ is as before but $2b$ is different, who prefers NN to SS, SS to being a chicken:





Asymmetric information: player 2 knows whether he is $2a$ or $2b$ but player 1 does not. However, player 1 knows that Prob$[2a]$ = Prob$[2b]$ =0.5.

If we regard $2a$ and $2b$ as two different players, the game tree becomes:



In this incomplete information game, player $2b$ has a dominant strategy, N. Hence, it can be reduced to a 2-person game. The pure strategy Bayesian equilibria are SNN and NSN. The mixed strategy equilibrium is different from the ordinary chicken game.

6.9.3 A duopoly model with unknown MC

2 firms, 1 and 2, with market demand $p = 2 - q_1 - q_2$.
$\mathrm{MC}_1 = 1$, $\pi_1 = q_1(1 - q_1 - q_2)$.
2 types of firm 2, a and b.
$\mathrm{MC}_{2a} = 1.25$, $\pi_{2a} = q_{2a}(0.75 - q_1 - q_{2a})$.
$\mathrm{MC}_{2b} = 0.75$, $\pi_{2b} = q_{2b}(1.25 - q_1 - q_{2b})$.

Asymmetric information: firm 2 knows whether he is $2a$ or $2b$ but firm 1 does not. However, firm 1 knows that Prob[$2a$] = Prob[$2b$] =0.5.

To find the Bayesian equilibrium, we regard the duopoly as a 3-person game with payoff functions:

$$\begin{aligned}
\Pi_1(q_1, q_{2a}, q_{2b}) &= 0.5q_1(1 - q_1 - q_{2a}) + 0.5q_1(1 - q_1 - q_{2b}) \\
\Pi_{2a}(q_1, q_{2a}, q_{2b}) &= 0.5q_{2a}(0.75 - q_1 - q_{2a}) \\
\Pi_{2b}(q_1, q_{2a}, q_{2b}) &= 0.5q_{2b}(1.25 - q_1 - q_{2b})
\end{aligned}$$

FOC are

$$0.5(1-2q_1-q_{2a})+0.5(1-2q_1-q_{2b}) = 0, \quad 0.5(0.75-q_1-2q_{2a}) = 0, \quad 0.5(1.25-q_1-2q_{2b}) = 0.$$

$$\begin{pmatrix} 4 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} q_1 \\ q_{2a} \\ q_{2b} \end{pmatrix} = \begin{pmatrix} 2 \\ 0.75 \\ 1.25 \end{pmatrix}.$$

The Bayesian equilibrium is $(q_1, q_{2a}, q_{2b}) = (\frac{1}{3}, \frac{5}{24}, \frac{11}{24})$.

7 Differentiated Products Markets

7.1 2-Differentiated Products Duopoly

2 Sellers producing differentiated products.

$$p_1 = \alpha - \beta q_1 - \gamma q_2,\ p_2 = \alpha - \beta q_2 - \gamma q_1,\ \ \beta > 0,\ \beta^2 > \gamma^2.$$

In matrix form,

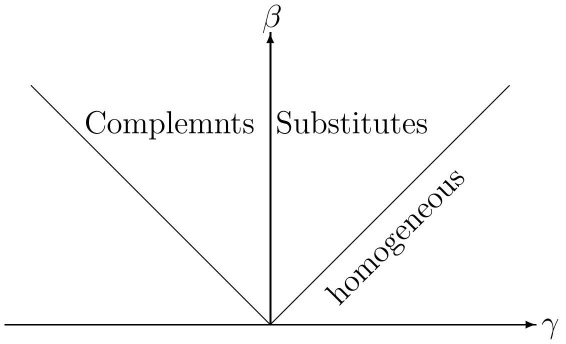
$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} \alpha \\ \alpha \end{pmatrix} - \begin{pmatrix} \beta & \gamma \\ \gamma & \beta \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, \Rightarrow \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} \beta & \gamma \\ \gamma & \beta \end{pmatrix}^{-1}\left[\begin{pmatrix} \alpha \\ \alpha \end{pmatrix} - \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}\right],$$

$$= \frac{1}{\beta^2 - \gamma^2}\left\{\begin{pmatrix} \alpha(\beta-\gamma) \\ \alpha(\beta-\gamma) \end{pmatrix} - \begin{pmatrix} \beta & -\gamma \\ -\gamma & \beta \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix}\right\} \equiv \begin{pmatrix} a \\ a \end{pmatrix} - \begin{pmatrix} b & -c \\ -c & b \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix},$$

where

$$a \equiv \frac{\alpha}{\beta + \gamma}, \quad b \equiv \frac{\beta}{\beta^2 - \gamma^2}, \quad c \equiv \frac{\gamma}{\beta^2 - \gamma^2}.$$

If $\gamma = 0$ ($c = 0$), the firms are independent monopolists. If $0 < \gamma < \beta$ ($0 < c < b$) the products are substitutes. When $\beta = \gamma \Rightarrow p_1 = p_2$, the products are perfect substitutable (homogenous).



Define $\delta \equiv \dfrac{\gamma^2}{\beta^2}$, degree of differentiation.

If δ (hence γ, c) $\to 0$, products are highly differentiated.

If $\delta \to \beta$ (hence $\gamma \to c$), products are highly homogenous.

Assume that $\mathrm{TC}_i(q_i) = c_i q_i$, $i = 1, 2$. In a Cournot quantity competition duopoly game, the payoffs are represented as functions of (q_1, q_2):

$$\pi_1^c(q_1, q_2) = (p_1 - c_1)q_1 = (\alpha - \beta q_1 - \gamma q_2 - c_1)q_1, \quad \pi_2^c(q_1, q_2) = (p_2 - c_2)q_2 = (\alpha - \beta q_1 - \gamma q_2 - c_2)q_2.$$

In a Bertrand price competition duopoly game, the payoffs are represented as functions of (p_1, p_2):

$$\pi_1^b(p_1, p_2) = (p_1 - c_1)q_1 = (p_1 - c_1)(a - bp_1 + cp_2), \quad \pi_2^b(p_1, p_2) = (p_2 - c_2)q_2 = (p_2 - c_2)(a - bp_2 + cp_1).$$

It seems that Cournot game and Bertrand game are just a change of variables of each other. However, the Nash equilibrium is totally different. A change of variables of a game also changes its Nash equilibrium.

7.1.1 Change of variables of a game

Suppose we have a game in (x_1, x_2):

$$\pi_1^a(x_1, x_2), \quad \pi_2^a(x_1, x_2).$$

FOCs of the x-game:

$$\frac{\partial \pi_1^a}{\partial x_1} = 0, \quad \frac{\partial \pi_2^a}{\partial x_2} = 0, \qquad \Rightarrow (x_1^a, x_2^a).$$

Change of variables:

$$x_1 = F(y_1, y_2), \qquad x_2 = G(y_1, y_2).$$

The payoff functions for the y-game is

$$\pi_1^b(y_1, y_2) = \pi_1^a(F(y_1, y_2), G(y_1, y_2)), \quad \pi_2^b(y_1, y_2) = \pi_2^a(F(y_1, y_2), G(y_1, y_2)).$$

FOCs of the y-game:

$$\frac{\partial \pi_1^b}{\partial y_1} = 0 = \frac{\partial \pi_1^a}{\partial x_1}\frac{\partial F}{\partial y_1} + \frac{\partial \pi_1^a}{\partial x_2}\frac{\partial G}{\partial y_1}, \quad \frac{\partial \pi_2^b}{\partial y_2} = 0 = \frac{\partial \pi_2^a}{\partial x_1}\frac{\partial F}{\partial y_2} + \frac{\partial \pi_2^a}{\partial x_2}\frac{\partial G}{\partial y_2}, \quad \Rightarrow (y_1^b, y_2^b).$$

Because the FOCs for the x-game is different from that of the y-game, $x_1^a \neq F(y_1^b, y_2^b)$ and $x_2^a \neq G(y_1^b, y_2^b)$ in general. Only when $\frac{\partial F}{\partial y_2} = \frac{\partial G}{\partial y_1} = 0$ will $x_1^a = F(y_1^b, y_2^b)$ and $x_2^a = G(y_1^b, y_2^b)$.

In case of the duopoly game, a Bertrand equilibrium is different from a Cournot equilibrium in general. Only when $\delta = \gamma = c = 0$ will the two equilibria the same. That is, if the two products are independent, the firms are independent monopolies and it does not matter whether we use prices or quantities as the strategical variables.

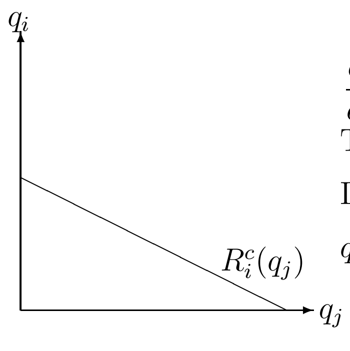
7.1.2 Quantity Game

Assume that $c_1 = c_2 = 0$. The payoff functions are

$$\pi_1^c(q_1, q_2) = (\alpha - \beta q_1 - \gamma q_2) q_1, \quad \pi_2^c(q_1, q_2) = (\alpha - \beta q_1 - \gamma q_2) q_2.$$

The FOC of firm i and its reaction function are

$$\alpha - 2\beta q_i - \gamma q_j = 0, \Rightarrow q_i = R_i^c(q_j) = \frac{\alpha - \gamma q_j}{2\beta} = \frac{\alpha}{2\beta} - \frac{\gamma}{2\beta} q_j \quad \left.\frac{dq_i}{dq_j}\right|_{R_i^c} = -\frac{\gamma}{2\beta} = -0.5\sqrt{\delta}.$$



$$\left.\frac{dq_i}{dq_j}\right|_{R_i^c} = -\frac{\gamma}{2\beta} = -0.5\sqrt{\delta}$$

The larger δ, the steeper the reaction curve.

If products are independent, $\delta = 0$,

q_i is independent of q_j.

In a symmetric equilibrium, $q_1 = q_2 = q^c$, $p_1 = p_2 = p^c$,

$$q^c = \frac{\alpha}{2\beta + \gamma} = \frac{\alpha}{\beta(2+\sqrt{\delta})}, \quad p^c = \alpha - (\beta+\gamma)q^c = \frac{\alpha\beta}{2\beta+\gamma} = \frac{\alpha}{2+\sqrt{\delta}},$$

and $\pi_1 = \pi_2 = \pi^c$,

$$\pi^c = \frac{\alpha^2\beta}{(2\beta+\gamma)^2} = \frac{\alpha^2}{\beta(2+\sqrt{\delta})^2}.$$

$$\frac{\partial q^c}{\partial \delta} < 0, \quad \frac{\partial p^c}{\partial \delta} < 0, \quad \frac{\partial \pi^c}{\partial \delta} < 0.$$

Therefore, when the degree of differentiation increases, q^c, p^c, and π^c will be increased. When $\delta = 1$, it reduces to the homogeneous case.

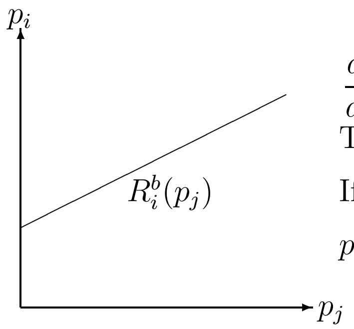
7.1.3 Price Game

Also assume that $c_1 = c_2 = 0$. The payoff functions are

$$\pi_1^b(p_1, p_2) = (a - bp_1 + cp_2)p_1, \quad \pi_2^b(p_1, p_2) = (a - bp_2 + cp_1)p_2.$$

The FOC of firm i and its reaction function are

$$a - 2bp_i + cp_j = 0, \Rightarrow p_i = R_i^b(p_j) = \frac{a + cp_j}{2b} = \frac{a}{2b} + \frac{c}{2b}p_j \quad \left.\frac{dp_i}{dp_j}\right|_{R_i^b} = \frac{c}{2b} = 0.5\sqrt{\delta}.$$



$$\left.\frac{dp_i}{dp_j}\right|_{R_i^b} = 0.5\sqrt{\delta}$$

The larger δ, the steeper the reaction curve.

If products are independent, $\delta = 0$,

p_i is independent of p_j.

In a symmetric equilibrium, $p_1 = p_2 = p^b$, $q_1 = q_2 = q^b$,

$$p^b = \frac{a}{2b+c} = \frac{a}{b(2+\sqrt{\delta})} = \frac{\alpha(\beta-\gamma)}{2\beta-\gamma}, \quad q^b = a - (b-c)p^b = \frac{ab}{2b-c} = \frac{a}{2-\sqrt{\delta}},$$

and $\pi_1 = \pi_2 = \pi^b$,

$$\pi^b = \frac{a^2b}{(2b-c)^2} = \frac{a^2}{b(2-\sqrt{\delta})^2} = \frac{\alpha^2(\beta-\gamma)\beta}{(2\beta-\gamma)^2(\beta+\gamma)} = \frac{\alpha^2(1-\sqrt{\delta})}{\beta(2-\sqrt{\delta})^2(1+\sqrt{\delta})}.$$

$$\frac{\partial p^b}{\partial \delta} < 0, \quad \frac{\partial q^b}{\partial \delta} < 0, \quad \frac{\partial \pi^b}{\partial \delta} < 0.$$

Therefore, when the degree of differentiation increases, p^b, q^b, and π^b will be increased. When $\delta = 1$, it reduces to the homogeneous case and $p^b \rightarrow 0$.

7.1.4 Comparison between quantity and price games

$$p^c - p^b = \frac{\alpha}{4\delta^{-1} - 1} > 0, \quad \Rightarrow q^c - q^b < 0.$$

As $\delta \to 0$, $p^c \to p^b$ and when $\delta = 0$, $p^c = p^b$.

Strategic substitutes vs Strategic complements: In a continuous game, if the slopes of the reaction functions are negative as in the Cournot quantity game, the strategic variables (e.g., quantities) are said to be strategic substitutes. If the slopes are positive as in the Bertrand price game, the strategic variables (e.g., prices) are said to be strategic complements.

7.1.5 Sequential moves game

The Stackelberg quantity leadership model can be generalized to the differentiated product case. Here we use an example to illustrate the idea. Consider the following Bertrand game:

$$q_1 = 168 - 2p_1 + p_2, \quad q_2 = 168 - 2p_2 + p_1; \quad \mathrm{TC} = 0.$$

$$\Rightarrow p_i = R_i^b(q_j) = 42 + 0.25p_j, \ p^b = 56, \ q^b = 112, \ \pi^b = 6272.$$

In the sequential game version, assume that firm 1 moves first:

$$\pi_1 = [168 - 2p_1 + (42 + 0.25p_1)]p_1 = [210 - 1.75p_1]p_1.$$

FOC: $210 - 3.5p_1 = 0, \ \Rightarrow p_1 = 60, \ p_2 = 57, \ q_1 = 105, \ q_2 = 114, \ \pi_1 = 6300, \ \pi_2 = 6498.$
$\Rightarrow p_1 > p_2, \ q_1 < q_2, \ \pi_2 > \pi_1 > \pi^b = 6272.$

The Cournot game is:

$$p_1 = 168 - \frac{2q_1 + q_2}{3}, \ p_2 = 168 - \frac{2q_2 + q_1}{3};$$

$$\Rightarrow q_i = R_i^c(q_j) = 126 - 0.25q_j, \ q = p^c = 100.8, \ p^c = 67.2, \ \pi^c = 6774.$$

In the sequential game version, assume that firm 1 moves first:

$$\pi_1 = (168 - \frac{2}{3}q_i - \frac{1}{3}q_j)q_i = (126 - \frac{7}{12}q_j)q_i.$$

FOC: $126 - \frac{7}{6}q_1 = 0, \ \Rightarrow q_1 = 108, \ q_2 = 81, \ p_1 = 69, \ p_2 = 78, \ \pi_1 = 7452, \ \pi_2 = 6318.$
$\Rightarrow p_1 < p_2, \ q_1 > q_2, \ \pi_1 > \pi^c > \pi_2.$

From the above example, we can see that in a quantity game firms prefer to be the leader whereas in a price game they prefer to be the follower.

7.2 Free entry/exit and LR equilibrium number of firms

So far we have assumed that the number of firms is fixed and entry/exit of new/existing firms is impossible. Now let us relax this assumption and consider the case when new/existing firms will enter/exit the industry if they can make a positive/negative profit. The number of firms is now an endogenous variable.

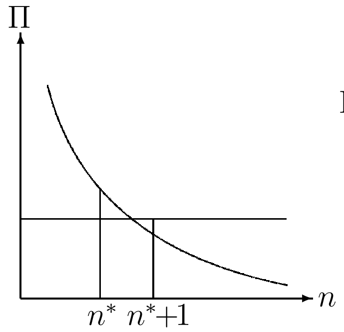
Assume that every existing and potential producer produces identical product and has the same cost function.
$\text{TC}_i(q_i) = F. \qquad P = A - Q$

Assume that there are n firms in the industry. The n-firm oligopoly quantity competition equilibrium is (see 6.1.2)

$$q_i = \frac{A}{n+1} = P, \quad \Rightarrow \pi_i = \left(\frac{A}{n+1}\right)^2 - F \equiv \Pi(n).$$

If $\Pi(n+1) > 0$, then at least a new firm will enter the industry. If $\Pi(n) < 0$, then the some of the existing firms will exit.
In LR equilibrium, the number of firms n^* will be such that $\Pi(n^*) \geq 0 \geq \Pi(n^*+1)$.

$$\Pi(n^*) = \left(\frac{A}{n+1}\right)^2 - F \geq 0 \quad \Rightarrow n^* = \left[\frac{A}{\sqrt{F}}\right] - 1.$$



$\Pi(n^*) > 0$ and $\Pi(n^*+1) < 0$.

The model above can be modified to consider the case when firms produce differentiated products:

$$p_i = A - q_i - \delta \sum_{j \neq i} q_j, \ \pi_i = \left(A - q_i - \delta \sum_{j \neq i} q_j\right) q_i - F, \ \Rightarrow \ \text{FOC:} \ A - 2q_i - \delta \sum_{j \neq i} q_j = 0,$$

where $0 < \delta < 1$. If there are N firms, in Cournot equilibrium, $q_i = q^*$ for all i, and

$$q^* = p^* = \frac{A}{2 + (N-1)\delta}, \ \pi^* = \left(\frac{A}{2 + (N-1)\delta}\right)^2 - F.$$

The equilibrium number of firms is

$$N^* = \left[\frac{A}{\delta\sqrt{F}} + 1 - \frac{2}{\delta}\right] = \left[\frac{1}{\delta}\left(\frac{A}{\sqrt{F}} - 1\right) + 1 - \frac{1}{\delta}\right].$$

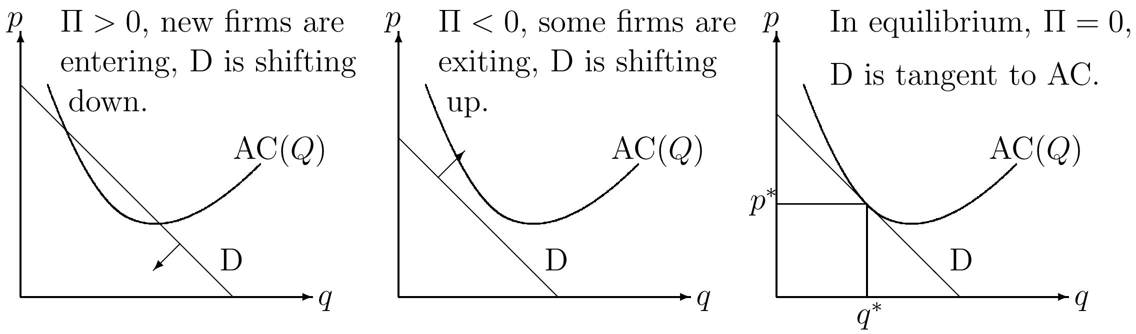
When $\delta \to 1$, it reduces to the homogenous product case. When δ decreases, N^* increases.

7.3 Monopolistic Competition in Differentiated Products

The free entry/exit model of last section assumes that firms compete in a oligopoly market, firms in the market are aware of the co-existent relationship. Now we consider a monopolistic competition market in which each firm regards itself as a monopoly firm.

7.3.1 Chamberlin Model

There are many small firms each produces a differentiated product and has a negatively sloped demand curve. The demand curve of a typical firm is affected by the number of firms in the industry. If existing firms are making positive profits $\Pi > 0$, new firms will enter making the demand curve shifts down. Conversely, if existing firms are negative profits $\Pi < 0$, some of them will exit and the curve shifts up. In the industry equilibrium, firms are making 0-profits, there is no incentive for entry or exit and the number of firms does not change.



7.3.2 Dixit-Stiglitz Model

Dixit-Stiglitz (*AER* 1977) formulates Chamberlin model.
There is a representative consumer who has I dollars to spend on all the brands available. The consumer has a CES utility function

$$U(q_1, q_2, \ldots) = \left[\sum_{i=1}^{\infty} (q_i)^\alpha\right]^{1/\alpha}, \quad 0 < \alpha < 1.$$

The consumer's optimization problem is

$$\max \sum_{i=1}^{N} (q_i)^\alpha, \text{ subject to } \sum_{i=1}^{N} p_i q_i = I, \Rightarrow \mathcal{L} = \sum_{i=1}^{N} (q_i)^\alpha + \lambda(I - \sum_{i=1}^{N} p_i q_i).$$

FOC is

$$\frac{\partial L}{\partial q_i} = \alpha(q_i)^{\alpha-1} - \lambda p_i = 0 \Rightarrow q_i = \left(\frac{\lambda p_i}{\alpha}\right)^{\frac{1}{\alpha-1}} = \left(\frac{I}{\sum_j p_j^{\frac{-\alpha}{1-\alpha}}}\right) p_i^{\frac{-1}{1-\alpha}} = A p_i^{\frac{-1}{1-\alpha}}.$$

As N is very large, $\dfrac{\partial A}{\partial p_i} \approx 0$ and the demand elasticity is approximately $|\eta| = \dfrac{1}{1-\alpha}$.

The cost function of producer i is $\text{TC}_i(q_i) = F + cq_i$.

$$\max_{q_i} p_i q_i - \text{TC}(q_i) = A^{1-\alpha} q_i^{\alpha} - F - cq_i.$$

The monopoly profit maximization pricing rule $\text{MC} = P(1 - \frac{1}{|\eta|})$ means:

$$p_i^* = c\frac{|\eta|}{|\eta| - 1} = \frac{c}{\alpha},\ \pi_i = (p_i^* - c)q_i - F = \frac{c(1-\alpha)}{\alpha}q_i - F.$$

In equilibrium, $\pi_i = 0$, we have (using the consumer's budget constraint)

$$q_i^* = \frac{\alpha F}{(1-\alpha)c},\ N^* = \frac{I}{p^* q^*} = \frac{(1-\alpha)I}{F}.$$

The conclusions are

1. $p^* = c/\alpha$ and Lerner index of each firm is $\dfrac{p-c}{p} = 1 - \alpha$.

2. Each firm produces $q^* = \dfrac{\alpha F}{(1-\alpha)c}$.

3. $N^* = \dfrac{(1-\alpha)I}{F}$.

4. **Variety effect**:

$$U^* = (N^* q^{*\alpha})^{1/\alpha} = N^{*1/\alpha} q^* = [\alpha^{\alpha}(1-\alpha)^{1-\alpha} I]^{1/\alpha} F^{1-\frac{1}{\alpha}} c^{-1}.$$

   The size of the market is measured by I. When I increases, N^* (the variety) increases proportionally. However, U^* increases more than proportionally.

Examples: Restaurants, Profesional Base Ball, etc.

If we approximate the integer number N by a continuous variable, the utility function and the budget constraint are now

$$U(\{q(t)\}_{0 \le t \le N}) = \left[\int_0^{\infty} [q(t)]^{\alpha} dt\right]^{1/\alpha}, \quad \int_0^N p(t)q(t)dt = I.$$

The result is the same as above.

7.3.3 Intra-industry trade

Heckscher-Ohlin trade theory: A country exports products it has **comparative advantage** over other countries.
Intra-industry trade: In real world, we see countries export and import the same products, eg., cars, wines, etc. It seems contradictory to Heckscher-Ohlin's comparative advantage theory.

Krugman (JIE 1979): If consumers have preferences for varieties, as in Dixit-Stiglitz monopolistic competition model, intra-industry trade is beneficiary to trading countries.
No trade: $p^* = c/\alpha$, $q^* = (1-\alpha)F/(\alpha c)$, $N^* = \dfrac{(1-\alpha)I}{F}$, $U^* \propto I^{1/\alpha}$.
Trade: $p^* = c/\alpha$, $q^* = (1-\alpha)F/(\alpha c)$, $N^* = \dfrac{(1-\alpha)2I}{F}$, $U^* \propto (2I)^{1/\alpha}$.

Gros (1987), with tariff.
Chou/Shy (1991), with non-tradable good sector.

7.4 Location Models

The models discussed so far assume that the product differentiation is exogenously determined. Location models provide a way to endogenize product differentiation.

7.4.1 Product characteristics 產品特徵

Different products are characterized by different characters. 位置, 甜度, 顏色, 品質.

Firms choose different product characters to differentiated each other.

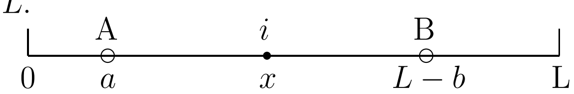
Vertical differentiation: Consumers' preferences are consistent w.r.t. the differentiated character, eg.,. 品質. (Ch12)

Horizontal differentiation: Different consumers have different tastes w.r.t. the differentiated character, eg., 位置, 甜度.

To determine the characteristics of a product is to locate the product on the space of product characteristics. 產品定位

7.4.2 Hotelling linear city model

Assume that consumers in a market are distributed uniformly along a line of length L.



Firm A is located at point a, P_A is the price of its product.

Firm B is located at point $L-b$, P_B is the price of its product.

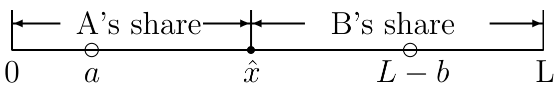
Each point $x \in [0, L]$ represents a consumer x. Each consumer demands a unit of the product, either purchases from A or B.

$$U_x = \begin{cases} -P_A - \tau|x-a| & \text{if x buys from A.} \\ -P_B - \tau|x-(L-b)| & \text{if x buys from B.} \end{cases}$$

τ is the transportation cost per unit distance. $|x-a|$ ($|x-(L-b)|$) is the distance between x and A (B).
The marginal consumer $\hat{x}$ is indifferent between buying from A and from B. The location of $\hat{x}$ is determined by

$$-P_A - \tau|x-a| = -P_B - \tau|x-(L-b)| \Rightarrow \hat{x} = \frac{L-b+a}{2} - \frac{P_A-P_B}{2\tau}. \qquad (1)$$

The location of $\hat{x}$ divids the market into two parts: $[0, \hat{x})$ is firm A's market share and $(\hat{x}, L]$ is firm B's market share.



Therefore, given (P_A, P_B), the demand functions of firms A and B are

$$D_A(P_A, P_B) = \hat{x} = \frac{L-b+a}{2} - \frac{P_A-P_B}{2\tau}, \quad D_B(P_A, P_B) = L - \hat{x} = \frac{L-a+b}{2} + \frac{P_A-P_B}{2\tau}.$$

Assume that firms A and B engage in price competition and that the marginal costs are zero. The payoff functions are

$$\Pi_A(P_A, P_B) = P_A\left[\frac{L-b+a}{2} - \frac{P_A-P_B}{2\tau}\right], \quad \Pi_B(P_A, P_B) = P_B\left[\frac{L-a+b}{2} + \frac{P_A-P_B}{2\tau}\right].$$

The FOCs are (the SOCs are satisfied)

$$\frac{L-b+a}{2} - \frac{2P_A-P_B}{2\tau} = 0, \quad \frac{L-a+b}{2} + \frac{P_A-2P_B}{2\tau} = 0. \qquad (2)$$

The equilibrium is given by

$$P_A = \frac{\tau(3L-b+a)}{3}, \; P_B = \frac{\tau(3L-a+b)}{3}, \; Q_A = \hat{x} = \frac{3L-b+a}{6}, \; Q_B = L - \hat{x} = \frac{3L-a+b}{6}.$$

$$\Pi_A = \frac{\tau(3L-b+a)^2}{18} = \frac{\tau(2L+d+2a)^2}{18}, \quad \Pi_B = \frac{\tau(3L-a+b)^2}{18} = \frac{\tau(2L+d+2b)^2}{18},$$

where $d \equiv L-b-a$ is the distance between the locations of A and B. the degree of product differentiation is measured by $d\tau$. When d or τ increases, the products are more differentiated, the competition is less intensive, equilibrium prices are higher and firms are making more profits.

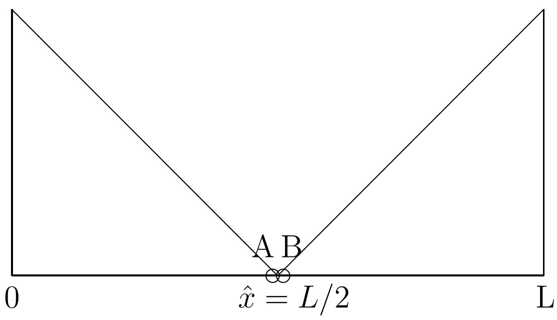
捷運開通使新光三越與 SOGO 競爭更劇烈.

$$\frac{\partial \Pi_A}{\partial a} > 0, \quad \frac{\partial \Pi_B}{\partial b} > 0.$$

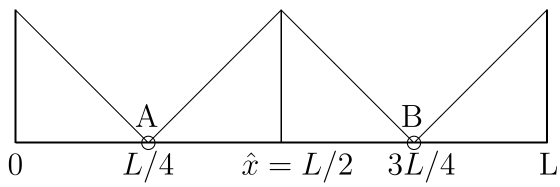
Moving towards the other firm will increase one's profits. In approximation, the two firms will end up locating at the mid-point $\frac{L}{2}$. In this model the differentiation is minimized in equilibrium.

Note: 1. In this model, we can not invert the demand function to define a quantity competition game because the Jocobian is singular.
2. The degree of homogeneity $\delta = \frac{\gamma}{\beta}$ is not definable either. Here we use the distance between the locations of A and B as a measure of differentiation.
3. The result that firms in the Hotelling model will choose to minimize product differentiation is so far only an approximation because the location of the marginal consumer $\hat{x}$ in (7) is not exactly described. It is actually an upper-semi continuous correspondence of (P_A, P_B). The reaction functions are discontinuous and the price competition equilibrium does not exist when the two firms are too close to each other. See Oz Shy's Appendix 7.5.

Welfare index: aggregate transportation costs



Equilibrium transportation cost curve



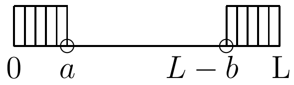
Social optimum transportation cost curve

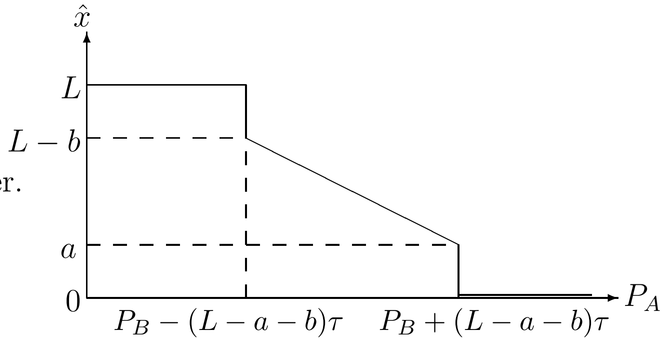
7.4.3 Digression: the exact profit function

In deriving the market demand, we regard the location of the marginal consumer $\hat{x}$ as the market dividing point. This is correct only when $a < \hat{x} < L - b$. When $\hat{x} \leq a$, all the consumers buy from B, $Q_A = 0$ and $Q_B = L$. The reason is, if the consumer located at a (the location of firm A) prefers to buy from firm B, since the transportation cost is linear, all consumers to the left of a would also prefer to buy from B. Similarily, when $\hat{x} \geq L - b$, all the consumers buy from A, $Q_A = L$ and $Q_B = 0$.

Consumers in $[0,a]$ move together.

Consumers in $[L-b,L]$ move together.





The true profit function of firm A is

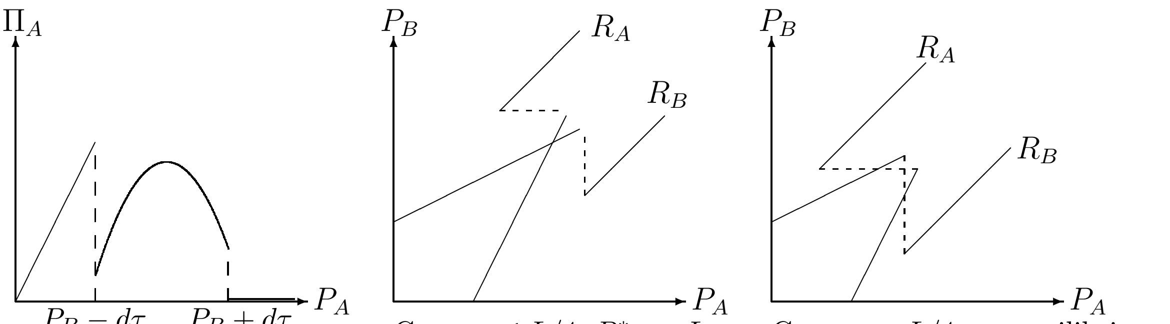
$$\Pi_A(P_A,P_B)=\begin{cases} P_A\left[\dfrac{L-b+a}{2}-\dfrac{P_A-P_B}{2\tau}\right] & a<\hat{x}<L-b \\ 0 & \hat{x}\le a \\ P_AL & L-b\ge\hat{x} \end{cases}$$

$$=\begin{cases} P_AL & P_A\le P_B-d\tau \\ P_A\left[\dfrac{L-b+a}{2}-\dfrac{P_A-P_B}{2\tau}\right] & P_B-d\tau<P_A<P_B+d\tau \\ 0 & P_A\ge P_B+d\tau, \end{cases}$$

where $d\equiv L-a-b$. Consider the case $a=b$, $d=L-2a$. The reaction function of firm A is (firm B's is similar):

$$P_A=R_A(P_B)=\begin{cases} 0.5(\tau L+P_B) & \frac{(\tau L+P_B)^2}{8\tau[P_B-d\tau]}\le L \text{ or } \frac{P_B}{\tau}\notin(3L-4\sqrt{La},3L+4\sqrt{La}) \\ P_B-(L-2a)\tau & \frac{(\tau L+P_B)^2}{8\tau[P_B-d\tau]}\ge L \text{ or } \frac{P_B}{\tau}\in[3L-4\sqrt{La},3L+4\sqrt{La}] \end{cases}$$

For $a\le L/4$, the reaction functions intersect at $P_A=P_B=\tau L$. When $a>L/4$, the reaction functions do not intersect.



Case: $a\le L/4$, $P^*=\tau L$, Case: $a>L/4$, no equilibrium.

7.4.4 Quadratic transportation costs

Suppose now that the transportation cost is proportional to the square of the distance.

$$U_x=\begin{cases} -P_A-\tau(x-a)^2 & \text{if x buys from A.} \\ -P_B-\tau[x-(L-b)]^2 & \text{if x buys from A.} \end{cases}$$

The marginal consumer $\hat{x}$ is defined by

$$-P_A - \tau(x-a)^2 = -P_B - \tau[x-(L-b)]^2 \Rightarrow \hat{x} = \frac{L-b+a}{2} - \frac{P_A - P_B}{2\tau(L-a-b)}. \quad (3)$$

The demand functions of firms A and B are

$$D_A(P_A, P_B) = \hat{x} = \frac{L-b+a}{2} - \frac{P_A - P_B}{2\tau(L-a-b)}, \; D_B(P_A, P_B) = L - \hat{x} = \frac{L-a+b}{2} + \frac{P_A - P_B}{2\tau(L-a-b)}.$$

The payoff functions are

$$\Pi_A(P_A, P_B) = P_A \left[\frac{L-b+a}{2} - \frac{P_A - P_B}{2\tau(L-a-b)}\right], \; \Pi_B(P_A, P_B) = P_B \left[\frac{L-a+b}{2} + \frac{P_A - P_B}{2\tau(L-a-b)}\right].$$

The FOCs are (the SOCs are satisfied)

$$\frac{L-b+a}{2} - \frac{2P_A - P_B}{2\tau(L-a-b)} = 0, \quad \frac{L-a+b}{2} + \frac{P_A - 2P_B}{2\tau(L-a-b)} = 0. \quad (4)$$

The equilibrium is given by

$$P_A = \frac{\tau(3L-b+a)(L-a-b)}{3}, \; Q_A = \hat{x} = \frac{3L-b+a}{6}, \Pi_A = \frac{\tau(3L-b+a)^2(L-a-b)}{18},$$

$$P_B = \frac{\tau(3L-a+b)(L-a-b)}{3}, \; Q_B = L - \hat{x} = \frac{3L-a+b}{6}, \Pi_B = \frac{\tau(3L-a+b)^2(L-a-b)}{18}.$$

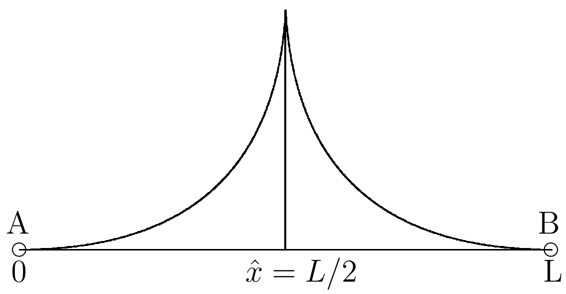
$$\frac{\partial \Pi_A}{\partial a} < 0, \quad \frac{\partial \Pi_B}{\partial b} < 0.$$

Therefore, both firms will choose to maximize their distance. The result is opposite to the linear transportation case.

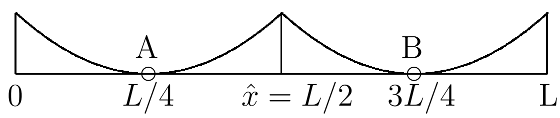
Two effects of increasing distance:
1. Increase the differentiation and reduce competition. $\Pi \uparrow$.
2. Reduce a firm's turf. $\Pi \downarrow$.

In the linear transportation case, the 2nd effect dominates. In the quadratic transportation case, the 1st effect dominates.
Also, $\hat{x}$ is differentiable in the quadratic case and the interior solution to the profit maximization problem is the global maximum.

Welfare comparison:



Equilibrium transportation cost curve

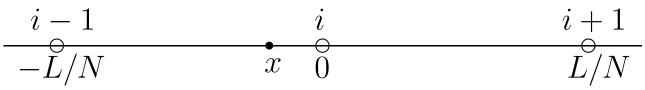


Social optimum transportation cost curve

7.5 Circular Market Model

It is very difficult to generalize the linear city model to more than 2 firms. Alternatively, we can assume that consumers are uniformly distributed on the circumference of a round lake. We assume further that the length of the circumference is L. First, we assume that there are N firms their locations are also uniformly distributed along the circumference. Then we will find the equilibrium number of firms if entry/exit is allowed.

Firm i faces two competing neighbers, firms $i-1$ and $i+1$. $\mathrm{TC}_i(Q_i) = F + cQ_i$.

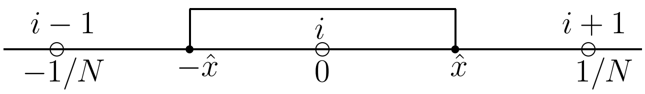


As in the linear city model, each consumer needs 1 unit of the product. The utility of consumer x, if he buys from firm j, is

$$U(x) = -P_j - \tau|x - l_j|, \quad j = i-1, i, i+1,$$

where l_j is the location of firm j. To simplify, let $L = 1$. Given the prices P_i and $P_{i-1} = P_{i+1} = P$, there are two marginal consumers $\hat{x}$ and $-\hat{x}$ with

$$\hat{x} = \frac{P - P_i}{2\tau} + \frac{1}{2N}, \quad \Rightarrow Q_i = 2\hat{x} = \frac{P - P_i}{\tau} + \frac{1}{N}.$$



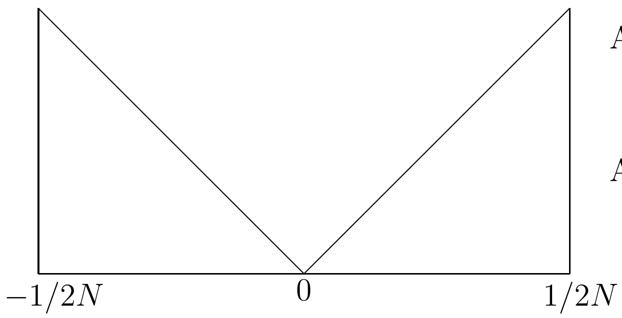
$$\Pi_i = (P_i - c)\left[\frac{P - P_i}{\tau} + \frac{1}{N}\right] - F, \quad \Rightarrow \text{ FOC: } \frac{c + P - 2P_i}{\tau} + \frac{1}{N} = 0.$$

In equilibrium, $P_i = P$,

$$P = c + \frac{\tau}{N}, \quad \Pi = \frac{\tau}{N^2} - F.$$

In a free entry/exit long run equilibrium, N is such that

$$\Pi(N) \geq 0,\ \Pi(N+1) \leq 0 \Rightarrow N^* = \left[\sqrt{\frac{\tau}{F}}\right],\ P^* = c + \sqrt{\tau F},\ Q^* = \frac{1}{N}.$$



Equilibrium transportation cost curve

Aggregate transportation costs:

$$T(N) = N\left(\frac{1}{2N}\frac{\tau}{2N}\right) = \frac{\tau}{4N}.$$

Aggregate Fixed Costs $= NF$.

Since each consumer needs 1 unit, TVC $= c$ is a constant. The total cost to the society is the sum of aggregate transportation cost, TVC, and fixed cost.

$$\text{SC}(N) = T(N) + c + NF = \frac{\tau}{4N} + c + NF, \quad \text{FOC: } -\frac{\tau}{4N^2} + F = 0, \; \Rightarrow N^s = 0.5\sqrt{\frac{\tau}{F}}.$$

The conclusion is that the equilibrium number of firms is twice the social optimum number.

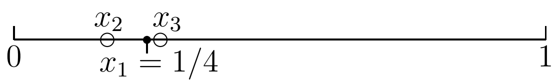
Remarks: 1. AC is decreasing (DRTS), less firms will save production cost.
2. $T(N)$ is decreasing with N, more firms will save consumers' transportation cost.
3. The monopolistic competition equilibrium ends up with too many firms.

7.6 Sequential entry in the linear city model

If firm 1 chooses his location before firm 2, then clearly firm 1 will choose $l_i = L/2$ and firm 2 will choose $l_2 = L/2^+$. If there are more than two firms, the Nash equilibrium location choices become much more complicated.

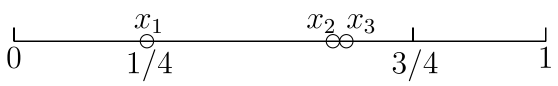
Assume that there are 3 firms and they enter the market (select locations) sequentially. That is, firm 1 chooses x_1, then firm 2 chooses x_2, and finally firm 3 chooses x_3. To simplify, we assume that firms charge the same price $p = 1$ and that $x_1 = 1/4$. We want to find the equilibrium locations x_2 and x_3.

1. If firm 2 chooses $x_2 \in [0, x_1) = [0, 1/4)$, then firm 3 will choose $x_3 = x_1 + \epsilon$. $\pi_2 = (x_2 - x_1)/2 < 1/4$.



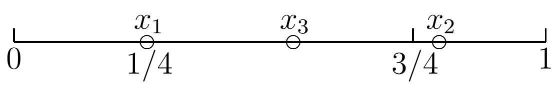
$\pi_2 = (x_2 + x_1)/2 < 1/4.$

2. If firm 2 chooses $x_2 \in (x_1, 3/4) = (1/4, 3/4)$, then firm 3 will choose $x_3 = x_2 + \epsilon$. $\pi_2 = x_2 - (x_2 + x_1)/2 = (x_2 - x_1)/2 < 1/4$.



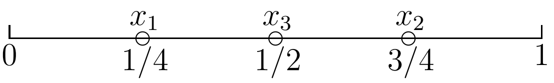
$\pi_2 = (x_2 - x_1)/2 < 1/4.$

3. If firm 2 chooses $x_2 \in [3/4, 1]$, then firm 3 will choose $x_3 = (x_2 + x_1)/2$. $\pi_2 = 1 - 0.5(x_2 + x_3)/2 = (15 - 12x_2)/16$.



$\pi_2 = (15 - 12x_2)/16.$

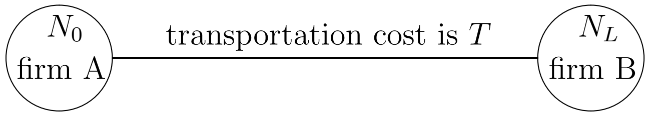
The subgame perfect Nash equilibrium is $x_2 = 3/4$, $x_3 = 1/2$ with $\pi_1 = \pi_2 = 3/8$ and $\pi_3 = 1/4$.



SPE: $\pi_1 = \pi_2 = 3/8,\ \pi_3 = 1/4$.

7.7 Discrete location model

Consider now that the consumers are concentrated on two points:



N_0 consumers live in city 0 where firm A is located.
N_L consumers live in city L where firm B is located.
The round trip transportation cost from city 0 to city L is T.
Given the prices (P_A, P_B), the utility of a consumer is

$$U_0 = \begin{cases} -P_A \\ -P_B - T \end{cases} \qquad U_L = \begin{cases} -P_A - T \\ -P_B. \end{cases}$$

n_A (n_B) is the number of firm A's (firm B's) consumers.

$$n_A = \begin{cases} 0 & P_A > P_B + T \\ N_0 & P_B - T < P_A < P_B + T \\ N_0 + N_L & P_A < P_B + T \end{cases} \qquad n_B = \begin{cases} 0 & P_B > P_A + T \\ N_L & P_A - T < P_B < P_A + T \\ N_0 + N_L & P_B < P_A + T. \end{cases}$$

Nash Equilibrium: (P_A^n, P_B^n) such that P_A^n maximizes $\Pi_A = P_A n_A$ and P_B^n maximizes $\Pi_B = P_B n_B$.

Proposition: There does not exist a Nash equilibrium.

Proof: 1. If $P_A^n - P_B^n > T$, then $\Pi_A = 0$, firm A will reduce P_A. Similarily for $P_B^n - P_A^n > T$.
2. If $|P_A^n - P_B^n| < T$, then firm A will increase P_A.
3. If $|P_A^n - P_B^n| = T$, then both firms will reduce their prices.

Undercut proof equilibrium (UE):$(P_A^u, P_B^u, n_A^u, n_B^u)$ such that
1. P_A^n maximizes Π_A subject to $\Pi_B = P_B^u n_B^u \geq (N_0 + N_L)(P_A^u - T)$.
2. P_B^n maximizes Π_B subject to $\Pi_A = P_A^u n_A^u \geq (N_0 + N_L)(P_B^u - T)$.

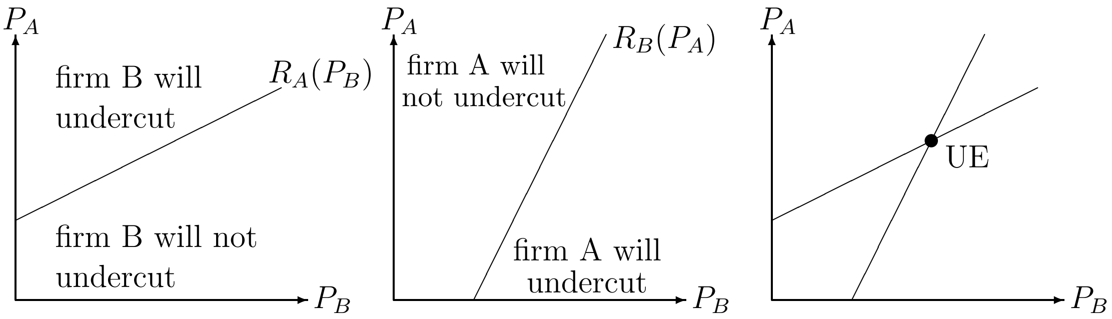
In choosing P_A, firm A believes that if P_A is too high, firm B will undercut its price to grab A's consumers and vice versa.

There is a undercut proof equilibrium: $n_A^u = N_0$, $n_B^u = N_L$,and (P_A^u, P_B^u) satisfies

$$P_B^u N_L = (N_0 + N_L)(P_A^u - T), \quad P_A^u N_0 = (N_0 + N_L)(P_B^u - T).$$

$$\Delta P = P_B^u - P_A^u = \frac{(N_0 + N_L)(N_0 - N_L)T}{N_0^2 + N_L^2 + N_0 N_L}, \ \Delta P \gtrless 0 \ \text{ if } \ N_0 \gtrless N_L.$$

In the symmetric case, $N_0 = N_L$ and $P_A^u = P_B^u = 2T$.



8 Concentration, Mergers, and Entry Barriers

產業生態: 新廠加入; 舊廠退出; 廠商合併; 廠商分解。
新興產業, 成熟產業, 夕陽產業。

集中度與利潤之關係;
政府對集中度的管制; 1. 直接干預: 強制分解, 審核廠商合併
2. 禁止舊廠採取進場阻擾手段

8.1 Concentration Measures

We want to define some measures of the degree of concentration of an industry in order to compare different industries or a similar industry in different countries.

$i = 1, 2, \ldots, N, \quad Q = q_1 + q_2 + \cdots + q_N.$

Market shares: $s_i \equiv \dfrac{q_i}{Q} \times 100\%$, $s_1 \geq s_2 \geq s_3 \geq \cdots \geq s_N$.

$I_4 \equiv s_1 + s_2 + s_3 + s_4, \qquad I_8 \equiv \sum_{i=1}^{8} s_i.$

Herfindahl-Hirshman Index: $I_{HH} \equiv \sum_{i=1}^{N} s_i^2$.

Gini coefficient: $G \equiv \dfrac{1}{N} \sum_{j=1}^{N} \sum_{i=1}^{j} s_i = \dfrac{1}{N} \sum_{1}^{N} (N-i) s_i$.

Entropy: $I_E \equiv \sum_{1}^{N} s_i \ln s_i$.

8.1.1 Relationship between I_{HH} and Lerner index

Consider a quantity competition oligopoly industry. $P(Q) = P(q_1 + q_2 + \cdots + q_N)$ is the market demand. λ_i is the conjecture variation of firm i, i.e., when firm i increases 1 unit of output, he expects that all other firms together will response by increasing λ_i units of output.

$$\pi_i = P(Q) q_i - c_i(q_i), \quad Q_{-i} \equiv \sum_{j \neq i} q_j, \;\; \lambda_i \equiv \frac{dQ_{-i}^e}{dq_i}.$$

FOC: $\dfrac{\partial \Pi_i^e}{\partial q_i} = \dfrac{\partial \pi i}{dq_i} + \dfrac{\partial \pi i}{dQ_{-i}} \dfrac{dQ_{-i}^e}{dq_i}$ or $\qquad P + q_i P'(1 + \lambda_i) = c_i'$.

Lerner index of firm i, L_i:

$$L_i \equiv \frac{P - c_i'}{P} = \frac{-q_i P'(1+\lambda_i)}{P} = \frac{-QP'}{P} \frac{q_i}{Q} (1 + \lambda_i) = \frac{1}{\epsilon_{QP}} s_i (1 + \lambda_i).$$

Industry average lerner index L:

$$L \equiv \sum_{i=1}^{N} s_i L_i = \frac{1}{\epsilon_{QP}} \sum_{i=1}^{N} s_i^2 (1 + \lambda_i) = \frac{I_{HH} + \sum_i s_i^2 \lambda_i}{\epsilon_{QP}}.$$

If $\lambda_i = \lambda$ for all i, then $L = \dfrac{I_{HH}(1+\lambda)}{\epsilon_{PQ}}$.

The case of collusion $\lambda_i = \dfrac{Q_{-i}}{q_i}$: FOC is MR = MC$_i$ and $L_i = L = \dfrac{1}{\epsilon_{PQ}}$.

8.2 Mergers

Mergers, takeovers, acquisitions, integration.

3 types:
Horizontal mergers: between the same industry
Vertical mergers: between upstream industry firms and down stream industry firms
Conglomerate mergers: other cases.

In US economic history there were 4 active periods:
1901: mostly horizontal and vertical
1920, 1968, 1980: other types, mostly influenced by changes in Anti-trust Law.

Purpose: (1) reduce competition, (2) IRTS, (3) differences in the prospective of firms between sellers and buyers,
(4) managers' intension to enlarge their own careers, (5) the insterests of the promoters.

8.2.1 Horizontal merger

合併 $\rightarrow$ concentration ratio $\uparrow$ $\rightarrow$ competition $\downarrow$ $\rightarrow$ Welfare $\downarrow$?
Not necessary. If high cost (inefficient) firms are taken-over, efficiency increases.

Example: In Cournot duopoly model, assume $c_1 = 1$, $c_2 = 4$, and $P = 10 - Q$.

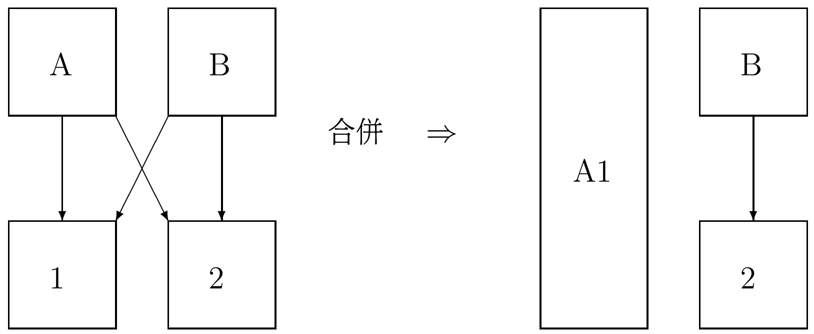
$$q_1^c = 4,\ q_2^c = 1,\ P^c = 5,\ \pi_1^c = 16,\ \pi_2^c = 1,\ \text{CS} = 12.5,\ W^c = 29.5.$$

If firms 1 and 2 merge to become a monopoly, the monopoly would shut down the production of firm 2 so that the MC of the monopoly is $c = 1$.

$$Q^m = 4.5,\ P^m = 5.5,\ \pi^m = 20.25,\ \text{CS} = 10.125,\ W^m = 30.375 > W^c.$$

Comparison of Welfare: $W^m > W^c$, it is the trade-off between the production efficiency and monopoly inefficiency.
Comparison of I_{HH}: $I^c_{HH} = 6,800$, $I^m_{HH} = 10,000$.
According to Anti-trust law, such a merger is prohibited, but it is good to the society.
However, if the market is originally in Bertrand price competition, the conclusion is totally different. In a Bertrand competition market, the industry is efficient.

8.2.2 Vertical merger



If both upstream and downstream industries produce homogeneous products and are Bertrand price competition markets, the merger does not affect anything.

Assumption: The upstream was originally in Bertrand competition and the downstream was in Cournot competition.
Downsteam market demand: $P = \alpha - q_1 - q_2$. $\quad \text{MC}_1 = c_1$, $\text{MC}_2 = c_2$.

$$\Rightarrow q_i = \frac{\alpha - 2c_i + c_j}{3},\ \pi_i = \frac{(\alpha - 2c_i + c_j)^2}{9}; \quad Q = \frac{2\alpha - c_1 - c_2}{3},\ P = \alpha - Q = \frac{\alpha + c_1 + c_2}{3}.$$

Upstream: Assume $\text{MC}_A = \text{MC}_B = 0$. $\quad$ Bertrand equilibrium: $p_A = p_B = c_1 = c_2 = 0$.

Pre-merge Equilibrium:

$$q_1 = q_2 = \frac{\alpha}{3},\ \pi_1 = \pi_2 = \frac{\alpha^2}{9},\ \pi_A = \pi_B = 0; \quad P = \frac{\alpha}{3},\ Q = \frac{2\alpha}{3}.$$

Post-merge: Assume that A1 does not sell raw material to 2. B becomes an upstream monopoly. We ignore the fact that 2 is also a downstream monopsony.

$$\pi_B = c_2 q_2 = \frac{c_2(\alpha - 2c_2 + c_1)}{3} = \frac{p_B(\alpha - 2p_B)}{3}, \quad \max_{p_B} \pi_B \Rightarrow p_B = c_2 = \frac{\alpha}{4}.$$

Post-merge Equilibrium:

$$q_{A1} = \frac{5\alpha}{12},\ q_2 = \frac{\alpha}{6},\ P = \frac{5\alpha}{12},\ Q = \frac{7\alpha}{12},\ \pi_{A1} = Pq_{A1} = \frac{25\alpha^2}{144},\ = \pi_2 = \frac{\alpha^2}{36},\ \pi_B = P_B q_2 = \frac{\alpha^2}{24}.$$

The effects of merge: $P \uparrow$, $q_1 \uparrow$, $q_2 \downarrow$, $\pi_2 \downarrow$, $\pi_B \uparrow$, $\pi_A + \pi_1 \uparrow$, $\pi_B + \pi_2 \downarrow$.
Firm 2 and consumers are the losers.

8.2.3 merger of firms producing complementary goods

Firm X produces PCs and Firm Y produces monitors.
A system is S = X + Y. $\quad P_s = P_x + P_y$.

Market demand: $Q_s = \alpha - P_s = \alpha - P_x - P_y$, $Q_x = Q_y = Q_s$.
Pre-merge:

$$\Pi_x = P_x Q_x = P_x(\alpha - P_x - P_y), \quad \Pi_y = P_y Q_y = P_y(\alpha - P_x - P_y).$$

FOC: $\alpha - 2P_x - P_y = 0$, $\alpha - P_x - 2P_y = 0$.

$$\Rightarrow P_x = P_y = \frac{\alpha}{3}, \quad Q_s = Q_x = Q_y = \frac{\alpha}{3}, \quad \Pi_x = \Pi_y = \frac{\alpha^2}{9}.$$

Post-merge:

$$\Pi_s = P_s Q_s = P_s(\alpha - P_s), \Rightarrow P_s = Q_s = \frac{\alpha}{2}, \ \Pi_s = \frac{\alpha^2}{4}.$$

The effects of merger:

$$P_s = \frac{\alpha}{2} < P_x + P_y = \frac{2\alpha}{3}, \ Q_s = \frac{\alpha}{2} > Q_x = Q_y = \frac{\alpha}{3}, \ \Pi_s = \frac{\alpha^2}{4} > \Pi_x + \Pi_y = \frac{2\alpha^2}{9}.$$

Therefore, one monopoly is better than two monopolies.

Remarks: 1. It is similar to the joint product monopoly situation. When a monopoly produces two complementary goods, the profit percentage should be lower than individual Lerner indices. In this case, a higher P_x will reduce the demand for Y and vice versa. After merger, the new firm internalizes these effects.
2. The model here is isomorphic to the Cournot duopoly model with price variables and quantity variables interchanged.
3. If there are 3 products, X, Y, and Z, and only X and Y merge, the welfare is not necessarily improving.

8.3 Entry Barriers 新廠進入的阻力

現有廠商 (Incumbent) 的優勢 <==> 新廠進入的阻力 ==> 超額利潤

1. Economy of scale, large fixed cost
2. Production differentiation advantages (reputation, good will)
3. Consumer loyalty, network externalities
4. Absolute cost advantages (learning experiences)
5. Location advantage (sequential entry)
6. Other advantages

Incumbents may also take entry deterrence (進場干擾) strategies.

8.3.1 Fixed cost and I_{HH}

In Dixit-Stiglitz monopolistic competition model, $N = \dfrac{(1-\alpha)I}{F}$:

$$I_{HH} = N\left(\frac{100}{N}\right)^2 = \frac{10,000}{N} = \frac{F}{(1-\alpha)}10,000, \quad \frac{\partial I_{HH}}{\partial F} > 0.$$

In quantity competition with free entry/exit model, $N \approx \dfrac{(A-c)^2}{\sqrt{bF}}$:

$$I_{HH} = \frac{10,000}{N} = \frac{\sqrt{bF}}{A-c}10,000, \quad \frac{\partial I_{HH}}{\partial F} > 0.$$

In the circular city model, $N = \sqrt{\dfrac{\tau}{F}}$, $I_{HH} = \dfrac{F}{\tau}10,000$.

8.3.2 Sunk cost

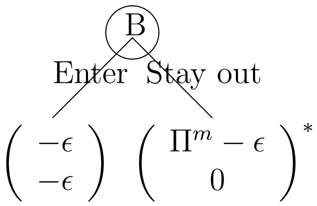
Sunk costs: Costs that cannot be reversed. 開辦費, 廣告費, 裝潢, firm specific equipments, etc.
Sunk costs 愈大, 新廠愈不敢進場競爭。

Stiglitz (1987): 只要有一點 Sunk Cost 存在, 即可使新廠商 (Potential Entrants) 裹足不前, 使現有廠商 (Incumberts) 能繼續賺到獨占利潤。

A: An Incumbent, B: A Potential entrant.

$$\Pi^A = \begin{cases} \Pi^m - \epsilon & \text{if no entry} \\ -\epsilon & \text{B enters,} \end{cases} \qquad \Pi^B = \begin{cases} 0 & \text{do not enter} \\ -\epsilon & \text{B enters,} \end{cases}$$

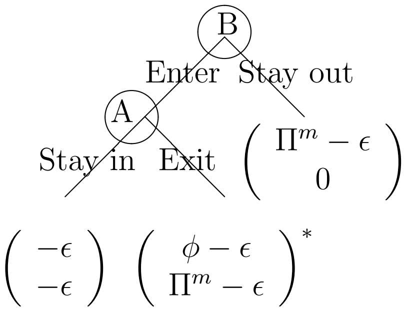
where Π^m is the monopoly profit and ϵ is the sunk cost.



Proposition: As long as $0 < \epsilon < \Pi^m$, there exists only one subgame perfect equilibrium, i.e., B stays out.
Conditions: 1. A and B produce homogeneous product with identical marginal cost.
2. Post-entry market is a Bertrand duopoly.
3. A cannot retreat.

If B can invest in product differentiation to avoid homogeneous product Bertrand competition or the post-entry market is a Cournot duopoly, then the proposition is not valid any more.

If A can resale some of its investments, say, recover $\phi > 0$. The game becomes

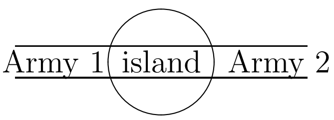


The only subgame-perfect equilibrium is that B enters and A exits. However, the result is just a new monopoly replacing an old one. The sunk cost ϵ can be regarded as the entry barrier as before.
Notice that there is a non-perfect equilibrium in which A chooses the incredible threat strategy of Stay in and B chooses Stay out.

8.4 Entry Deterrence

8.4.1 Burning one's bridge strategy (Tirole CH8)

Two countries wishing to occupy an island located between their countries and connected by a bridge to both. Each army prefers letting its opponent have the island to fighting. Army 1 occupies the island and burns the bridge behind it. This is the paradox of commitment.



8.4.2 Simultaneous vs. Sequential Games

Consider a 2-person game:

$$\Pi_1(x_1, x_2, y_1, y_2), \quad \Pi_2(x_1, x_2, y_1, y_2),$$

where (x_1, y_1) is firm 1's strategy variables and (x_2, y_2) is firm 2's strategy variables.

Simultaneous game: Both firms choose (x, y) simultaneously.
FOC: $\dfrac{\partial \Pi_1}{\partial x_1} = \dfrac{\partial \Pi_1}{\partial y_1} = 0, \qquad \dfrac{\partial \Pi_2}{\partial x_2} = \dfrac{\partial \Pi_2}{\partial y_2} = 0.$

Sequential game: In $t = 1$ both firms choose x_1 and x_2 simultaneously and then in $t = 2$ both firms choose y_1 and y_2 simultaneously.
To find a subgame perfect equilibrium, we solve backward:

In $t = 2$, x_1 and x_2 are given, the FOC are

$$\frac{\partial \Pi_1}{\partial y_1} = 0 \text{ and } \frac{\partial \Pi_2}{\partial y_2} = 0, \quad \Rightarrow y_1 = f(x_1, x_2) \text{ and } y_2 = g(x_1, x_2).$$

In $t = 1$, the reduced game is:

$$\pi_1(x_1, x_2) = \Pi_1(x_1, x_2, f(x_1, x_2), g(x_1, x_2)), \quad \pi_2(x_1, x_2) = \Pi_2(x_1, x_2, f(x_1, x_2), g(x_1, x_2)).$$

The FOC is

$$\frac{\partial \pi_1}{\partial x_1} = \frac{\partial \Pi_1}{\partial x_1} + \frac{\partial \Pi_1}{\partial y_1}\frac{\partial f}{\partial x_1} + \frac{\partial \Pi_1}{\partial y_2}\frac{\partial g}{\partial x_1} = 0, \text{ and } \frac{\partial \pi_2}{\partial x_2} = \frac{\partial \Pi_2}{\partial x_2} + \frac{\partial \Pi_2}{\partial y_1}\frac{\partial f}{\partial x_2} + \frac{\partial \Pi_2}{\partial y_2}\frac{\partial g}{\partial x_2} = 0.$$

Since $\frac{\partial \Pi_1}{\partial y_1} = \frac{\partial \Pi_2}{\partial y_2} = 0$, the FOC becomes

$$\frac{\partial \pi_1}{\partial x_1} = \frac{\partial \Pi_1}{\partial x_1} + \frac{\partial \Pi_1}{\partial y_2}\frac{\partial g}{\partial x_1} = 0, \text{ and } \frac{\partial \pi_2}{\partial x_2} = \frac{\partial \Pi_2}{\partial x_2} + \frac{\partial \Pi_2}{\partial y_1}\frac{\partial f}{\partial x_2} = 0.$$

Compare the FOC of the sequential game with that of the simultaneous game, we can see that the equilibria are not the same. In $t = 1$, both firms try to influence the $t = 2$ decisions of the other firms. For the simultaneous game, there is no such considerations.
$\frac{\partial \Pi_1}{\partial y_2}\frac{\partial g}{\partial x_1}$ and $\frac{\partial \Pi_2}{\partial y_1}\frac{\partial f}{\partial x_2}$ are the strategic consideration terms.

Entry deterrence application: In $t = 1$, only firm 1 exists:

$$\Pi_1(x_1, y_1, y_2), \quad \Pi_2(x_1, y_1, y_2).$$

Firm 1 is the incumbent and tries to influence the entry decision of firm 2.

8.4.3 Spence (1977) entry deterrence model

In this model the incumbent attempts to use strategic capacity investment to deter the entry of a potential entrant.

$t = 1$: Firm 1 decides its capacity-output level k_1.
$t = 2$: Firm 2 decides its capacity-output level k_2.
If $k_2 > 0$, firm 2 enters. If $k_2 = 0$, firm 2 stays out.

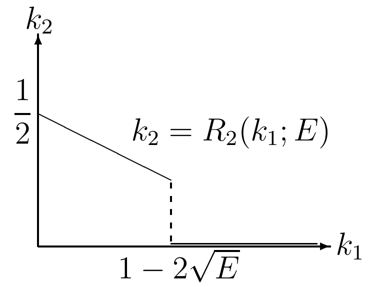
$$\pi_1(k_1, k_2) = k_1(1 - k_1 - k_2), \quad \pi_2(k_1, k_2) = \begin{cases} k_2(1 - k_1 - k_2) - E & \text{enter} \\ 0 & \text{stay out,} \end{cases}$$

where E is the fixed cost if firm 2 enters.
Back induction: In $t = 2$, k_1 is given. If firm 2 enters, its profit maximization FOC is

$$\frac{\partial \pi_2}{\partial k_2} = 1 - k_1 - 2k_2 = 0, \Rightarrow k_2 = \frac{1 - k_1}{2}, \ \pi_2 = \frac{1 - k_1}{2}\left(1 - k_1 - \frac{1 - k_1}{2}\right) - E = \frac{(1 - k_1)^2}{4} - E.$$

If $\dfrac{(1-k_1)^2}{4} - E < 0$, firm 2 will choose not to enter. Therefore, firm 2's true reaction function is

$$k_2 = R_2(k_1, E) = \begin{cases} \dfrac{1-k_1}{2} & \text{if } k_1 < 1 - 2\sqrt{E} \\ 0 & \text{if } k_1 > 1 - 2\sqrt{E}. \end{cases}$$



In $t = 1$, firm 1 takes into consideration firm 2's discontinuous reaction function. If $1 - 2\sqrt{E} \le \frac{1}{2}$ ($\Rightarrow E \ge \frac{1}{16}$), then firm 1 will choose monopoly output $k_1 = \frac{1}{2}$ and firm 2 will stay out. This is the case of **entry blockaded**.

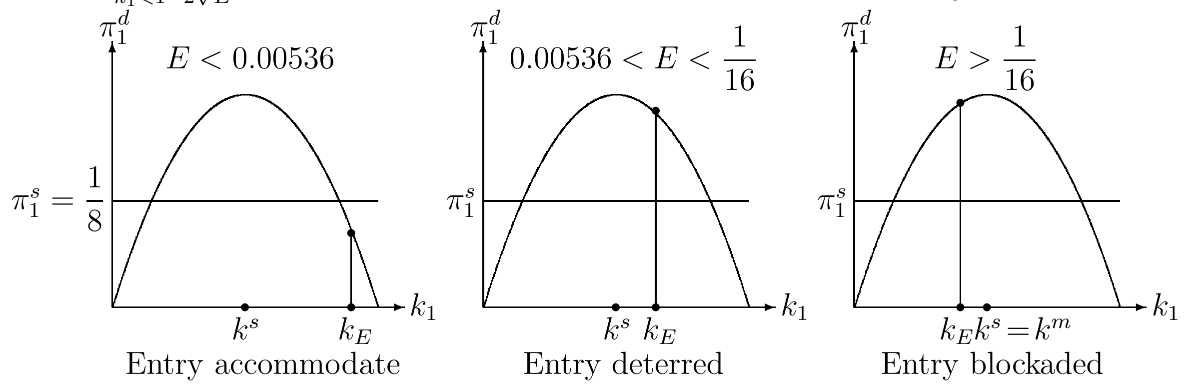
Next, we consider the case $E < \frac{1}{16}$. If firm 1 chooses monopoly output, firm 2 will enter. Firm 1 is considering whether to choose $k_1 \ge 1 - 2\sqrt{E}$ to force firm 2 to give up or to choose $k_1 < 1 - 2\sqrt{E}$ and maximizes duopoly profit.

1. **entry deterrence:** If firm 2 stays out ($k_1 \ge 1 - 2\sqrt{E}$ and $k_2 = 0$), firm 1 is a monopoly by deterrence:

$$\pi_1^d(E) = \max_{k_1 \ge 1-2\sqrt{E}} k_1(1-k_1) = k_E(1-k_E), \quad \text{where } k_E \equiv 1 - 2\sqrt{E}.$$

2. **entry accommodate:** If firm 2 enters ($k_1 < 1 - 2\sqrt{E}$ and $k_2 > 0$), firm 1 is the leader of the Stackelberg game:

$$\pi_1^s(E) = \max_{k_1 < 1-2\sqrt{E}} k_1(1-k_1-k_2) = \frac{1}{2}k_1(1-k_1), \quad \Rightarrow \pi_1^s = \frac{1}{2}k^s(1-k^s) = \frac{1}{8} \text{ where } k^s \equiv \frac{1}{2}.$$



$$\pi_1^d(E) = (1-2\sqrt{E})[1-(1-2\sqrt{E})] \gtrless \pi_1^s(E) = \frac{1}{8} \text{ if } E \gtrless \frac{(1-\sqrt{1/2})^2}{16} \approx 0.00536.$$

In summary, if $E < 0.0536$, firm 1 will accommodate firm 2's entry; if $0.00536 < E < \frac{1}{16}$, firm 1 will choose $k_1 = k_E$ to deter firm 2; if $E > F116$, firm 1 is a monopoly and firm 2's entry is blockaded.

Spence model is built on the so called Bain-Sylos style assumptions:
1. Firm 2 (entrant) believes that firm 1 (incumbent) will produce $q_1 = k_1$ after firm 2's entry is deterred. However, it is not optimal for firm 1 to do so. Therefore, the equilibrium is not subgame perfect.
2. Firm 2 has sunk costs but not firm 1. The model is not symmetrical.

8.4.4 Friedman and Dixit's criticism

1. Incumbent's pre-entry investment should have no effects on the post-entry market competition. The post-entry equilibrium should be determined by post-entry market structure.
2. Therefore, firm 2 should not be deterred.
3. Firm 1's commitment of $q_1 = k_1$ is not reliable. Also firm 2 can make commitment to threat firm 1. The first-mover advantage does not necessarily belong to incumbent.

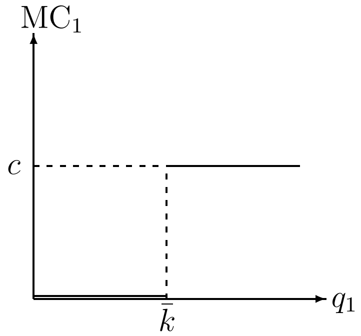
8.4.5 Dixit 1980

If firm 2 is not convinced that $k_1 = q_1$ if firm 2 enters, then firm 1 cannot choose k_E to deter firm 2's entry. Consider a 2-period model:

$t = 1$: Firm 1 chooses $\bar{k}$.
$t = 2$ (Cournot competition): Firms 1 and 2 determine q_1, q_2 simultaneously.

Firm 1's MC curve in $t = 2$ is

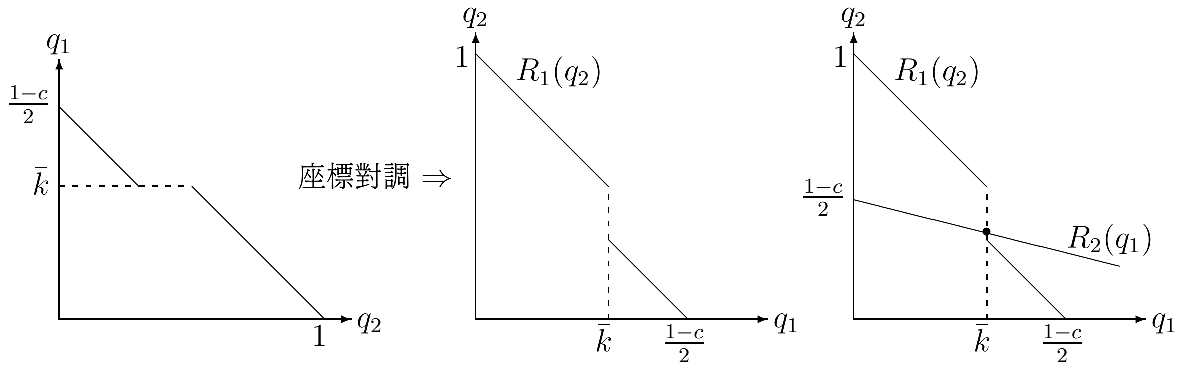
$$\mathrm{MC}_1 = \begin{cases} 0 & \text{if } q_1 \le \bar{k} \\ c & \text{if } q_1 > c \end{cases}$$



In $t = 2$, Firm 1's FOC is $1 - 2q_1 - q_2 = \mathrm{MC}_1$. Its reaction function is

$$q_1 = R_1(q_2) = \begin{cases} \dfrac{1-q_2}{2} & \text{if } q_1 \le \bar{k} \\ \dfrac{1-c-q_2}{2} & \text{if } q_1 > \bar{k} \end{cases} = \begin{cases} (1-q_2)/2 & \text{if } q_2 \ge 1-2\bar{k} \\ \bar{k} & \text{if } 1-c-2\bar{k} < q_2 < 1-2\bar{k} \\ (1-c-q_2)/2 & \text{if } q_2 < 1-c-2\bar{k}. \end{cases}$$

Firm 2's reaction function $R_2(q_1) = \dfrac{1-c-q_1}{2}$ has nothing to do with $\bar{k}$.
Firm 1's reaction function is affected by $\bar{k}$.
Therefore, the Cournot equilibrium in $t = 2$ is affected by $\bar{k}$.

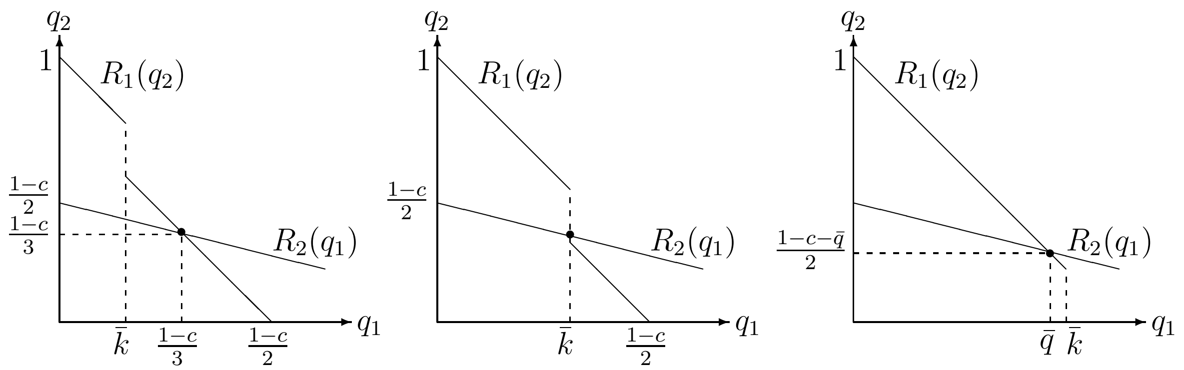


In $t = 1$, firm 1 will choose $\bar{k}$ to affect the Cournot equilibrium in $t = 2$. There are 3 cases:

(1) $\bar{k} \leq q^c \equiv \dfrac{1-c}{3}$: $q_1 = q_2 = q^c$, $\pi_1^* = \pi^c \equiv \dfrac{(1-c)^2}{9}$, i.e., Cournot equilibrium.

(2) $q^c < \bar{k} < \bar{q} \equiv \dfrac{1+c}{3}$: $q_1 = \bar{k}$, $q_2 = \dfrac{1-c-\bar{k}}{2}$, $p = \dfrac{1+c-\bar{k}}{2}$, $\pi_1 = (p-c)q_1 = \dfrac{(1-c-\bar{k})\bar{k}}{2}$.

(3) $\bar{q} \leq \bar{k}$: $q_1 = \bar{q}$, $q_2 = \dfrac{1-c-\bar{q}}{2}$, $p = \dfrac{1+c-\bar{q}}{2}$, $\pi_1^* = p\bar{q} - c\bar{k} = \dfrac{(1-c-\bar{q})\bar{q}}{2} - c(\bar{k} - \bar{q})$.

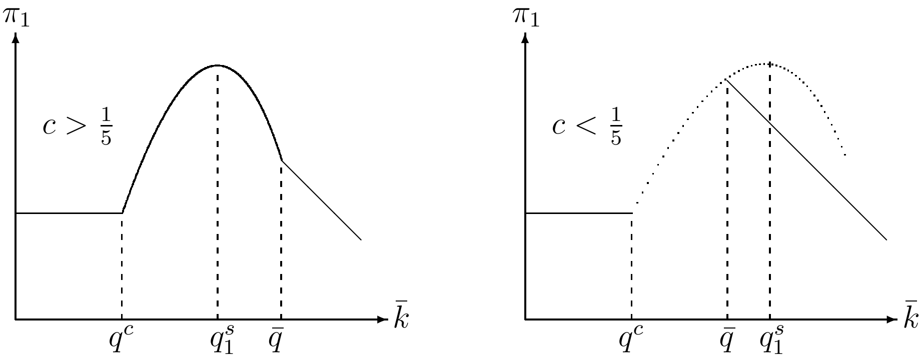


The reduced profit function of firm 1 is

$$\pi_1(\bar{k}) = \begin{cases} \dfrac{(1-c)^2}{9} & \bar{k} \leq q^c \\ \dfrac{(1-c-\bar{k})\bar{k}}{2} & q^c < \bar{k} < \bar{q} \\ \dfrac{(1-c-\bar{q})\bar{q}}{2} - c(\bar{k} - \bar{q}) & \bar{q} \leq \bar{k}. \end{cases}$$

Let q_1^s be the Stackelberg leadership quantity of firm 1: $q_1^s \equiv \dfrac{1-c}{2}$.

1. If $c > \dfrac{1}{5}$, then $q_1^s < \bar{q}$ and firm 1 will choose $\bar{k}^* = q_1^s$.
2. If $c < \dfrac{1}{5}$, then $q_1^s > \bar{q}$ and firm 1 will choose $\bar{k}^* = \bar{q}$.



In Stackelberg model, firm 1 can choose any point on firm 2's reaction curve $R_2(q_1)$ to maximize π_1. In Dixit model, firm 1's choice is restricted to the section of $R_2(q_1)$ such that $\bar{k} \in [0, \bar{q}]$. When $\bar{q} \geq q_1^s$, the result is the same as Stackelberg model. When $\bar{q} < q_1^s$, firm 1 can only choose $\bar{k} = \bar{q}$.

In both cases $q_1 = \bar{k}$. Therefore, firm 1 does not over-invest (choose $\bar{k} > q_1$) to threaten firm 2's entry.

8.4.6 Capital replacement model of Eaton/Lipsey (1980)

It is also possible that an incumbent will replace its capital before the capital is complete depreciated as a commitment to discourage the entry of a potential entrant.

$t = -1, 0, 1, 2, 3, \ldots$.
In each period t, if only one firm has capital, the firm earns monopoly profit H.
If both firms have capital, each earns duopoly profit L.
Each firm can make investment in each period t by paying F.
Denote by R_t^i (C_t^i) the profit (cost) of firm i in period t.

$$\Pi_i = \sum_{t=0}^{\infty} \rho^t (R_t^i - C_t^i), \quad R_t^i = \begin{cases} 0 & \text{no capital} \\ L & \text{duopoly} \\ H & \text{monopoly} \end{cases} \quad C_t^i = \begin{cases} 0 & \text{no invest (NI)} \\ F & \text{invest (INV).} \end{cases}$$

Assumption 1: An investment can be used for 2 periods with no residual value left.
Assumption 2: $2L < F < H$.
Assumption 3: $F/H < \rho < \sqrt{(F-L)/(H-L)}$.

Firm i's strategy in period t is $a_t^i \in \{\text{NI}, \text{INV}\}$. Assume that $a_{-1}^1 = \text{INV}$. We consider only Markov stationary equilibrium.

If firm 2 does not exist, firm 1 will choose to invest (INV) in $t = 1, 3, 5, \ldots$. Given the threat of firm 2's possible entry, in a subgame-perfect equilibrium, firm 1 will invest in every period and firm 2 will not invest forever.

The symmetrical SPE strategy is such that an incumbent firm invests and a potential entrant does not:

$$a_t^i = \begin{cases} \text{INV} & \text{if } a_{t-1}^j = \text{NI} \\ \text{NI} & \text{otherwise.} \end{cases}$$

Proof that the above strategy is optimal if the oppoent plays the same strategy:
$\Pi_1 = \dfrac{H-F}{1-\rho}$, $\Pi_2 = 0$.
If firm 1 deviates and chooses $a_0^1 = \text{NI}$, Π_1 becomes $H < (H-F)/(1-\rho)$ (by Assumption 3), because firm 2 will invest and become the monopoly.
If firm 2 deviates and chooses $a_0^2 = \text{INV}$, then $\Pi_2 = (1+\rho)(L-F) + \dfrac{\rho^2(H-F)}{1-\rho} < 0$ (also by Assumption 3).

8.4.7 Judo economics 小廠進場安身立命之道

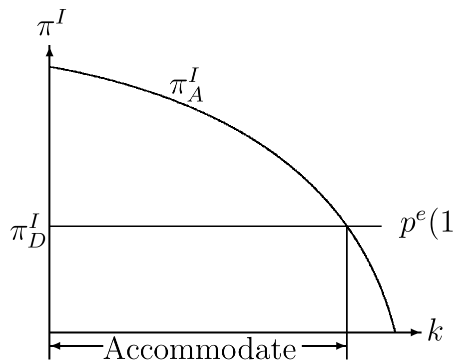
The (inverse) market demand is $P = 100 - Q$.
$t = 1$: Firm 2 (entrant) determines whether to enter and if enters, its capacity level k and price p^e.
$t = 2$: Firm 1 (incumbent) determines its price p^I. Assume that firm 1 has unlimited capacity and, if $p^I = p^e$, all consumers will purchase from firm 1.

$$q^I = \begin{cases} 100 - p^I & p^I \le p^e \\ 100 - k - p^I & p^I > p^e \end{cases} \qquad q^e = \begin{cases} k & p^e < p^I \\ 0 & p^e \ge p^I \end{cases}$$

Backward induction: At $t = 2$, (k, p^e) is given.
1. If firm 1 decides to deter entry, he chooses $p^I = p^e$ and $\pi_D^I = p^e(100 - p^e)$.
2. If firm 1 decides to accommodate firm 2, he chooses to maximize $\pi_A^I = p^I(100 - k - p^I)$.

$$\max_{p^I > p^e} p^I(100-k-p^I), \ \Rightarrow \text{FOC: } 0 = 100-k-2p^I, \ \Rightarrow p_A^I = \frac{100-k}{2} = q_A^I \Rightarrow \pi_A^I = \frac{(100-k)^2}{4}.$$

Firm 1 will accommodate firm 2 if $\pi_A^I > \pi_D^I$, or if $\dfrac{(100-k)^2}{4} \ge p^e(100-p^e)$, whence $\pi^e = p^e k > 0$.
If firm 2 chooses a small k and a large enough p^e, firm 1 will accommodate.



At $t = 1$, firm 2 will choose (k, p^e) such that

$$\max p^e k \text{ subject to } \frac{(100-k)^2}{4} \geq p^e(100-p^e).$$

8.4.8 Credible spatial preemption 強行進場佔有

Suppose that the incumbent is a monopoly in two markets selling substitute products $j = 1, 2$. If an entrant enters into one (say, product 1) of the two markets, the incumbent will give up the market in order to protect the monopoly profit of the other market (product 2).
Reason: If the incumbent stays in market 1, the Bertrand competition will force p_1 down to its marginal cost. As discussed in the monopoly chapter, the demand of product 2 will be reduced.

Example: Suppose that firm 1 has a Chinese restaurant C and a Japanese restaurant J in a small town, both are monopoly.
There are two consumers (assumed to be price takers), c and j.

$$U^c = \begin{cases} \beta - P^C & \text{dine at C} \\ \beta - \lambda - P^J & \text{dine at J,} \end{cases} \quad U^j = \begin{cases} \beta - \lambda - P^C & \text{dine at C} \\ \beta - P^J & \text{dine at J,} \end{cases} \quad \beta > \lambda > 0,$$

β is the utility of dinner and λ is the disutility if one goes to a less preferred restaurant.
Monopoly equilibrium: $P^C = P^J = \beta$, $\pi_1 = 2\beta$.

Suppose now that firm 2 opens a Chinese restaurant in the same town.

1. If firm 1 does not close its Chinese restaurant, the equilibrium will be $P^C = 0$, $P^J = \lambda$, $\pi_1 = \lambda$, $\pi_2 = 0$.
2. If firm 1 closes its Chinese restaurant, the equilibrium will be $P^C = \beta = P^J$, $\pi_1 = \beta = \pi_2$.

The conclusion is that firm 1 will close its Chinese restaurant.

8.5 Contestable Market of Baumol/Panzar/Willig (1982)

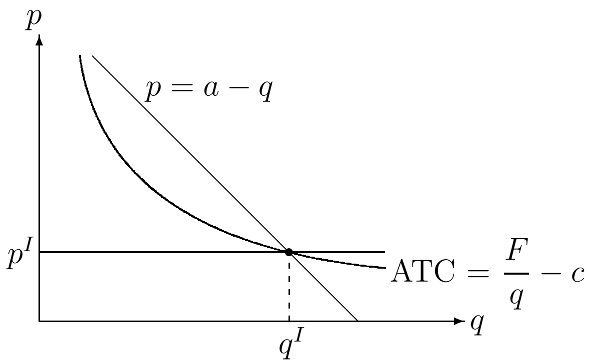
Deregulation trend in US in later 1970s.
Airline industries: Each line is an individual industry.

Contestable market: In certain industries entry does not require any sunk cost. Incumbent firms are constantly faced by threats of hit-and-run entry and hence behave like competitive firms (making normal profits).

Assumption: Potential entrants and an incumbent produce a homogenous product and have the same cost function $\mathrm{TC}(q_i) = F + cq_i$. The market demand is $p = a - Q$.

An industry configuration: (p^I, q^I).
Feasibility: (1) $p^I = a - q^I$. (2) $\pi^I = p^I q^I - (F + cq^I) \geq 0$.
Sustainability: $\nexists (p^e, q^e)$ such that $p^e \leq p^I$, $q^e \leq a - p^e$, $\pi^e = p^e q^e - (F + cq^e) > 0$.

A contestable-market equilibrium: A feasible, sustainable configuration.



Extension: 1. More incumbent firms. 2. More than 1 products.

Comments: 1. If there are sunk costs, the conclusions would be reversed. See Stiglitz (1987) discussed before.
2. If incumbents can respond to hit-and-run entries, they can still make some positive profits.

8.6 A Taxonomy (分類法) of Business Strategies

Bulow, Geanakoplos, and Klemperer (1985), "Multimarket Oligopoly: Strategic Substitutes and Complements," *JPE*.
A 2-period model: Firm 1 chooses K_1 at $t = 1$ and firms 1 and 2 choose x_1, x_2 simultaneously at $t = 2$.

$$\Pi^1 = \Pi^1(K_1, x_1, x_2), \quad \Pi^2 = \Pi^2(K_1, x_1, x_2).$$

The reduced payoff functions at $t = 1$ are

$$\Pi^1 = \Pi^1(K_1, x_1^*(K_1), x_2^*(K_1)), \quad \Pi_2 = \Pi_2(K_1, x_1^*(K_1), x_2^*(K_1),$$

where $x_1^*(K_1)$ and $x_2^*(K_1)$ are the NE at $t = 2$, given K_1, $\dfrac{\partial \Pi^1}{dx_1} = \dfrac{\partial \Pi^2}{dx_2} = 0$.

top dog (霸道, 裝腔作勢): be big or strong to look tough or aggressive.
puppy dog (嬌小, 低姿勢): be small or weak to look soft or inoffensive.
lean and hungry look (裝出吃不飽餓不死, 小氣): be small or weak to look tough or aggressive.
fat cat (故示大方): be big or strong to look soft or inoffensive.

8.6.1 Deterence case

If firm 1 decides to deter firm 2, firm 1 will choose K_1 to make $\Pi^2 = \Pi^2(K_1, x_1^*(K_1), x_2^*(K_1)) = 0$.

$$\frac{d\Pi^2}{dK_1} = \frac{\partial \Pi^2}{\partial K_1} + \frac{\partial \Pi^2}{\partial x_1}\frac{dx_1^*}{dK_1}.$$

Assume that $\partial\Pi^2/\partial K_1 = 0$ so that only strategic effect exists.
$d\Pi^2/dK_1 = \dfrac{\partial \Pi^2}{\partial x_1}\dfrac{dx_1^*}{dK_1} < 0$: The investment makes firm 1 tough.
$d\Pi^2/dK_1 = \dfrac{\partial \Pi^2}{\partial x_1}\dfrac{dx_1^*}{dK_1} > 0$: The investment makes firm 1 soft.

To deter entry, firm 1 will overinvest in case of tough investment (top dog strategy 裝腔作勢的看門狗, entrant 一進來就要咬.)

and underinvest in case of soft investment (lean and hungry look 裝出吃不飽餓不死的樣子, 使 entrant 以爲無利可圖).

8.6.2 Accommodation case

If firm 1 decides to accommodate firm 2, firm 1 considers maximizing $\Pi^1 = \Pi^1(K_1, x_1^*(K_1), x_2^*(K_1))$.

$$\frac{d\Pi^1}{dK_1} = \frac{\partial \Pi^1}{\partial K_1} + \frac{\partial \Pi^1}{\partial x_2}\frac{dx_2^*}{dK_1} = \frac{\partial \Pi^1}{\partial K_1} + \frac{\partial \Pi^1}{\partial x_2}\frac{dx_2^*}{dx_1}\frac{dx_1^*}{dK_1}.$$

Assume that $\partial\Pi^1/\partial x_2$ and $\partial\Pi^2/\partial x_1$ have the same sign and that $\partial\Pi^2/\partial K_1 = 0$.

$$\text{sign}\left(\frac{\partial \Pi^1}{\partial x_2}\frac{dx_2^*}{dx_1}\frac{dx_1^*}{dK_1}\right) = \text{sign}\left(\frac{\partial \Pi^2}{\partial x_1}\frac{dx_1^*}{dK_1}\right) \times \text{sign}(R_2').$$

There are 4 cases:

1. Tough investment $\dfrac{\partial \Pi^2}{\partial x_1}\dfrac{dx_1^*}{dK_1} < 0$ with negative R_2': "top dog" strategy, be big or strong to look tough or aggressive. 裝腔作勢以壓制 entrant 氣勢.
2. Tough investment with positive R_2': "puppy dog" strategy, be small or weak to look soft or inoffensive. 低姿勢, 不要招惹 entrant 以免引起競爭.
3. Soft investment $\dfrac{\partial \Pi^2}{\partial x_1}\dfrac{dx_1^*}{dK_1} > 0$ with negative R_2': "lean and hungry look" strategy, be small or weak to look tough or aggressive. 裝出吃不飽餓不死的樣子以打擊 entrant 士氣.
4. Soft investment with positive R_2': "fat cat" strategy, be big or strong to look soft or inoffensive. 故示大方以緩和 entrant 鬥志.

top dog: 鬥狗時壓在上面的狗, 支配者.
puppy: 自負的傻小子, 令人討厭的年輕人.
fat cat: (政黨之) 重要的資助人, 金主. 有財有勢之大亨. 自鳴得意的懶人.

8.7 Limit Pricing as Cost Signaling, Milgrom/Roberts (1982)

Incumbent 用低價策略來表示自己爲低成本, 高效率之廠商, 目的是嚇阻 a potential entrant.

8.7.1 Assumptions of the model

$t = 1, 2$. Each period's demand is $P = 10 - Q$.
Firm 1 is the incumbent, a monopoly in $t = 1$.
Firm 2 decides whether to enter in $t = 2$. If firm 2 enters, the market becomes Cournot competition.
$c_2 = 1$, $F_2 = 9$, i.e., $\mathrm{TC}_2(q_2) = 9 + q_2$ if $q_2 > 0$
$c_1 = 0$ with 50% probability and $c_1 = 4$ with 50% probability.
Firm 1 knows whether $c_1 = 0$ or $c_1 = 4$ but firm 2 does not.
It is an incomplete information game: Firm 1 knows both its and firm 2's payoff functions but firm 2 knows only its own payoff functions.

8.7.2 Complete information case

If there is no uncertainty, firm 1 will choose the monopoly quantity at $t = 1$, i.e.,

$$q_1(c_1 = 0) = 5,\ p_1(c_1 = 0) = 5,\ \pi_1(c_1 = 0) = 25; \quad q_1(c_1 = 4) = 3,\ p_1(c_1 = 4) = 7,\ \pi_1(c_1 = 0) = 9.$$

At $t = 2$, the duopoly equilibrium for firm 1 with $c_1 = 0$ and firm 2 ($c_2 = 1$) is

$$p = 11/3,\ q_1(c_1 = 0) = 11/3,\ q_2 = 8/3,\ \pi_1(c_1 = 0) = 121/9,\ \pi_2 = \frac{64}{9} - 9 = -17/9.$$

The duopoly equilibrium for firm 1 with $c_1 = 4$ and firm 2 ($c_2 = 1$) is

$$p = 5,\ q_1(c_1 = 4) = 1,\ q_2 = 4,\ \pi_1(c_1 = 4) = 1,\ \pi_2 = 16 - 9 = 7.$$

Therefore, firm 2 will enter if $c_1 = 4$ and not enter if $c_1 = 0$.
The high cost firm 1 has incentives to confuse firm 2.

8.7.3 The case when firm 2 has no information

If firm 2 does not know the type of firm 1, firm 2's expected profit when enters is $0.5(7) + 0.5(-17/9) = 23/9 > 0$. Therefore firm 2 will choose to enter.
Firm 1 has incentives to let firm 2 know firm 1's type.

8.7.4 Separating equilibrium

Firm 2 will use the monopoy price at $t = 1$ to make inference about the type of firm 1, i.e., firm 2 will not enter if $p = 5$ will enter if $p = 7$.
However, this is not an equilibrium. If the low cost firm 1 chooses the monopoly quantity $q_1(c_1 = 0) = 5$, the high cost firm 1 will have incentives to imitate.
To avoid being imitated, the low cost firm 1 will choose a lower price $p = 4.17$ ($q_1 = 5.83$). In such case, the high cost firm 1 cannot gain by imitating because the gain at

$t = 2$ is $9 - 1 = 8$ whereas the lose at $t = 1$ due to imitation is $9 - 5.83(4.17 - 4) \approx 8$. The separating equilbrium is as follows. At $t = 1$,

$$q_1(c_1 = 0) = 5.83,\ p_1(c_1 = 0) = 4.17,\ \pi_1(c_1 = 0) = 24.31;$$

$$q_1(c_1 = 4) = 3,\ p_1(c_1 = 4) = 7,\ \pi_1(c_1 = 0) = 9.$$

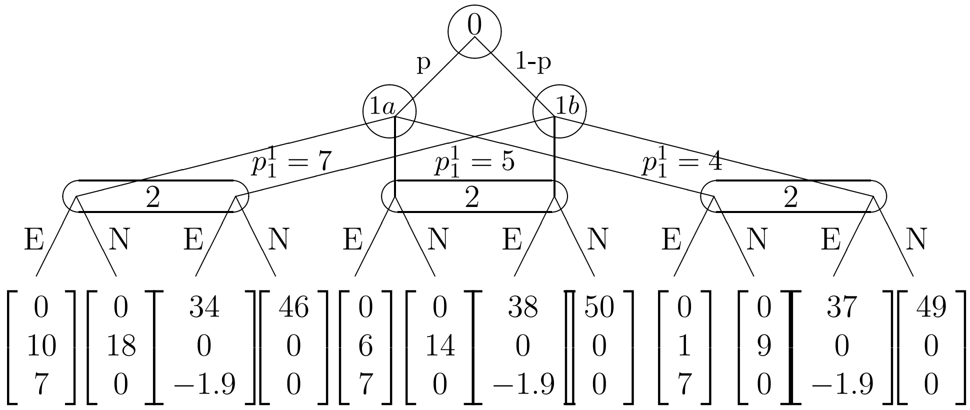
At $t = 2$, firm 2 will enter only if $p_1 > 4.17$.

8.7.5 Pooling equilibrium

If the standard distribution of c_1 is smaller, a separating equilibrium will not exist. Instead, the high cost firm 1 will immitate the low cost firm 1 and firm 2 will not enter.
For example: Prob$[c_1 = 0] = 0.8$ and Prob$[c_1 = 4] = 0.2$.

8.7.6 A finite version

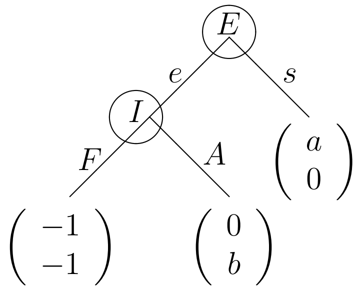
Assume that $p_1^1 \in \{7, 5, 4\}$, the game tree is



There is a separating equilbrium $p_{1a}^1 = 4$, $p_{1b}^1 = 7$ and firm 2 chooses not to enter (N) if $p_1^1 = 4$ and enter (E) if $p_1^1 \in \{7, 5\}$.
For $p > 7/8.9$, there is also a pooling equilibrium $p_{1a}^1 = p_{1b}^1 = 5$ and firm 2 chooses not to enter (N) if $p_1^1 \in \{5, 4\}$ and enter (E) if $p_1^1 = 7$.

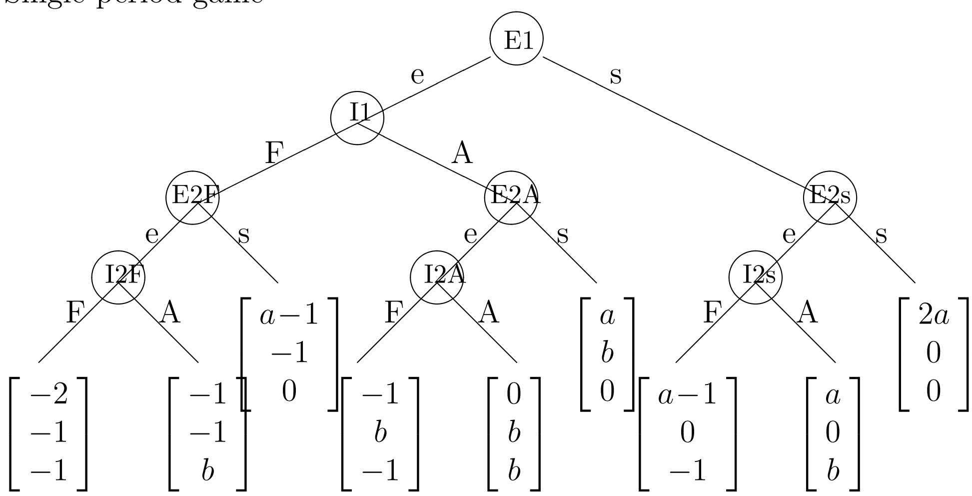
8.8 Chain-Store Game

Selton (1978), "The chain-store paradox," *Theory and Decision.*
A single long-run incumbent firm (I) faces potential entry by a series of short-run firms, each of which plays only once but observes all previous play. Each period, a potentail entrant (E) decides whether to enter (e) or stay out (s) of a single market. If s, I enjoys a monopoly in that market; if e, I must choose whether to fight (F) or to accommodate (A).



Single period game

In the 1-period game, the only SPNE is $\{A, e\}$.
In the T-period game, using backward induction, the only SPNE is also $\{A, e\}$ in each period.
In the ∞-period game, there is a SPNE such that $\{F, s\}$ is chosen in each period.



2-period game

Paradox: When T is large, the incumbent is tempered to fight to try to deter entry. The SPNE above is counterintuitive.

8.8.1 Incomplete information and reputation

Kreps and Wilson (1982), "Reputation and imperfect information," *JET.*
Milgrom and Roberts (1982), "Predation, reputation and entry deterrence," *JET.*
With probability p^0 the incumbent is "tough", ie., will fight for sure.
With probability q^0 an entrant is "tough", ie., will enter for sure. The events that entrants being "taugh" are independent.
δ: the discount factor.
Since the strategies of both "tough" type I and E are given, we need only analyze the equilibrium strategies of "normal" type.

$T = 1$: It is easily shown that the NE, $\{\mu^*(I), \mu^*(E)\}$, is

$$\mu^*(I) = a, \qquad \mu^*(E) = \begin{cases} e & \text{if } p^0 < \dfrac{b}{1+b} \equiv \bar{p} \\ s & \text{if } p^0 > \bar{p}. \end{cases}$$

$T = 2$: There are three cases.
(1) $q^0 > \dfrac{a\delta - 1}{a\delta} \equiv \bar{q}$: It is not worthwhile for I to fight to deter entry:

$$\mu^*(I1) = \mu^*(I2) = a, \quad \mu^*(E1) = \mu^*(E2s) = \begin{cases} e & \text{if } p^0 < \bar{p} \\ s & \text{if } p^0 > \bar{p}, \end{cases}$$

$$\mu^*(E2e) = \begin{cases} e & \text{if } I \text{ accommodates at } t = 1 \\ s & \text{if } I \text{ fights at } t = 1. \end{cases}$$

(2) $q^0 < \bar{q}$ and $p^0 > \bar{p}$: Fighting will deter entry and $\mu^*(I1) = F$.
(3) $q^0 < \bar{q}$ and $p^0 > \bar{p}$: Both fighting and accommodating are not equilibrium. I will randomize. Let $\beta \equiv Prob[F]$. β is such that $E2$'s posterior probability that I is "tough" equals $\bar{q}$:

$$Prob[\text{“tough”}|F] = \frac{p^0}{p^0 + \beta(1 - p^0)} = \bar{q} \Rightarrow \beta = \frac{p^0}{(1 - p^0)b}.$$

The total probability of fighting at $t = 1$ for $E1$ is

$$p^0 + (1 - p^0)\beta = p^0(b + 1)/b.$$

Therefore, $E1$ will enter if $p^0 > \bar{p}^2$ and stay out otherwise.

$T = 3$: (a) $p^0 > \bar{p}^2$, I will fight and $E1$ will stay out at $t = 1$.
(b) $\bar{p}^3 < p^0 < \bar{p}^2$, I will randomize at $t = 1$.
(c) $p^0 < \bar{p}^3$, I will accommodate and $E1$ will enter $t = 1$.

$T > 3$: Entrants will stay out until $t = k$ such that $p^0 < \bar{p}^k$.

When $\delta = 1$: (a) $q^0 < a/(1 + a)$, I will accommodate at first entry and reveal its type. Hence, $\lim_{T\to\infty} \pi/T = 0$.
(b) $q^0 > a/(1+a)$, there exists an $n(p^0)$ such that I will fight until there are no more than $n(p^0)$ entrants remaining. Hence, $\lim_{T\to\infty} \pi/T = (1 - q^0)a - q^0$.

9 Research and Development (R&D)

R&D影響產業生態最劇烈。

1. 新生產方法改變廠商成本結構, 使產業市場佔有率重新調整。
2. 新產品開拓新領域。
3. R&D 本身爲廠商競爭策略。
4. 技術擴散問題。
5. R&D, 模仿, 與經濟發展。
6. R&D 與 Merger activities 之關係。

R&D佔營業額之比率 (OECD 1980):

航太 23%, 計算機 18%, 電子 10%, 醫藥 9%, 食品, 煉油, 印刷, 傢俱, 紡織 $<$ 1%。

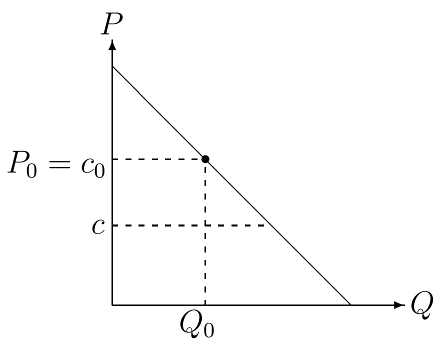
Production and cost functions are black boxes created by economists. Investigating R&D processes helps us to open the boxes.

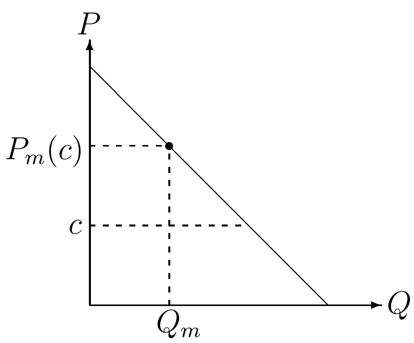
Process innovation: An innovation that reduces the production cost of a product.
Product innovation: An innovation that creats a new product.

The distinction is not essential. A process innovation can be treated as the creation of new intermediate products that reduce the production costs. On the other hand, a product innovation can be regarded as an innovation that reduces the production cost of a product from infinity to a finite value.

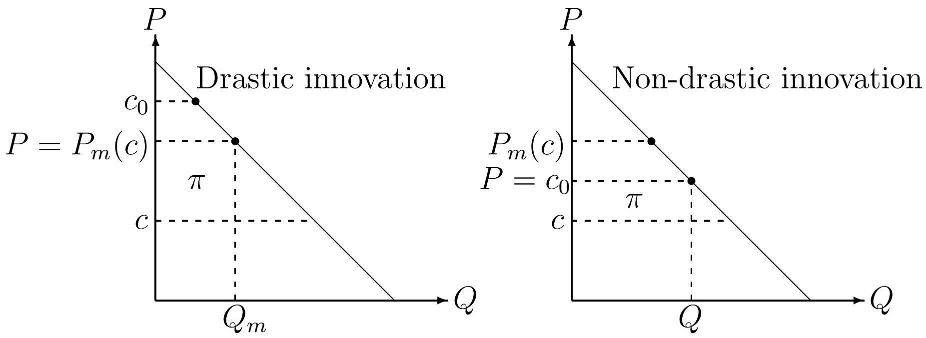
9.1 Classification of Process Innovations

Consider a Bertrand competition industry.
Inverse demand function: $P = a - Q$.
In the beginning, all firms have the same technology and $P_0 = C_0$. Suppose that an inventor innovates a new production procedure so that the marginal production cost reduces to $c < c_0$.





2 Cases:
Drastic innovation (large or major innovation): If $P_m(c) \equiv \dfrac{a+c}{2} < c_0 = P_0$, then the innovator will become a monopoly.
Non-drastic innovation (small or minor innovation): If $P_m(c) \equiv \dfrac{a+c}{2} > c_0 = P_0$, then the innovator cannot charge monopoly price and has to set $P = c_0 - \epsilon$.



A drastic innovation will reduce the market price. A non-drastic innovation will not change the Bertrand equilibrium price. In both cases the innovator makes positive profits.

9.2 Innovation Race

發明 (innovation): 本來不存在的商品, 經由 R&D 把它創造出來。
模仿 (immitation): 抄襲別人的發明; 通常經由逆向工程 (backward engineering)。
重複別人的發明 (duplication): 不知情之下, 經由 R&D 重複別人的發明。

Patent right (專利權): 頒授給發明家之獨家專賣權; 不論有意模仿或不知情重複, 都是侵犯專利權。
專利分類: 新產品, 新方法, 新成分, 新設計。
取得專利之條件: 有用 (Usefulness), 複雜性 (non-triviality), 新奇性 (novelty)。但是數學公式不能取得專利。
專利迴避: 爲避免侵犯別人的專利, 發明家必須採取新的研發路線。

Innovation race (專利競賽): 多位發明家競相研發爭取某一商品之專利權。最先發明者取得專利, 但落後者可採取專利迴避手段研發替代產品。不過最先發明者通常會有較多之消費者忠誠度 (Consumer Loyalty)。

問題: 是否會造成過度競爭, 浪費太多之研發資源?

Assumptions:

1. 2 firms compete to innovate a product.

2. The value of the patent right to the product is $ V.

3. To compete, each firm has to spend \$ I to establish a research lab.
4. The probability of firm i innovating the product is α. The events of firms being successful is independent.
5. If only one firm successes, the firm gains \$ V. If both success, each gains 0.5 V. If a firm fails, it gains 0.
6. The entry is sequential. Firm 1 decides first and then firm 2 makes decision.

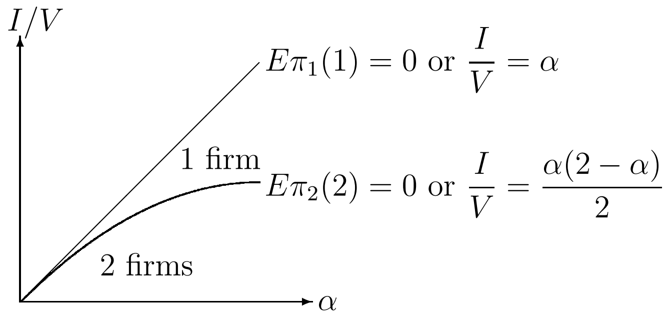
9.2.1 Market equilibrium

$E\pi_k(n)$: The expected profit of firm k if there are n firms competing.
i_k: The investment expenditure of firm k, $i_k \in \{0, I\}$.
$n = 1$:

$$E\pi_1(1) = \alpha V - I \quad \Rightarrow \; i_1 = \begin{cases} I & \text{if } \alpha V \geq I \\ 0 & \text{if } \alpha V < I. \end{cases}$$

$n = 2$:

$$E\pi_2(2) = \frac{\alpha(2-\alpha)}{2} V - I \; \Rightarrow \; i_2 = \begin{cases} I & \text{if } \dfrac{\alpha(2-\alpha)}{2} V \geq I \\ 0 & \text{if } \dfrac{\alpha(2-\alpha)}{2} V < I. \end{cases}$$

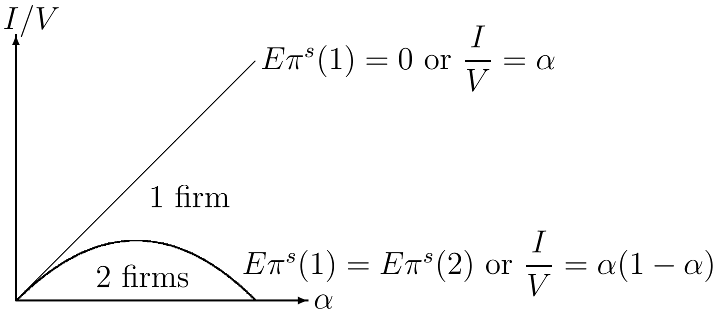


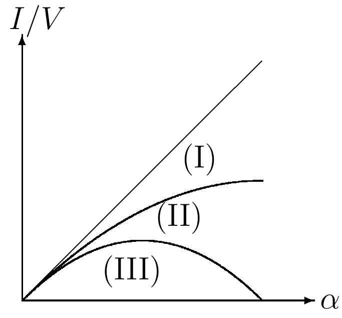
9.2.2 Social Optimal

If there are more firms, the probability of success is higher; On the other hand the investment expenditure will be also higher. $E\pi^s(n)$: The expected social welfare if there are n firms attempting.

$$E\pi^s(1) = E\pi_1(1) = \alpha V - I \quad E\pi^s(2) = 2\alpha(1-\alpha)V + \alpha^2 V - 2I,$$

$$E\pi^s(2) \geq E\pi^s(1) \quad \text{if and only if } \alpha(1-\alpha) \geq I.$$





(I): Social optimal is 1 firm, the same as market equilibrium number of firms.
(II): Social optimal is 1 firm, market equilibrium has 2 firms.
(III): Social optimal is 2 firms, the same as market equilibrium number of firms.

Area (II) represents the market inefficient area.

9.2.3 Expected date of discovery

Suppose that the R&D race will continue until the discovery.

$ET(n)$: Expected date of discovery if there are n firms.

$$ET(1) = \alpha + (1-\alpha)\alpha 2 + (1-\alpha)^2\alpha 3 + \ldots = \alpha \sum_{t=0}^{\infty}(1-\alpha)^{t-1}t = \frac{\alpha}{[1-(1-\alpha)]^2} = \frac{1}{\alpha}.$$

$$ET(2) = \alpha(2-\alpha)+(1-\alpha)^2\alpha(2-\alpha)2+\ldots = \alpha(2-\alpha)\sum_{t=0}^{\infty}(1-\alpha)^{2(t-1)}t = \frac{1}{\alpha(2-\alpha)} < ET(1).$$

9.3 Cooperation in R&D

Firms' cooperation in price setting is against anti-trust law. Cooperation in R&D activities usually is not illegal. Therefore, firms might use cooperation in R&D as a substitute for cooperation in price setting.
In this subsection we investigate the effects of firms' cooperation in R&D on social welfare.

A 2-stage duopoly game with R&D
$t = 1$: Both firms decide R&D levels, x_1 and x_2, simultaneously.
$t = 2$: Both firms engage in Cournot quantity competition.
Market demend is $P = 100 - Q$.
Firm i's R&D cost: $\mathrm{TC}_i(x_i) = x_i^2/2$.
Firm i's unit production cost: $c_i(x_i, x_j) = 50 - x_i - \beta x_j$.
$\beta \gtreqless 0$; if $\beta > 0$, it represents the spillover effect of R&D; if $\beta < 0$, it is the interference effect.

9.3.1 Noncooperative R&D equilibrium

When firms do not cooperate in R&D, they decide the R&D levels independent of each other. We solve the model backward.

At $t = 2$, x_1 and x_2 are determined. The Cournot equilibrium is such that

$$\Pi_i(c_i, c_j) = \frac{(100 - 2c_i + c_j)^2}{9} - \mathrm{TC}_i(x_i).$$

Substituting the unit cost functions, we obtain the reduced profit function of $t = 1$:

$$\begin{aligned} \Pi_i(x_i, x_j) &= \frac{[100 - 2(50 - x_i - \beta x_j) + (50 - 2x_j - \beta x_i)]^2}{9} - \frac{x_i^2}{2} \\ &= \frac{[50 + (2 - \beta)x_i + (2\beta - 1)x_j]^2}{9} - \frac{x_i^2}{2}. \end{aligned}$$

At $t = 1$, firm i chooses x_i to maximize $\Pi_i(x_i, x_j)$. FOC is

$$\frac{\partial \Pi_i}{\partial x_i} = 0 = \frac{2(2 - \beta)[50 + (2 - \beta)x_i + (2\beta - 1)x_j]}{9} - x_i.$$

In a symmetric equilibrium, $x_i = x_j = x^{nc}$:

$$x_1 = x_2 = x^{nc} = \frac{50(2 - \beta)}{4.5 - (2 - \beta)(1 + \beta)}, \quad c_1 = c_2 = \frac{50[4.5 - 2(2 - \beta)(1 + \beta)]}{4.5 - (2 - \beta)(1 + \beta)},$$

$$P^{nc} - c^{nc} = Q^{nc} = \frac{75}{4.5 - (2 - \beta)(1 + \beta)}, \quad \Pi_1 = \Pi_2 = \Pi^{nc} = \frac{25^2[9 - 2(2 - \beta)]}{[4.5 - (2 - \beta)(1 + \beta)]^2}.$$

9.3.2 Cooperative R&D equilibrium

When firms cooperate in R&D, they choose $x_1 = x_2 = x$ so that

$$\Pi_i = \Pi_j = \Pi(x) = \frac{[50 + (1 + \beta)x]^2}{9} - \frac{x^2}{2}.$$

Then they decide the level of x to maximize $\Pi(x)$. FOC is

$$\frac{\partial \Pi}{\partial x} = 0 = \frac{2(1 + \beta)[50 + (1 + \beta)x]}{9} - x.$$

Denote by x^c the optimal level of x,

$$x_1 = x_2 = x^c = \frac{50(1 + \beta)}{4.5 - (1 + \beta)^2}, \quad c_1 = c_2 = \frac{50[4.5 - 2(1 + \beta)^2]}{4.5 - (1 + \beta)^2},$$

$$P^c - c^c = Q^c = \frac{75}{4.5 - (1 + \beta)^2}, \quad \Pi_1 = \Pi_2 = \Pi^c = \frac{25^2[9 - 2(1 + \beta)^2]}{[4.5 - (1 + \beta)^2]^2}.$$

Conclusions:

1. $\Pi^c > \Pi^{nc}$.
2. If $\beta > 0.5$ then $x^c > x^{nc}$ and $Q^c > Q^{nc}$.
3. If $\beta < 0.5$ then $x^c < x^{nc}$ and $Q^c < Q^{nc}$.

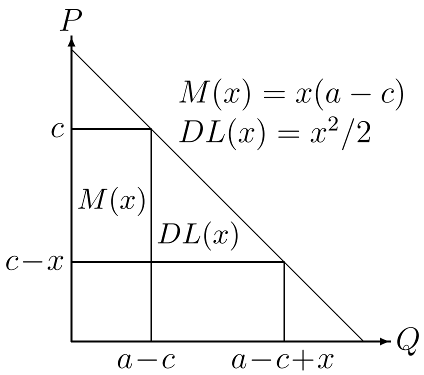
When $\beta > 0.5$, consumers will be better off to allow R&D cooperation; social welfare will definitely increase. When $\beta < 0.5$, consumers will be worse off to allow R&D cooperation; but the social welfare also depends on the change in firms' profits.

9.4 Patents

發明之直接價值: 創造生產者利潤及消費者剩餘。
發明之間接價值: 啓發新發明, 比較不易衡量。
專利權: 社會給發明家之報酬, 用以鼓勵創新。
專利造成獨占, 但是沒有專利制度則沒有足夠誘因來鼓勵發明。
在專利制度出現之前, 發明家只能用保密的方法來保護自己的權利。事實上, 現代的發明家亦有用保密的方法來取得比專利更長的獨占利潤。
例如, Stradivarius Violin, Coca Cola。
da Vinci 則爲個人興趣而發明。
專利權長度: 美國 17 年, 歐洲 20 年, 台灣 20 年。
版權與專利權長度不同。數學公式不能申請專利, 但可保密。電腦軟體屬於版權。
問題: 專利權長度要多少才能在鼓勵創新與獨占資源扭曲之間取得最適折衷?

9.4.1 Nordhous 1969 partial equilibrium model

$P = a - Q$: Demand function of a Bertrand competition market.
c: Unit production cost before R&D.
x: R&D magnitude. $\mathrm{TC}(x) = x^2/2$: R&D expenditure.
$c - x$: Unit production cost of the innovator after R&D.
Assume that the innovation is non-drastic.



T: Patent length (duration).
$M(x)$: Innovator's expected profit per period during periods $T = 1, 2, \ldots, T$.
$DL(x)$: Deadweight Loss due to monopoly (Bertrand competition).
$\rho = 1/(1+r)$: discount factor. (r is market interest rate.)

Innovator's problem:

$$\max_x \ \pi(x:T) = \sum_{t=1}^{T} \rho^{t-1} M(x) - \text{TC}(x) = \frac{1-\rho^T}{1-\rho} M(x) - \text{TC}(x) = \frac{1-\rho^T}{1-\rho}(a-c)x - \frac{x^2}{2}.$$

FOC: $\frac{1-\rho^T}{1-\rho}(a-c) - x \quad \Rightarrow \quad x = \frac{1-\rho^T}{1-\rho}(a-c).$

Comparative statics: $\frac{\partial x}{\partial T} > 0, \qquad \frac{\partial x}{\partial a} > 0, \qquad \frac{\partial x}{\partial c} < 0, \qquad \frac{\partial x}{\partial \rho} > 0.$

Social optimal duration of patents:

$$W(T) = \left[\sum_{t=1}^{T} \rho^{t-1} M(x)\right] + \left[\sum_{t=T+1}^{\infty} \rho^{t-1} DL(x)\right] - \frac{x^2}{2} = \frac{(a-c)x}{1-\rho} - \frac{1-\rho^T}{1-\rho}\frac{x^2}{2}.$$

$$\max_{x,T} \ \frac{(a-c)x}{1-\rho} - \frac{1-\rho^2}{1-\rho}\frac{x^2}{2} \qquad \text{subject to} \qquad x = \frac{1-\rho^T}{1-\rho}(a-c).$$

Eliminating x:

$$\max_T \ \frac{(a-c)}{1-\rho}\frac{1-\rho^T}{1-\rho}(a-c) - \frac{1-\rho^2}{1-\rho}\frac{1}{2}\left[\frac{1-\rho^T}{1-\rho}(a-c)\right]^2 = \left(\frac{a-c}{1-\rho}\right)^2 \left[1-\rho^T - \frac{(1-\rho^T)^3}{2(1-\rho)}\right].$$

Make change of variable $z \equiv 1-\rho^T$, or $T = \frac{\ln(1-z)}{\ln \rho}$. The problem becomes

$$\max_z \ z - \frac{z^3}{2(1-\rho)} \qquad \text{FOC:} \quad 1 - \frac{3z^2}{2(1-\rho)} = 0, \quad \Rightarrow \quad z^* = \sqrt{2(1-\rho)/3}$$

$$\Rightarrow \quad T^* = \frac{\ln(1-\sqrt{2(1-\rho)/3})}{\ln \rho}.$$

9.4.2 General equilibrium models

K. Judd (1985) "On performance of patent," *Econometrica* is a general equilibrium model. His conclusion is that $T^* = \infty$:
1. All products are monopoly priced with the same mark-up ratio and therefore there is no price distortion.
2. The R&D costs of an innovation should be paid by all consumers benefited from it to avoid intertemporal allocation distortion. Therefore, infinite duration of patents is needed.

C. Chou and O. Shy (1991) "Optimal duration of patents," *Southern Economic Journal*: If R&D has DRTS, optimal duration of patents may be finite. There are also many nonsymmetrical factors, eg., some products are competitively priced, demand elasticities are different, etc.

9.5 Licencing 專利授權

More that 80% of innovators licence their patents to other firms to collect licencing fees rather than produce products and make monopoly profits.
Kamien 1992
Consider a Cournot duopoly market with demand $P = a - Q$.
Firm 1 invents a new procedure to reduce the unit production cost from c to $c_1 = c - x$.
Firm 2's unit cost is $c_2 = c$ if no licencing.

9.5.1 Equilibrium without licencing

$$q_1 = \frac{a-c+2x}{3}, \quad q_2 = \frac{a-c-x}{3}, \quad P = \frac{a+2c-x}{3},$$
$$\pi_1 = \frac{(a-c+2x)^2}{9}, \quad \pi_2 = \bar{\pi}_2 = \frac{(a-c-x)^2}{9}.$$

9.5.2 Equilibrium with per-unit fee licencing

Firm 1 can make more profit by licencing the new procedure to firm 2 and changing per-unit fee for every unit sold by firm 2.
The maximum fee is $\phi = c_2 - c_1 = x$.
Firm 2's total cost per unit is still c_2 $(= c_1 + x)$. Therefore, the equilibrium is the same as without licencing except that firm 1 now collects x dollars per unit of q_2:

$$\pi_1^{\phi} = \frac{(a-c+2x)^2}{9} + \frac{(a-c-x)x}{3}, \quad \pi_2 = \bar{\pi}_2 = \frac{(a-c-x)^2}{9}.$$

9.5.3 Equilibrium with fixed-fee licencing

Firm 1 can also choose to charge firm 2 a fixed amount of money F, independent of q_2.
Firm 2's total cost per unit becomes $c_2 = c_1 = c - x$. Therefore, the equilibrium is now

$$q_1^F = q_2^F = \frac{a-c+x}{3}, \quad P^F = \frac{a+2c-2x}{3}, \quad \pi_1^F = \frac{(a-c+x)^2}{9} + F, \quad \pi_2^F = \frac{(a-c+x)^2}{9} - F.$$

The (maximum) F is such that $\pi_2^F = \bar{\pi}_2$. Therefore

$$F = \frac{(a-c+x)^2}{9} - \frac{(a-c-x)^2}{9} = \frac{(a-c)4x}{9}, \quad \Rightarrow \pi_1^F = \frac{(a-c+x)^2}{9} + \frac{(a-c)4x}{9}.$$

9.5.4 Comparison between π_1^{ϕ} and π_1^F

$$9 \times (\pi_1^{\phi} - \pi_1^F) = (a-c)x > 0.$$

Therefore, firm 1 will prefer per-unit fee licencing.
The reason is: In the case of fixed-fee licencing, $q_1 + q_2$ $\uparrow$ and P $\downarrow$ and therefore firm 1's total profit is smaller than that of per-unit fee licencing.

9.6 Governments and International R&D Race

9.6.1 Subsidizing new product development

Sometimes governmental subsidies can have very substantial strategical effects.
Krugman (1986), *Strategical Trade Policy and the New International Economics.*
Boeing (I, a US firm) and Airbus (II, an EU firm) are considering whether to develop super-large airliners.

Without intervention, the game is:

I \ II	Produce	Don't Produce
Produce	(-10, -10)	(50, 0)
Don't Produce	(0, 50)	(0, 0)

There are two equilibria: (Produce, Don't) and (Don't, Produce).

If EU subsidizes 15 to Airbus to produce, the game becomes:

I \ II	Produce	Don't Produce
Produce	(-10, 5)	(50, 0)
Don't Produce	(0, 65)	(0, 0)

There is only one equilibrium: (Don't, Produce).
In this case, by subsidizing product development, a governmental can secure the world dominance of the domestic firm.

9.6.2 Subsidizing process innovation

If we regard the R&D levels x_1 and x_2 in the R&D cooperation model as the amount of R&D sponsored by governments 1 and 2, it becomes a model of government subsidy competition.

9.7 Dynamic Patent Races

Tirole section 10.2.
Reinganum (1982) "A dynamic game of R&D," *Econometrica.*

9.7.1 Basic model

2 firms compete in R&D to win the patent of a new product.
x_i: the size of R&D lab established (incurring a continuous cost of x_i per unit of time) by firm i, $i = 1, 2$.
V: the value of the patent per unit of time. r: interest rate.
T_i: firm i's discovery time.
Assumption: T_1 and T_2 are independent exponential random variable:

$$T_i \sim 1 - e^{-h(x_i)T_i}, \quad \text{density function: } h(x_i)e^{-h(x_i)T_i},$$

where $[h(x_i)]^{-1}$ is expected discovery time of firm i.

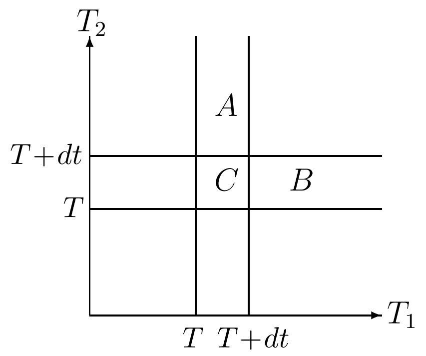
$$E(T_i) = [h(x_i)]^{-1}, \quad h(x_i) > 0,\ h'(x_i) > 0,\ h''(x_i) < 0.$$

Industry discovery time: $\hat{T} \equiv \min\{T_1, T_2\} \sim 1 - e^{-[h(x_1)+h(x_2)]\hat{T}}$ because

$$\text{Prob}\{\hat{T} > T\} = \text{Prob}\{T_1 > T,\ T_2 > T\} = e^{-h(x_1)T} e^{-h(x_2)T} = e^{-[h(x_1)+h(x_2)]T}.$$

Firm 1's winning probability: $\text{Prob}[T_1 = T, T_2 > T \mid \hat{T} = T] = \dfrac{h(x_1)}{h(x_1)+h(x_2)}$:

$$\frac{\text{Prob}[T_1, \hat{T} \in (T, T+dt)]}{\text{Prob}[\hat{T} \in (T, T+dt)]} \approx \frac{h(x_1)e^{-h(x_1)T}dt[1-(1-e^{-h(x_2)T})]}{[h(x_1)+h(x_2)]e^{-[h(x_1)+h(x_2)]T}dt} = \frac{h(x_1)}{h(x_1)+h(x_2)}.$$



$$\{\hat{T} \in (T, T+dt)\} = A \cup B \cup C$$
$$A = \{T_1 \in (T, T+dt),\ T_2 \geq T+dt\}$$
$$B = \{T_2 \in (T, T+dt),\ T_1 \geq T+dt\}$$
$$C = \{T_1, T_2 \in (T, T+dt)\},\ \text{Prob}[C] \approx 0.$$

Given (x_1, x_2), the expected payoff of firm 1, $\Pi_1(x_1, x_2)$ is (Π_2 is similar)

$$\begin{aligned}
& \int_0^\infty \left[\frac{h(x_1)}{h(x_1)+h(x_2)} \int_{\hat{T}}^\infty e^{-rt} V dt - \int_0^{\hat{T}} e^{-rt} x_1 dt \right] [h(x_1)+h(x_2)] e^{-[h(x_1)+h(x_2)]\hat{T}} d\hat{T} \\
= \ & \int_0^\infty \left[\frac{h(x_1)}{h(x_1)+h(x_2)} \frac{e^{-r\hat{T}}V}{r} - \frac{(1-e^{-r\hat{T}})x_1}{r} \right] [h(x_1)+h(x_2)] e^{-[h(x_1)+h(x_2)]\hat{T}} d\hat{T} \\
= \ & \frac{h(x_1)V/r}{h(x_1)+h(x_2)+r} - \frac{x_1}{r} + \frac{[h(x_1)+h(x_2)]x_1/r}{h(x_1)+h(x_2)+r} = \frac{h(x_1)V - rx_1}{r[h(x_1)+h(x_2)+r]}.
\end{aligned}$$

FOC for symmetric NE with $x_1 = x_2 = x$:

$$[2h(x)+r][h'(x)V-r]-[h(x)V-rx]h'(x) = h(x)h'(x)V+rh'(x)(x+V)-2rh(x)-r^2 = 0.$$

Social welfare when $x_1 = x_2 = x$:

$$\begin{aligned}
W(x) &= \int_0^\infty \left(\int_{\hat{T}}^\infty e^{-rt} V dt - \int_0^{\hat{T}} e^{-rt} 2x dt \right) 2h(x) e^{-2h(x)\hat{T}} d\hat{T} \\
&= \frac{1}{r} \int_0^\infty \left[e^{-r\hat{T}} V - (1-e^{-r\hat{T}}) 2x \right] 2h(x) e^{-2h(x)\hat{T}} d\hat{T} \\
&= \frac{2h(x)V/r}{2h(x)+r} - \frac{2x}{r} + \frac{2h(x)x/r}{2h(x)+r} = \frac{2[h(x)V - rx]}{r[2h(x)+r]}.
\end{aligned}$$

FOC for social optimal:

$$[2h(x)+r][h'(x)V-r]-2[h(x)V-rx]h'(x)=rh'(x)(2x+V)-2rh(x)-r^2=0.$$

Example: $h(x)=2\sqrt{x}$, $h'(x)=1/\sqrt{x}$.
FOC for NE:

$$2V+\frac{r(x+V)}{\sqrt{x}}-4r\sqrt{x}-r^2=0, \quad \Rightarrow \sqrt{x_n}=\frac{2V-r^2+\sqrt{(2V-r^2)^2+12r^2V}}{6r}.$$

FOC for social optimal:

$$\frac{r(2x+V)}{\sqrt{x}}-4r\sqrt{x}-r^2=0, \quad \Rightarrow \sqrt{x_s}=\frac{-r^2+\sqrt{r^4+8r^2V}}{4r}.$$

Derivation: Let $z_n \equiv \sqrt{x_n}$, $z_s \equiv \sqrt{x_s}$, $f_n(z) \equiv 3rz^2+(r^2-2V)z-rV$, and $f_s(z) \equiv 2rz^2+r^2z-rV$. $f_n(z_n)=0$ and $f_s(z_s)=0$. Direct computation shows that $f_n(z_s)<0$, $\lim_{z\to\infty} f_n(z)=\infty>0$. Hence $z_n>z_s$ and therefore

$$\frac{\sqrt{x_n}}{\sqrt{x_s}}=\frac{2}{3}\frac{k-1+\sqrt{k^2+4k+1}}{-1+\sqrt{1+4k}}>1,\ k=\frac{2V}{r^2}.$$

Therefore, the equilibrium R&D level is greater than the social optimal level.

Extensions: 1. $h(x)=\lambda\bar{h}(x/\lambda)=\lambda^{1-a}x^a$.
2. When there are $n>2$ firms.
3. n is endogenouse and optimal x and n.

9.7.2 R&D race between an incumbent and a potential entrant

2 firms compete in R&D to win the patent on a new procedure with unit cost c.
Firm 1: Incumbent with initial unit production cost $\bar{c}>c$.
Firm 2: A potential entrant.
$\Pi^m(\bar{c})$: Firm 1's monopoly profit before the discovery of the new procedure.
$\Pi^m(c)$: Firm 1's monopoly profit if firm 1 wins.
$\Pi_1^d(\bar{c},c)$: Firm 1's duopoly profit if firm 2 wins.
$\Pi_2^d(\bar{c},c)$: Firm 2's duopoly profit if firm 2 wins.

Assumption 1: $\Pi^m(c)\geq \Pi_2^d(\bar{c},c)+\Pi_1^d(\bar{c},c)$.
Assumption 2: The patent length is ∞.

Using the same derivation as basic model,

$$\begin{aligned} V_1(x_1,x_2) &= \frac{h(x_1)\Pi^m(c)+h(x_2)\Pi_1^d(\bar{c},c)+r[\Pi^m(\bar{c})-x_1]}{r[h(x_1)+h(x_2)+r]}, \\ V_2(x_1,x_2) &= \frac{h(x_2)\Pi_2^d(\bar{c},c)-rx_2}{r[h(x_1)+h(x_2)+r]}. \end{aligned}$$

Comparing the payoff functions reveals that firm 2's payoff function is essentially the same as that of the basic model. Further comparison between firm 1 and firm 2's payoff functions reveals that firm 1's incentives are different in two ways:

Efficiency effect: $\Pi^m(c) - \Pi_1^d(\bar{c}, c) \geq \Pi_2^d(\bar{c}, c)$, firm 1 has more incentives to win the race. and therefore x_1 tends to be greater than x_2 in this aspect.

Replacement effect: If firm 1 wins, he replaces himself with a new monopoly. Therefore, firm 1 tends to delay the discovery date. $\dfrac{\partial^2 V_1}{\partial \Pi^m(\bar{c}) \partial x_1} < 0$ tends to make x_1 smaller.

The net effect depends on which one dominates. Following are two extreme cases:

Drastic innovation: $\Pi_1^d(\bar{c}, c) = 0$ and $\Pi_2^d(\bar{c}, c) = \Pi^m(c)$. No efficiency effect and $x_1 < x_2$.

Almost linear $h(x)$ case: $h = \lambda h(x/\lambda)$, $\lambda \to \infty$, and $h(x) \approx h'(0)x$.
In this case $h(x_1)$ and $h(x_2)$ are very large and firm 1 is more concerned with his winning the race rather than replacing itself. Therefore, replacement effect dominates and $x_1 > x_2$.

10 Network Effects, Compatibility, and Standards 網路效果, 產品並容性, 產品規格標準

人類是社會性動物。不論生產或消費都有外部性 (externalities)。
生產面: 分工合作, 互相干擾。
消費面: 群居, 社會交往, 交換訊息, 互相認同。

$$U^i = U^i(x^i) \Rightarrow U^i = U^i(x^i, x^{-i}, y), \qquad F^j(y^j) = 0 \Rightarrow F^j(y^j, y^{-j}, x) = 0.$$

Arrow-Debreu general equilibrium model assumes no externalities. With production and/or consumption externalities, we have to modify Arrow-Debreu model.

生產, 消費外部性與網路效果, 產品並容性有關。經濟行爲人必須互相配合才能發揮經濟效率, 提升生活品質。例如

語言文字, 鍵盤排列 (qwerty,dvorak)
度量衡 (英制, 公制)
交通規則 (靠右邊, 靠左邊, 車輻, 路寬)
法律制度
本位貨幣 (交易中介, 記帳單位)

秦始皇統一中國接著也統一這些規則, 亦即標準化 (standardization), 完全並容性。

問題: 人類在進步, 知識不斷在累積, 新產品不斷出現, 太過強調標準化會阻擾發展。

世界經濟越來越全球化 (globalization), 並容性與標準化越來越成爲重要課題。各國廠商都希望自己的產品規格能成爲世界標準, 以取得並容性, 發揮最大之網路效果, 爭取最大之市場佔有率。
例如: 高解析度電視 HDTV, 電腦及其周邊產品, 各種網路產品。

10.0.3 3 concepts

1. Compatibility (並容性): 不同廠牌零件可以互換使用 (有一共同標準)。
   Standardization (標準化): 所有廠牌零件都可以互換使用。

2. Downward-compatibility: 新產品可以替代舊產品, 但舊產品不一定 可以替代新產品。例如 Pentium III vs 486 。

3. Network externalities (網路外部性): 消費者效用隨同廠牌使用人數 增加而增加。

可並容產品之例: 音響組合。
不可並容產品之例: 照相機機體與鏡頭組合。
有些情況, 可並容產品形成一個個小集團, 集團內爲策略聯盟廠商。
VHS vs β, 唱機, CD player, DVD, MO, etc.
ASCII 爲 7-bit 標準化字碼。
Extended ASCII 爲 8-bit 沒有標準化字碼 (有許多不同標準)。

10.0.4 A standardization game

Firm A (USA) and firm B (Canada) are choosing a standard for their product.
α-standard (英制規格, 或靠右邊行駛, etc.) β-standard (公制規格, 或靠左邊行駛, etc.)

A \ B	α	β
α	(a, b)	(c, d)
β	(d, c)	(b, a)

1. If $a, b > \max\{c, d\}$ (battle of the sexes), then (α, α) and (β, β) are both NE. They choose the same standard (有標準化).

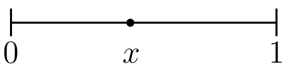
2. If $c, d > \max\{a, b\}$, then (α, β) and (β, α) are both NE. They choose the different standards (各行其是).

10.1 Network Externalities

10.1.1 Rohlfs phone company model

Rohlfs 1974, "A Theory of Interdependent Demand for a Communication Service," *Bell Journal of Economics.*

Consumers are distributed uniformly along a line, $x \in [0, 1]$.

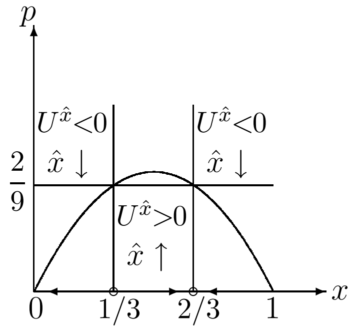


Consumers indexed by a low x are those who have high willingness to pay to subscribe to a phone system.

$$U^x = \begin{cases} n(1-x) - p & \text{if } x \text{ subscribes to the phone system} \\ 0 & \text{if } x \text{ does not,} \end{cases}$$

n, $0 < n < 1$: the total number of consumers who actually subscribe.
p: the price of subscribing.
$\hat{x}$: the marginal consumer who is indifference between subscribing and not.

In a rational expectation equilibrium, $\hat{x} = n$ and $U^{\hat{x}} = n(1 - \hat{x}) - p = 0$, $\Rightarrow$
Inverse demand function: $p = \hat{x}(1 - \hat{x})$.
Demand function: $\hat{x} = \dfrac{1 \pm \sqrt{1 - 4p}}{2}$.



If $0 < p < \frac{1}{4}$, then there are two possible marginal consumers, $\hat{x} = \frac{1 \pm \sqrt{1-4p}}{2}$. The smaller is unstable. The diagram uses $p = \frac{2}{9}$ to illustrate.

The phone company maximizes its profits:

$$\max_{\hat{x}} p\hat{x} = \hat{x}^2(1-\hat{x}), \quad \text{FOC: } 2\hat{x} - \hat{x}^2 = 0, \quad \Rightarrow \hat{x}^* = \frac{2}{3},\ p^* = \frac{2}{9}.$$

Dynamic model and critical mass (臨界基本客戶數):

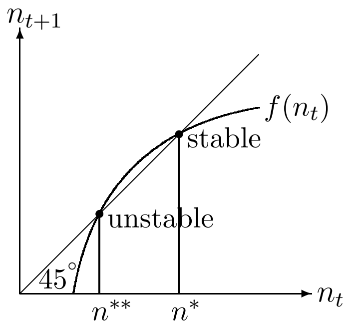
Assumption 1: The phone company sets $p = p^* = \frac{2}{9}$.
Assumption 2: At t, consumers expect that $n_t = \hat{x}_{t-1}$.

$$n_t(1-\hat{x}_t) = p = \frac{2}{9}, \quad \Rightarrow \quad n_t(1-n_{t+1}) = p = \frac{2}{9}, \quad \Rightarrow \quad n_{t+1} = 1 - \frac{2}{9n_t} \equiv f(n_t).$$

There are two equilibria: $n^{**} = \frac{1}{3}$ and $n^* = \frac{2}{3}$. n^{**} is unstable and n^* is stable:

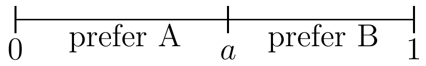
$$f'(n) = \frac{2}{9n^2} \quad f(n^{**}) = 2 > 1, \quad f(n^*) = \frac{1}{2} < 1.$$

If the initial subscription is $n_0 < 1/3$, $n_t \to 0$.
If the initial subscription is $n_0 > 1/3$, $n_t \to 2/3$. 1/3 is the critical mass.



10.1.2 The standardization-variety tradeoff 標準化或多元化?

Consumers are distributed uniformly along a line, $x \in [0, 1]$.



2 brands/standards, A (用右手) and B (用左手).
$a > 0$ consumers prefer A-standard.
$b = 1 - a > 0$ consumers prefer B-standard.
x_A: number of consumers using A-standard.
x_B: number of consumers using B-standard.
δ: the disutility of using a less prefered standard.

$$U^A = \begin{cases} x_A & \text{use A-standard} \\ x_B - \delta & \text{use B-standard,} \end{cases} \qquad U^B = \begin{cases} x_A - \delta & \text{use A-standard} \\ x_B & \text{use B-standard.} \end{cases}$$

Consumer distribution: (x_A, x_B) such that $x_A, x_B \geq 0$ and $x_A + x_B = 1$.

A-standard distribution: A distribution such that $(x_A, x_B) = (1, 0)$.

B-standard distribution: A distribution such that $(x_A, x_B) = (0, 1)$.

Incompatible AB-standards distribution A distribution with $x_A, x_B > 0$.

Equilibrium: (x_A, x_B) such that none wants to switch to a different brand/standard.

Proposition 10.3: If $\delta < 1$, then both A-standard and B-standard are equilibrium. If $\delta > 1$, then both A-standard and B-standard are not equilibrium.

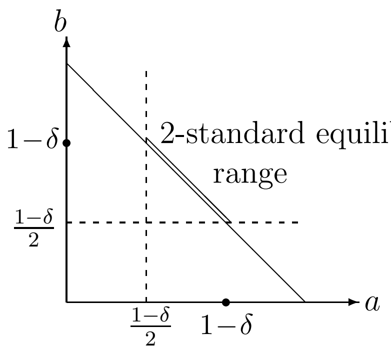
Proof: If $\delta > 1$ and every one chooses the same brand (either A or B), then none wants to switch to a different brand. If $\delta < 1$, then the cost of switching to a preferred brand is less than the benefit and therefore a single standard equilibrium cannot exist.

Proposition 10.4: If $a, b > \dfrac{1 - \delta}{2}$, then $(x_A, x_B) = (a, b)$ is an equilibrium.

Proof: Given the distribution $(x_A, x_B) = (a, b)$, the utility levels are

$$U^A = \begin{cases} a & \text{use A br.} \\ b - \delta = 1 - a - \delta < a & \text{use B br.} \end{cases} \qquad U^B = \begin{cases} a - \delta = 1 - b - \delta < b & \text{use A br.} \\ b & \text{use B br.} \end{cases}$$

Therefore, none will switch to a different brand.



Social welfare: $W(x_A, x_B) = aU^a + bU^b$, where U^a (U^b) is the utility level of A-prefered consumers (B-prefered consumers).

$W(A) = W(1,0) = a + b(1-\delta) = 1 - b\delta$.
$W(B) = W(0,1) = a(1-\delta) + b = 1 - a\delta$.
$W(AB) = W(a,b) = a^2 + b^2 = (1-b)^2 + b^2 = 1 - b - b(1-2b) = 1 - 2ab$.

Proposition 10.5: If $a > b$, then $W(A) > W(B)$.

Proof: If $W(A) - W(B) = a + b(1-\delta) - [a(1-\delta) - b] = (a-b)\delta > 0$.

Proposition 10.6: 1. If $\delta < 1$, then $W(AB) < \max\{W(A), W(B)\}$.
2. If $\delta > 1$ and $\frac{\delta}{2} > \max\{a, b\}$, then $W(AB) > \max\{W(A), W(B)\}$.

Proof: Assume that $a > b$ (or $1 - 2b > 0$). (case $b > a$ is similar.)
1. If $\delta < 1$, then $\max\{W(A), W(B)\} = W(A) = 1 - b\delta > 1 - b > 1 - b - b(1-2b) = W(AB)$.
2. If $\delta > 2a > 1$, then $W(AB) - W(A) = b(\delta - 2a) > 0$.

Proposition 10.7: If $\delta < 1$, then market failure can happen.

Remark: When $a > b$ and $\delta < 1$, the social optimal is A-standard. However, both the incompatible standards and B-standard can also be equilibrium.

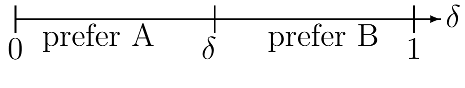
10.2 Supporting Services and Network Effects

Network effects can occur even there is no network externalities. For example, when there is a complementary supporting industry exhibiting increasing returns to scale such as PC industry.

10.2.1 Basic model

Chou/Shy (1990), "Network effects without network externalities," *International Journal of Industrial Organization.*

A PC industry with 2 brands, A and B and prices P_A and P_B.
Consumers are distributed uniformly along a line, $\delta \in [0, 1]$.



Let N_A and N_B be the numbers of software pieces available to computers A and B, respectively.

The utility of consumer δ is

$$U^{\delta} = \begin{cases} (1-\delta)\sqrt{N_A} & \text{if } \delta \text{ buys A-system} \\ \delta\sqrt{N_B} & \text{if } \delta \text{ buys B-system.} \end{cases}$$

In the above, $\sqrt{N_i}$ can easily be generalized to N_i^{α}.

Marginal consumer $\hat{\delta}$:

$$U^{\hat{\delta}}(A) = (1-\hat{\delta})\sqrt{N_A} = U^{\hat{\delta}}(B) = \hat{\delta}\sqrt{N_B}, \quad \Rightarrow \hat{\delta} = \frac{\sqrt{N_A}}{\sqrt{N_A}+\sqrt{N_B}}.$$

Market shares: $\delta_A = \hat{\delta}$ and $\delta_B = 1-\hat{\delta}$.
If N_A increases (or N_B decreases), $\hat{\delta}$ will decrease, A's market share will increase and B's market share will decrease.

$$\delta_A = \hat{\delta} = \frac{\sqrt{N_A}}{\sqrt{N_A}+\sqrt{N_B}}, \quad \delta_B = 1-\hat{\delta} = \frac{\sqrt{N_B}}{\sqrt{N_A}+\sqrt{N_B}}, \quad \frac{\delta_B}{\delta_A} = \frac{1-\hat{\delta}}{\hat{\delta}} = \frac{\sqrt{N_B}}{\sqrt{N_A}}.$$

In this model, there are two monopolistic competition software industries, A-software and B-software.
Assume that each consumer has Y dollars to spend on a computer system. If a consumer chooses i-stytem, he has $E_i \equiv Y - P_i$ to spend on software.
There is a variety effect in each software industry and the number of software pieces is propotional to the aggregate expenditure spent on them:

$$N_A = k\delta_A E_A = k\hat{\delta}(Y-P_A), \quad N_B = k\delta_B E_B = k(1-\hat{\delta})(Y-P_B).$$

$$\frac{1-\hat{\delta}}{\hat{\delta}} = \frac{\sqrt{N_B}}{\sqrt{N_A}} = \sqrt{\left(\frac{1-\hat{\delta}}{\hat{\delta}}\right)\left(\frac{Y-P_B}{Y-P_A}\right)}, \quad \Rightarrow \frac{1-\hat{\delta}}{\hat{\delta}} = \frac{Y-P_B}{Y-P_A} = \frac{E_B}{E_A}.$$

Therefore, the equilibrium market shares are

$$\delta_A = \hat{\delta} = \frac{E_A}{E_A+E_B} = \frac{Y-P_A}{2Y-P_A-P_B}, \quad \delta_B = 1-\hat{\delta} = \frac{E_B}{E_A+E_B} = \frac{Y-P_B}{2Y-P_A-P_B}.$$

Network effects: When $\hat{\delta}$ goes down, δ_A goes down (δ_B goes up), which in turn will reduce N_A (increase N_B). Finally, A-users' utility levels will decrease (B-users' utility levels will increase).
The network effect here is the same as the variety effects in Dixit/Stiglitz monopolistic competition model.

Duopoly price competition:

The profit functions of firms A and B are

$$\Pi_A(P_A,P_B) = \delta_A P_A = \frac{P_A(Y-P_A)}{2Y-P_A-P_B}, \quad \Pi_B(P_A,P_B) = \delta_B P_B = \frac{P_B(Y-P_B)}{2Y-P_A-P_B}.$$

The price competition equilibrium is derived in Chou/Shy (1990).

10.2.2 Partial compatibility

Chou/Shy (1993) "Partial compatibility and supporting services", *Economic Letters.*

In the basic model, A-computers and B-computers are incompatible in the sence that A-computers use only A-software and B-computers use only B-software. The model can be generalized to the case when computer firms design their machines in such a way that some fraction of B-software can be used in A-machines and vice versa.
Let ρ_A (ρ_B) be the proportion of B-software (A-software) that can be run on A-computers (B-computers).
Incompatibility: $\rho_A = \rho_B = 0$.
Mutual compatibility: $\rho_A = \rho_B = 1$.
One-way compatibility: $\rho_A = 1$, $\rho_B = 0$ or $\rho_A = 0$, $\rho_B = 1$.

n_A: number of software pieces written for A-computers.
n_B: number of software pieces written for B-computers.

$$N_A = n_A + \rho_A n_B, \quad N_B = n_B + \rho_B n_A, \quad \Rightarrow n_A = \frac{N_A - \rho_A N_B}{1 - \rho_A \rho_B} \quad n_B = \frac{N_B - \rho_B N_A}{1 - \rho_A \rho_B}. \tag{5}$$

$\delta_i E_i = \delta_i (Y - P_i)$: Aggregate expenditure on software from i-computer users.
$\frac{n_i}{N_i}\delta_i E_i + \frac{\rho_j n_j}{N_j}\delta_j E_j$: Aggregate expenditure on i-software.

As in the basic model, the number of i-software, n_i, is proportional to the aggregate expenditure on i-software:

$$n_A = k\left(\frac{n_A}{N_A}\delta_A E_A + \frac{\rho_B n_A}{N_B}\delta_B E_B\right), \quad n_B = k\left(\frac{\rho_A n_B}{N_A}\delta_A E_A + \frac{n_B}{N_B}\delta_B E_B\right),$$

$$\Rightarrow \quad N_A = \frac{(1-\rho_A\rho_B)\delta_A E_A}{k(1-\rho_B)}, \quad N_B = \frac{(1-\rho_A\rho_B)\delta_B E_B}{k(1-\rho_A)}. \tag{6}$$

As in the basic model,

$$\frac{1-\hat{\delta}}{\hat{\delta}} = \frac{\sqrt{N_B}}{\sqrt{N_A}} = \sqrt{\left(\frac{1-\hat{\delta}}{\hat{\delta}}\right)\frac{(1-\rho_B)E_B}{(1-\rho_A)E_A}},$$

$$\Rightarrow \quad \frac{1-\hat{\delta}}{\hat{\delta}} = \frac{(1-\rho_B)E_B}{(1-\rho_A)E_A} = \frac{(1-\rho_B)(Y-P_B)}{(1-\rho_A)(Y-P_A)}.$$

Other things being equal, if firm A increases the degree of compatibility ρ_A, the number of software pieces run on A-computers will decrease and hence its market share $\delta_A = \hat{\delta}$ will also decrease.

10.3 The Components Model

Matutes/Regibeau (1988), "Mix and Match: Product Compatibility Without Network Externalities," *RAND Journal of Economics*.
Economides (1989), "Desirability of Compatibility in the Absence of Network Externalities," *American Economic Review*.

AS1 2 firms, A and B, producing X_A, Y_A, X_B, Y_B.

AS2 Marginal costs are 0.

AS3 X and Y are completely complementary.

AS4 3 consumers: AA, AB, BB. You need an X and a Y to form a system S.

2 situations:
1. Incompatibility: A and B's products are not compatible. You have to buy X_AY_A or X_BY_B.
2. Compatibility: A and B's products are compatible. There are 4 possible systems: X_AY_A, X_AY_B, X_BY_A, and X_BY_B.

Consumer ij's utility, $ij = AA, AB, BB$, is

$$U^{ij} = \begin{cases} 2\lambda - (P^x_{i'} + P^y_{j'}) & i'j' = ij, \text{ i.e., } X,\, Y \text{ 都合意。} \\ \lambda - (P^x_{i'} + P^y_{j'}) & i' = i \text{ or } j' = j \text{ but } i'j' \neq ij, \text{ i.e., } X,\, Y \text{ 一項合意。} \\ -(P^x_{i'} + P^y_{j'}) & i' \neq i \text{ and } j' \neq j, \text{ i.e., } X,\, Y \text{ 都不合意。} \\ 0 & \text{不消費。} \end{cases}$$

10.3.1 Incompatibility

There are only 2 systems: A-system (X_AY_A) and B-system (X_BY_B).
$P_A = P^x_A + P^y_A$, $P_B = P^x_B + P^y_B$: Price of system A and system B, respectively.

Equilibrium: $(P^I_A, P^I_B; q^I_A, q^I_B)$ such that
1. P^I_i maximizes $\Pi_i(P_i, P^I_j)$.
2. (q^I_A, q^I_B) are the aggregate demand of the consumers at price (P^I_A, P^I_B).

Lemma 10.1. In an equilibrium, consumer AA (consumer BB) purchases A-system (B-system).

Proof: If in an equilibrium consumer AA purchases B-system, it must be $P_B = 0$. In that case, firm A can set $0 < P_A < 2\lambda$ to attract consumer AA.

Proposition 10.13. There are 3 different equilibria:

1. $(P^I_A, P^I_B; q^I_A, q^I_B) = (\lambda, 2\lambda; 2, 1)$. AA and AB purchase A-system and BB purchases B-system, $\Pi_A = \Pi_B = 2\lambda$, CS $= \lambda$, social welfare is 5λ.

2. $(P_A^I, P_B^I; q_A^I, q_B^I) = (2\lambda, \lambda; 1, 2)$. AA purchases A-system and BB and AB purchase B-system. $\Pi_A = \Pi_B = 2\lambda$, CS $= \lambda$, social welfare is 5λ.

3. $(P_A^I, P_B^I; q_A^I, q_B^I) = (2\lambda, 2\lambda; 1, 1)$. AA purchases A-system and BB purchases B-system. AB chooses to do without. $\Pi_A = \Pi_B = 2\lambda$, CS $= 0$, social welfare is 4λ.

10.3.2 Compatibility

4 systems: AA-system $(X_A Y_A)$, AB-system $(X_A Y_B)$, BA-system $(X_B Y_A)$, and BB-system $(X_B Y_B)$.

Equilibrium: $(P_{Ax}^c, P_{Ay}^c, P_{Bx}^c, P_{By}^c; q_{Ax}^c, q_{Ay}^c, q_{Bx}^c, q_{By}^c)$ such that
1. (P_{ix}^c, P_{iy}^c) maximizes $\Pi_i(P_{ix}, P_{iy}; P_{jx}^c, P_{jy}^c)$.
2. $(q_{Ax}^c, q_{Ay}^c, q_{Bx}^c, q_{By}^c)$ are the aggregate demand of the consumers at price $(P_{Ax}^c, P_{Ay}^c, P_{Bx}^c, P_{By}^c)$.

Proposition 10.14. There exists an equilibrium such that $P_{Ax}^c = P_{Ay}^c = P_{Bx}^c = P_{By}^c = \lambda$, $q_{Ax}^c = q_{By}^c = 2$, $q_{Ay}^c = q_{Bx}^c = 1$, $\Pi_A^c = \Pi_B^c = 3\lambda$, $U^{AA} = U^{AB} = U^{BB} = 0$, and social welfare is 6λ.

10.3.3 Comparison

1. Consumers are worse off under compatibility.

2. Firms are better off under compatibility.

3. Social welfare is higher under compatibility.

Extension to a 2-stage game: If at $t = 1$ firms determine whether to design compatible components and at $t = 2$ they engage in price competition, then they will choose compatibility.

11 Advertising

Advertising is defined as a form of providing information about prices, quality, and location of goods and services.
2% of GNP in developed countries.
vegetables, etc., < 2% of sales.
cosmetics, detergent, etc., 20-60 % of sales.
In 1990, GM spent $63 per car, Ford $130 per car, Chrysler $113 per car.

What determines advertising in different industries or different firms of the same industry?
Economy of scale, advertising elasticity of demand, etc.

Kaldor (1950), "The Economic Aspects of Advertising," *Review of Economic Studies.*
Advertising is manipulative and reduces competition.
1. Wrong information about product differentiations ⇒ increases cost.
2. An entry-deterring mechanism ⇒ reduces competition.

Telser (1964), "Advertising and Competition," *JPE*
Nelson (1970),"Information and Consumer Behavior," *JPE*
Nelson (1974),"Advertising as Information," *JPE*
Demsetz (1979), "Accounting for Advertising as a Barrier to Entry," *J. of Business.*
Positive sides of advertising: It provides produt information.

Nelson:
Search goods: Quality can be identified when purchasing. ⇒ 不需廣告。
Experience goods: Quality cannot be identified until consuming. ⇒ 廣告可影響需要。

Persuasive advertising: Intends to enhance consumer tastes, eg diamond.
Informative advertising: Provides basic information about the product.

11.1 Persuasive Advertising

$$Q(P,A) = \beta A^{\epsilon_a} P^{\epsilon_p}, \quad \beta > 0,\ 0 < \epsilon_a < 1,\ \epsilon_p < -1.$$

A: expenditure on advertising.
ϵ_a: Advertising elasticity of demand.
ϵ_p: Price elasticity of demand.
c: Unit production cost.

$$\max_{P,A} \Pi = PQ - cQ - A = (P-c)\beta A^{\epsilon_a} P^{\epsilon_p} - A.$$

FOC with respect to P:

$$\frac{\partial \Pi}{\partial P} = \beta A^{\epsilon_a}\left[(\epsilon_p+1)P^{\epsilon_p} - c\epsilon_p P^{\epsilon_p}\right] = 0, \quad \Rightarrow P^m = \frac{c\epsilon_p}{\epsilon_p+1}, \ \frac{P^m - c}{P^m} = \frac{\epsilon_p}{-1}.$$

FOC with respect to A:

$$\frac{\partial \Pi}{\partial A} = \epsilon_a \beta A^{\epsilon_a - 1} P^{\epsilon_p}(P-c) - 1 = 0, \quad \Rightarrow \frac{P^m - c}{P^m} = \frac{A}{PQ}\frac{1}{\epsilon_a}, \ \frac{\epsilon_a}{\epsilon_p} = \frac{A}{PQ}.$$

Proposition: The propotion of advertising expenditure to total sales is equal to the ratio of advertising elasticity to price elasticity.

11.1.1 Example: $\beta = 64, \epsilon_a = 0.5, \epsilon_p = -2, c = 1$

$$Q = 64\sqrt{A}P^{-2}, \quad P = 8A^{1/4}Q^{-1/2}, \quad \Rightarrow P^m = 2, \ Q^m = 16\sqrt{A}, \ A^m = 64, \ \Pi = 16\sqrt{A},$$

$$\Rightarrow \quad \mathrm{CS}(A) = \int_0^{16\sqrt{A}} P(Q)dQ - P^m Q^m = \int_0^{16\sqrt{A}} 8A^{1/4}Q^{-1/2}dQ - 32\sqrt{A} = 32\sqrt{A}.$$

Social welfare: $W(A) \equiv \mathrm{CS}(A) + \Pi(A) - A = 48\sqrt{A} - A$.
Social optimal: $W'(A) = 0 = \dfrac{24}{\sqrt{A}} - 1, \ \Rightarrow A = 24^2 = 576 > A^m = 64.$

Remark: 1. Does CS(A) represent consumers' welfare? If it is informative advertising, consumers' utility may increase when A increases. However, if consumers' are just persuased to make unnecessary purchases, the demand curve does not really reflect consumers' marginal utility.
2. Crowding-out effect: Consumption for other goods will decrease.
3. A can be interpreted as other utility enhancing factors.

11.2 Informative Advertising

Benham (1972), "The effects of Advertising on the Price of Eye-glasses," *J. of Law and Economics.*
在美國, 禁止眼鏡廣告的州眼鏡價格通常較高。

Consumers often rely on information for their purchases. The problem is whether there is too little or too much informative advertising.

Butters (1977), "Equilibrium Distributions of Sales and Advertising Prices," *Review of Economic Studies.*
Informative Advertising level under monopolistic competition equilibrium is social optimal.

Grossman/Shapiro (1984), "Informative Advertising with Differentiated Prdoducts," *Review of Economic Studies.*
In a circular market, informative advertising level is too excessive.

Meurer/Stahl (1994), "Informative Advertising and Product Match," *IJIO.*
In the case of 2 differentiated products, the result is uncertain.

11.2.1 A simple model of informative advertising

1 consumer wants to buy 1 unit of a product.
p: the price. m: its value.

$$U = \begin{cases} m - p & \text{買。} \\ 0 & \text{不買。} \end{cases}$$

If the consumer does not receive any advertisement, he will not purchase.
If he receives an advertisement from a firm, he will purchase from the firm.
If he receives 2 advertisements from 2 firms, he will randomly choose one to buy.

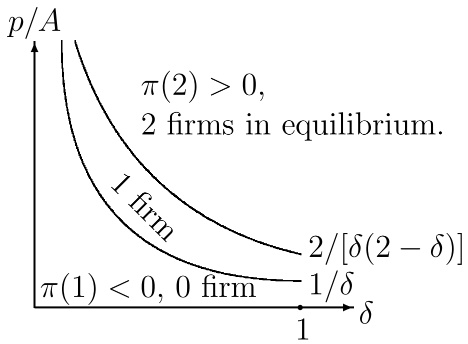
2 firms, unit production cost is 0, informative advertising cost is A.
Each chooses either to advertise its product or not to advertise it.

$$\pi_i = \begin{cases} p - A & \text{if only firm } i\text{'s ad is received.} \\ \frac{p}{2} - A & \text{if both firms' ad are received.} \\ -A & \text{if firm } i\text{'s ad is not received.} \\ 0 & \text{if firm } i \text{ chooses not to advertise.} \end{cases}$$

Let δ be the probability that an advertisement is received by the consumer.

$$E\pi_i = \begin{cases} \delta(1-\delta)(p-A) + \delta^2(\frac{p}{2} - A) - (1-\delta)A \equiv \pi(2) & \text{if both choose to advertise.} \\ \delta(p-A) - (1-\delta)A \equiv \pi(1) & \text{if only firm } i \text{ chooses to advertise.} \\ 0 & \text{if firm } i \text{ chooses not to advertise.} \end{cases}$$

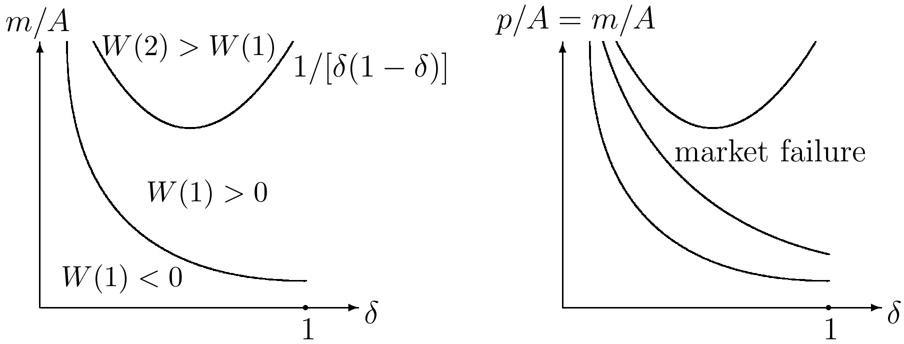
If $p/A > 1/\delta$, $\Rightarrow \pi(1) > 0$, $\Rightarrow$ at least one firm will choose to advertise.
If $p/A > 2/[\delta(2-\delta)]$, $\Rightarrow \pi(2) > 0$, $\Rightarrow$ both firms will choose to advertise.



Welfare comparison:

$$EW = \begin{cases} \delta(2-\delta)m - 2A \equiv W(2) & \text{if 2 firms advertise.} \\ \delta m - A \equiv W(1) & \text{if only one firm advertises.} \\ 0 & \text{if no firm advertises.} \end{cases}$$

If $\frac{m}{A} > \frac{1}{\delta}$, $\Rightarrow W(1) > 0$, $\Rightarrow$ social optimal is at least one firm advertises.
If $\frac{m}{A} > \frac{1}{\delta(1-\delta)}$, $\Rightarrow W(2) > W(1)$, $\Rightarrow$ social optimal is both firms advertise.



廣告技術提高使 δ 增加。When $\delta \to 1$, one firm would be enough. However, if $m/A > 1$, both firms will advertise.

11.3 Targeted Advertising 針對性廣告

(1) Consumers are heterogeneous with different tastes. (2) Large scale advertising is costly. (3) Intensive advertising will result in price competition.
$\Rightarrow$ 無法討好所有的消費者, 不同廠商採用不同性質之針對性廣告, 針對不同之消費者族群。

11.3.1 The model

2 firms, $i = 1, 2$, producing differentiated products.
2 groups of consumers: E experienced consumers and N inexperienced consumers.
θE of experienced consumers are brand 1 oriented, $0 < \theta < 1$.
$(1 - \theta)E$ of experienced consumers are brand 2 oriented.

2 advertising methods: P (persuasive) and I (informative).
Each firm can choose only one method.

AS1: Persuasive advertising attracts only inexperienced consumers. If only firm i chooses P, then all N inexperienced consumers will purchase brand i. If both firms 1 and 2 choose P, each will have $N/2$ inexperienced consumers.

AS2: Informative advertising attracts only the experienced consumers who are oriented toward the advertised brand, i.e., if firm 1 (firm 2) chooses I, θE ($(1-\theta)E$) experienced consumers will purchase brand 1 (brand 2).

AS3: A firm earns \$1 from each customer.

From the assumptions we derive the following duopoly advertising game:

firm 1 \ firm 2	P	I
P	$(N/2, N/2)$	$(N, (1-\theta)E)$
I	$(\theta E, N)$	$(\theta E, (1-\theta)E)$

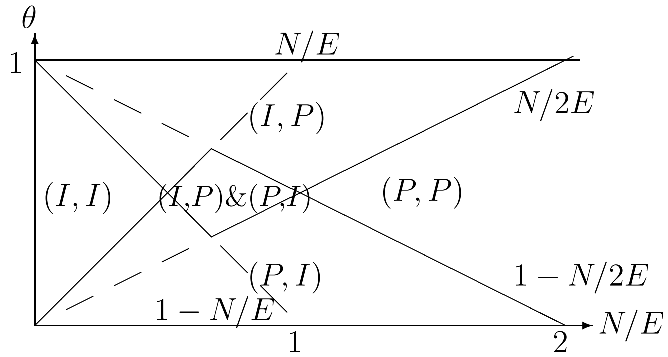
11.3.2 Proposition 11.5

1. (P,P) is a NE if only if $N/2 \geq \theta E$ and $N/2 \geq (1-\theta)E$ or $1 - \frac{N}{2E} \leq \theta \leq \frac{N}{2E}$. (If strict inequality holds, the NE is unique.)

2. (I,I) is a NE if only if $N \leq \theta E$ and $N \leq (1-\theta)E$ or $\frac{N}{E} \leq \theta \leq 1 - \frac{N}{E}$. (If strict inequality holds, the NE is unique.)

3. (P,I) is a NE if only if $N/2 \leq (1-\theta)E$ and $N \geq \theta E$ or $\theta \leq \min\{1 - \frac{N}{2E}, \frac{N}{E}\}$.

4. (I,P) is a NE if only if $N/2 \leq \theta E$ and $N \geq (1-\theta)E$ or $\theta \geq \max\{1 - \frac{N}{E}, \frac{N}{2E}\}$.



11.4 Comparison Advertising

Comparison advertising: The advertised brand and its characteristics are compared with those of the competing brand.
It became popular in the printed media and broadcast media in the early 1970s.
EEC Legal conditions: Material (具體) and verifiable (可證實) details, no misleading (沒有誤導), no unfair (公正).

Advantages of comparison ads:
1. Provide consumers with low-cost means of evaluating available products.
2. Makes consumers more conscious of comparison before buying.
3. Forces the manufacturers to build into the products attributes consumers want.

Negative points:
1. Lack of objectivity.
2. Deception and consumer confusion due to information overload.

Muehling/Stoltman/Grossbart (1990 J of Advertising): 40% of ads are comparison.

Pechmann/Stewart (1990 J of Consumer Research): Majority of ads (60%) are indirect comparison; 20% are direct comparison.

11.4.1 Application of the targeted ad model to comparison ad

Plain ad. => Persuasive ad, aiming at inexperienced consumers.
Comparison ad. => Targeted ad, aiming at experienced consumers.

Applying Proposition 11.5 and the diagram, we the following results:
1. Both firms will use comparison ad only if $E > 2N$.
2. If $2E < N$, both firms will use plain ad.
3. Comparison ad is used by the popular firm and plain ad is used by the less popular firm in other cases in general.

11.5 Other Issues

11.5.1 Can information be transmitted via advertising?

Search goods: False advertising is unlikely.
Experience goods: Producers will develop persuasive methods to get consumers to try their products.

Facts: 1. Due to **assymmetry** of information about quality, consumers can not simply rely on ads.
2. High-quality experience products buyers are mostly experienced consumers.

Schmalensee (1978), "A Model of Advertising and Product Quality," *JPE*.
Low-quality brands are more frequently purchased and firms producing low-quality brands advertise more intensively. $\Rightarrow$ There is a negative correlation between advertising and the quality of advertised products.

Kihlstrom/Riordan (1984), "Advertising as a Signal," *JPE*. High-quality firms have an incentive to advertise in order to trap repeated buyers. $\Rightarrow$ the correlation between ad and quality is positive.

Milgrom/Roberts (1986), "Price and Advertising Signals of Product Quality," *JPE*.
A signalling game model with ad as a signal sent by high-quality firms.

Bagwee (1994), "Advertising and Coordination," *Review of Economic Studies.* and Bagwell/Ramey (1994), "Coordination Economics, Advertising, and Search Behavior in Retail Markets," *AER*. Efficient firms with IRTS tend to spend large amount on advertising to convince buyers that large sales will end up with lower prices. $\Rightarrow$ ad is a signal to reveal low cost.

11.5.2 Advertising and concentration

Is there a positive correlation between advertising and concentration ratio?
Perfect competition industry: Individual firms have no incentives to advertise their products due to free rider effect. Collectively the industry demand can be increased

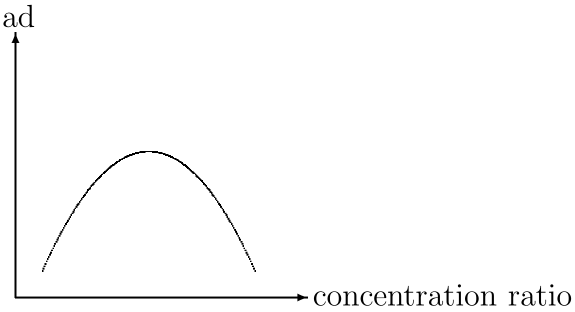
by advertising. However, there is the problem of free rider.
Monopoly industry: Due to scale economy, monopoly firms may have more incentives to advertise.

Kaldor (1950): In an industry, big firms advertise more.

Telser (1964) "Advertising and Competition," *JPE*. Very little empirical support for an inverse relationship between advertising and competition.

Orenstein (1976), "The Advertising - Concentration Controversy," *Southern Economic Journal*, showed very little evidence that there is increasing returns in advertising.

Sutton (1974), "Advertising Concentration, Competition," *Economic Journal*. The relationship between scale and advertising is not monotonic. Both perfect competition and monopoly firms do not have to advertise but oligopoly firms have to.



11.5.3 A simple model of dvertising and prices

Cost: $\text{TC}(Q) = \begin{cases} c_H Q & \text{if } Q \le Q^* \\ c_L Q & \text{if } Q > Q^*, \end{cases}$ Demand: $P = \begin{cases} a_1 - Q & \text{if advertising} \\ a_0 - Q & \text{if not advertising.} \end{cases}$

No Ad equilibrium: $Q_0 = (a_0 - c_H)/2$, $P_0 = (a_0 + c_H)/2$.
Ad equilibrium: $Q_1 = (a_1 - c_L)/2$, $P_1 = (a_1 + c_L)/2$.
If $c_H - c_L > a_1 - a_0$, then $P_1 < P_0$, i.e., advertising reduces the monopoly price.

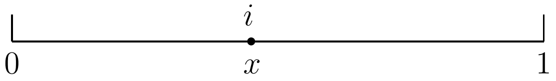
12 Quality

12.1 Vertical Differentiation in Hotelling Model

Quality is a vertical differentiation character.

2-period game:
At $t = 1$, firms A and B choose the quality levels, $0 \leq a < b \leq 1$, for their products.
At $t = 2$, they engage in price competition.

Consumers in a market are distributed uniformly along a line of unit length.



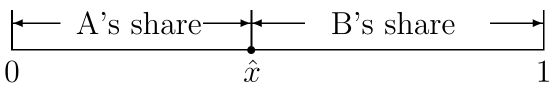
Each point $x \in [0, 1]$ represents a consumer x.

$$U_x = \begin{cases} ax - P_A & \text{if x buys from A.} \\ bx - P_B & \text{if x buys from B.} \end{cases}$$

The marginal consumer $\hat{x}$ is indifferent between buying from A and from B. The location of $\hat{x}$ is determined by

$$a\hat{x} - P_A = b\hat{x} - P_B \Rightarrow \hat{x} = \frac{P_B - P_A}{b - a}. \tag{7}$$

The location of $\hat{x}$ divids the market into two parts: $[0, \hat{x})$ is firm A's market share and $(\hat{x}, 1]$ is firm B's market share.



Assume that the marginal costs are zero. The payoff functions are

$$\Pi_A(P_A, P_B; a, b) = P_A\hat{x} = P_A \left[\frac{P_B - P_A}{b - a}\right], \quad \Pi_B(P_A, P_B; a, b) = P_B(1 - \hat{x}) = P_B \left[1 - \frac{P_B - P_A}{b - a}\right].$$

The FOCs are (the SOCs are satisfied)

$$\frac{\partial \Pi_A}{\partial P_A} = \frac{P_B - 2P_A}{b - a} = 0, \quad \frac{\partial \Pi_B}{\partial P_B} = 1 - \frac{2P_B - P_A}{b - a} = 0. \tag{8}$$

The equilibrium is given by

$$P_A = \frac{b - a}{3}, \; P_B = \frac{2(b - a)}{3}, \quad \Rightarrow \hat{x} = \frac{1}{3}.$$

The reduced profit functions at $t = 1$ are

$$\Pi_A = \frac{(b - a)}{9}, \quad \Pi_B = \frac{4(b - a)}{9}.$$

Both profit functions increase with $b-a$. Moving away from each other will increase both firm's profits. The two firms will end up with maximum product differentiation $a=0$ and $b=1$ in equilibrium.

Modifications: 1. High quality products are associated with high unit production cost.
2. Consumer distribution is not uniform.

12.2 Quality-Signalling Games 限量發行

There are one unit of identical consumers each with utility function

$$U = \begin{cases} H-P & \text{如果買到高品質產品。} \\ L-P & \text{如果買到低品質產品。} \\ 0 & \text{不買。} \end{cases}$$

$C_H > C_L \geq 0$: Unit production costs of producing high- and low-quality product.

AS1 The monopolist is a high-quality producer.

AS2 $H > L > C_H$.

Signalling equilibrium (限量發行): $P^m = H$ and $Q^m = \dfrac{L-C_L}{H-C_L}$.

Proof: 1. For a low-quality monopolist, to imitate the high-quality monopolist is not worthwhile:

$$\Pi_L(P^m, Q^m) = (P^m - C_L)Q^m = (H-C_L)\frac{L-C_L}{H-C_L} = L - C_L = \Pi_L(L,1).$$

2. The high-quality monopolist has no incentives to imitate a low-quality monopolist:

$$\Pi_H(P^m, Q^m) = (P^m - C_H)Q^m = (H-C_H)\frac{L-C_L}{H-C_L} > \Pi_H(L,1) = L - C_H.$$

The last inequality is obtained by cross multiplying.

The high-quality monopolist has to reduce its quantity to convince consumers that the quality is high.
If the information is perfect, he does not have to reduce quantity.
The quantity reduction is needed for signalling purpose.

12.3 Warranties 品質保證書

Spence (1977), "Consumer Misperceptions, Product Failure, and Producer Liability," *Review of Economic Studies.*
Higher-quality firms offer a larger warranty than do low-quality firms.

Grossman (1980), "The Role of Warranties and Private Disclosure about Product Quality," Journal of Law and Economics.
A comprehensive analysis of a monopoly that can offer a warranty for its product.

12.3.1 Symmetric information model

ρ: The probability that the product is operative.
V: The value to the consumer if the product is operative.
Symmetric information: ρ is known to both the seller and the buyer.
P: Price. C: unit production cost. Assumption: $\rho V > C$.

$$U = \begin{cases} V - P & \text{有品質保證書} \\ \rho V - P & \text{無品質保證書} \\ 0 & \text{不買.} \end{cases}$$

Monopoy equilibrium **without** warranty: $P^{nw} = \rho V$ and $\Pi^{nw} = \rho V - C$.

Expected cost of a unit with warranty: $C^w = C + (1-\rho)C + (1-\rho)^2 C + \cdots = \dfrac{C}{\rho}$.
(On average, $1/\rho$ units will end up with one operative unit.)

Monopoy equilibrium **with** warranty: $P^w = V$ and $\Pi^w = V - \dfrac{C}{\rho}$.

It seems that the monopoly makes a higher profit by selling the product with a warranty. However, if the consumer purchases $1/\rho$ units to obtain an operative unit, the result is the same. (Oz Shy's statement is not accurate unless the consumer is risk averse.)

12.3.2 Asymmetric information with warranty as a quality signal

ρ_L: The operative probability of a low-quality product.
$\rho_H > \rho_L$: The operative probability of a high-quality product.
Asymmetric information: The quality of a product is known only to the seller.

Bertrand equilibrium **without** warranty: $P^{nw} = C$ and $\Pi_i^{nw} = 0$, $i = H, L$.

Bertrand equilibrium **with** warranty: $P^w = C/\rho_L$, $Q_H = 1$, $Q_L = 0$, $\Pi_H^w = P^w - \dfrac{C}{\rho_H} > 0$, $\Pi_L^w = P^w - \dfrac{C}{\rho_L} = 0$, $U = V - P^w > 0$.

The high-quality firm has a lower unit production cost of the warranty product. In the market for warranty product, the low-quality firm cannot survive.

13 Pricing Tactics

13.1 Two-Part Tariff

Oi (1971), "A Disneyland Dilemma: Two-Part Tariffs for a Mickey Mouse," *QJE*.
In addition to the per unit price, a monopoly firm (amusement parks 遊樂場, sports clubs 健身俱樂部) can set a second pricing instrument (membership dues 年費) in order to be able to extract more consumer surplus.

P: price, ϕ: membership dues, m: consumption of other goods.
Budget contraint: $m + \phi + PQ = I$. Utility function: $U = m + 2\sqrt{Q}$.

$$\max_Q U = I - \phi - PQ + 2\sqrt{Q} \quad \Rightarrow \text{demand function: } P = \frac{1}{Q^d},\ Q^d = \frac{1}{P^2}.$$

13.1.1 No club annual membership dues

Club capacity: K.
Club profit: $\Pi(Q) = PQ = \sqrt{Q}$

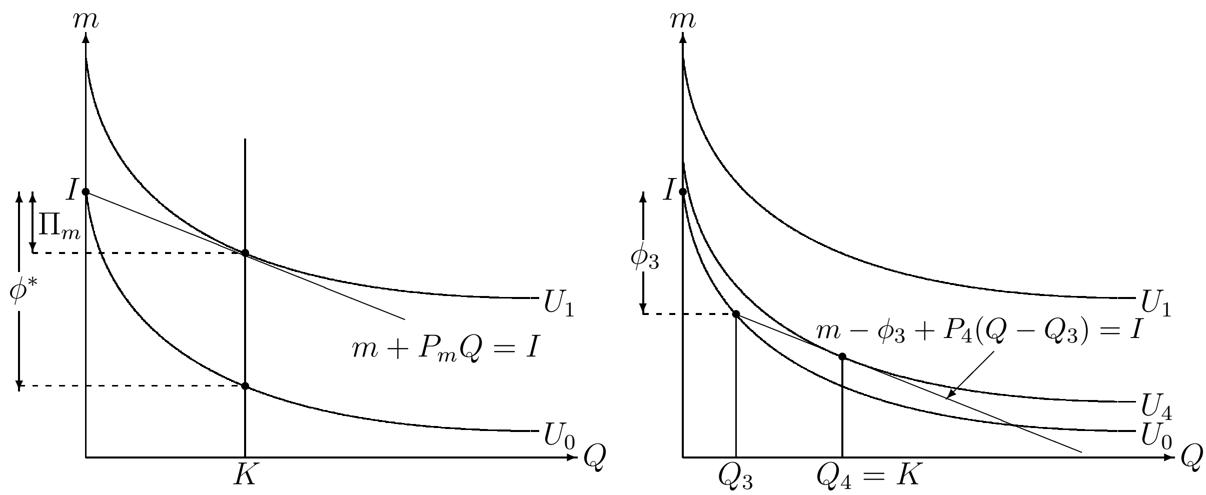
$\max_Q \Pi(Q) \quad \Rightarrow Q_m = K,\ P_m = \dfrac{1}{\sqrt{K}},\ \Pi_m = \sqrt{K}.$

13.1.2 Annual membership dues

$$\max_{\phi} \Pi_a(\phi) = \phi \quad \text{subject to } I - \phi + 2\sqrt{K} > I = I_0,$$

$\Rightarrow \phi^* = 2\sqrt{K} = \Pi_a > \Pi_m.$

Using annual membership dues, the monopoly extracts all the consumer surplus, like the 1st degree price discrimination.



13.1.3 Two-part tariff

There are two problems with membership dues:
1. It is difficult to estimate consumers' utility function and the profit maximizing ϕ^*. If the monopoly sets ϕ too high, the demand would be 0.
2. Consumers are heterogeneous.

Therefore, the monopoly offers a "package" of $Q_3 < K$ and annual fee $\phi_3 < \phi^*$. In addition, the monopoly offers an option to purchase additional quantity for a price P_4.

13.2 Peak-Load Pricing 尖峰, 離峰差別取價

High- and Low-Seasonal Demand Structure: $P^H = A^H - Q^H$, $P^L = A^L - Q^L$, $A^H > A^L > 0$.

Cost Structure: $\text{TC}(Q^H, Q^L, K) = c(Q^H + Q^L) + rK$ for $0 \leq Q^L, Q^H \leq K$.
c: unit variable cost, K: capacity, r: unit capacity cost.

$$\max_{Q^H, Q^L, K} \Pi = P^H Q^H + P^L Q^L - c(Q^H + Q^L) - rK \quad \text{subject to } 0 \leq Q^L, Q^H \leq K.$$

FOC:

$$\text{MR}^H(Q^H) = c + r, \quad \text{MR}^L(Q^L) = c, \quad Q^L < Q^H = K.$$

$$\Rightarrow P^H = \frac{A^H + c + r}{2} > P^L = \frac{A^L + c}{2}.$$

Regulation for efficiency: $P^H = c + r > P^L = c$.

n-period case: $\text{MR}^H(Q^H) = c + \dfrac{r}{n}, \quad \text{MR}^L(Q^L) = c.$

Modification: Substitutability between high- and low-seasonal demand.

14 Marketing Tactics: Bundling, Upgrading, and Dealerships

14.1 Bundling (量販) and Tying (搭售)

Bundling: Firms offer for sale packages containing more than one unit of the product. It is a form of nonlinear pricing (2nd degree price discrimination).

Tying: Firms offer for sale packages containing at least two different (usually complementary) products.
Examples: Car and car radio, PC and software, Book and T-shirt.

14.1.1 How can bundling be profitable?

Monopoly demand: $Q(P) = 4 - P$, MC $= 0$.

Monopoly profit maximization: $P^m = 2 = Q^m$, $\Pi^m = 4$.

Bundling 4-unit package for \$8:
(1) The consumer will have no choice but buying the package.
(2) The monopoly profit becomes $\Pi = 8 > \Pi^m$.

The monopoly in this case uses bundling tactics to extract all the consumer surplus.

14.1.2 How can tying be profitable?

A monopoly sells goods X and Y.
2 consumers, $i = 1, 2$ who have different valuations of X and Y.
Valuations: $V_x^1 = H$, $V_y^1 = L$; $V_x^2 = L$, $V_y^2 = H$, $H > L > 0$.
Assume that consumers do not trade with each other.

Equilibrium without tying:

$$P_x^{nt} = P_y^{nt} = \begin{cases} H & \text{if } H > 2L \\ L & \text{if } H < 2L \end{cases} \quad \text{and} \quad \Pi^{nt} = \begin{cases} 2H & \text{if } H > 2L \\ 4L & \text{if } H < 2L. \end{cases}$$

Equilibrium with tying, $P_T = P_{x\&y}$, $Q_T = Q_{x\&y}$:

$$P_T^t = H + L, \quad \text{and} \quad \Pi^t = 2(H + L) > \Pi^{nt}.$$

14.1.3 Mixed tying

Adams/Yellen (1976), "Commodity Bundling and the Burden of Monopoly," *QJE*.

A monopoly sells goods X and Y.
3 consumers, $i = 1, 2, 3$ who have different valuations of X and Y.
Valuations: $V_x^1 = 4$, $V_y^1 = 0$; $V_x^2 = 3$, $V_y^2 = 3$, $V_x^3 = 0$, $V_y^3 = 4$.

Assume that consumers do not trade with each other.

Equilibrium without tying:
(1) $P_x = P_y = 3$, $Q_x = Q_y = 2$, $\Pi(1) = 12$.
(2) $P_x = P_y = 4$, $Q_x = Q_y = 1$, $\Pi(2) = 8 < \Pi(1)$.
Therefore, $P_x^{nt} = P_y^{nt} = 3$, $Q_x^{ny} = Q_y^{nt} = 2$, $\Pi^{nt} = 12$.

Equilibrium with pure tying, $P_T = P_{x\&y}$, $Q_T = Q_{x\&y}$:
(1) $P_T = 4$, $Q_T = 3$, $\Pi(1) = 12$.
(2) $P_T = 6$, $Q_T = 1$, $\Pi(2) = 6 < \Pi(1)$.
Therefore, $P_T^t = 4$, $Q_T^t = 3$, $\Pi_T^t = 12$.

Equilibrium with mixed tying:
$P_x^{mt} = P_y^{mt} = 4$, $P_T^{mt} = 6$, $Q_x^{mt} = Q_y^{mt} = 1$, $Q_T^{mt} = 1$, $\Pi^{mt} = 14 > \Pi^t = 12$.

But mixed tying is not always as profitable as pure tying.

14.1.4 Tying and foreclosure (拒斥)

US antitrust laws prohibit bundling or tying behavior whenever it leads to a reduced competition. What is the connection between tying and reduced competition?

2 computer firms, X and Y, and a monitor firm Z (compatible with X and Y).
2 consumers $i = 1, 2$ with utility functions

$$U^1 = \begin{cases} 3 - P_x - P_z & \text{buys X and Z} \\ 1 - P_y - P_z & \text{buys Y and Z} \\ 0 & \text{buys nothing,} \end{cases} \qquad U^2 = \begin{cases} 1 - P_x - P_z & \text{buys X and Z} \\ 3 - P_y - P_z & \text{buys Y and Z} \\ 0 & \text{otherwise,} \end{cases}$$

Bertrand equilibrium with 3 independent firms:
(1) $P_x = P_y = 2$, $P_z = 1$, $Q_x = Q_y = 1$, $Q_z = 2$, $\Pi_x = \Pi_y = \Pi_z = 2$.
(2) Other equilibria: $(P_x, P_y, P_z) = (1, 1, 2) = (0, 0, 3) = (3, 3, 0)$.

Assume that firm X buys firm Z and sells X and Z tied in a single package.

Total foreclosure equilibrium:
$P_{xz}^{tf} = 3$, $Q_{xz}^{tf} = 1$, $Q_y^{tf} = 0$, $\Pi_{xz}^{tf} = 3 < \Pi_x + \Pi_z = 4$, $\Pi_y^{tf} = 0$.
P_y does not matter. Consumer 2 is not served. The industry aggregate profit is lower under total foreclosure.

ϵ-foreclosure equilibrium:
$P_{xz}^{\epsilon} = 3 - \epsilon$, $Q_{xz}^{\epsilon} = 2$, $P_y^{\epsilon} = \epsilon$, $Q_y^{\epsilon} = 1$, $\Pi_{xz}^{\epsilon} = 2(3 - \epsilon)$, $\Pi_y^{\epsilon} = \epsilon$.
Consumer 2 buys one X&Z and one Y and discards X.

14.1.5 Tying and International markets segmentation

Government trade restrictions like tarriffs, quotas, etc., help firms to engage in price discrimination across international boundaries.

A two countries, $k = 1, 2$, with one consumer in each country.
A world-monopoly producer sells X.
It can sell directly to the consumer in each country or open a dealership in each country selling the product tied with service to the consumer.
The utility of the consumer in each county (also denoted by $k = 1, 2$) is

$$U^1 = \begin{cases} B_1 + \sigma - P_1^s & \text{if 1 buys X \& service} \\ B_1 - P_1^{ns} & \text{if 1 buys X only} \\ 0 & \text{if 1 does not buy,} \end{cases} \qquad U^2 = \begin{cases} B_2 + \sigma - P_2^s & \text{if 2 buys X \& service} \\ B_2 - P_2^{ns} & \text{if 2 buys X only} \\ 0 & \text{if 2 does not buy,} \end{cases}$$

where P_k^s (P_k^{ns}) are the price with service (without service) in country k, $k = 1, 2$, and $\sigma > 0$ is the additional value due to service.

AS1 $B_1 > B_2$.

AS2 Marginal production cost is 0.

AS3 Unit cost of service provided by the dealership is $w \geq 0$.

No attempts to segment the market:

$$P^{ns} = \begin{cases} B_2 & \text{if } B_1 < 2B_2 \\ B_1 & \text{if } B_1 > 2B_2 \end{cases} \qquad \Pi^{ns} = \begin{cases} 2B_2 & \text{if } B_1 < 2B_2 \\ B_1 & \text{if } B_1 > 2B_2 \end{cases}$$

Segmenting the market:

$$P_k^s = B_k + \sigma, \quad \Pi^s = B_1 + B_2 + 2(\sigma - w).$$

Nonarbitrage condition: $B_1 - B_2 < \sigma$.