Dynamic Collusion

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Collusion in Cournot Competition

- In a one-shot Cournot game, the firms receive payoffs corresponding to the Cournot equilibrium.
- However, this equilibrium does not maximize joint profit.
- The firms can collude, essentially by reducing outputs, to produce a joint output that corresponds to monopolistic output, and obtain joint-monopolistic profit.
- This collusion is unattainable, as collusion does not constitute a NE.

Simple Example

- Two firms, 1 and 2; each produces an output $q_i, i = 1, 2$.
- production cost is 0.
- Linear market demand:

$$P = a - bQ;$$

  where $Q = q_1 + q_2, a, b > 0$, and P is market price.
- Cournot outputs: $q_1^c = q_2^c = a/3b$.
- Price under Cournot equilibrium: $P^c = a/3$.

Simple Example

- Monopoly output, however, is $Q^m = a/2b$, and $P^m = a/2$.
- Therefore, if each firm produces $a/4b$, then profit of each firm is $\pi_i^* = a^2/8b > \pi_i^c, i = 1, 2$.
- This is a standard scenario for collusive pricing.
- However, in a one-shot setting, this scenario cannot work, as the firms face a prisoner's dilemma.

	collusive	non-cooperative
collusive	$\frac{a^2}{8b}, \frac{a^2}{8b}$	$\frac{3a^2}{32b}, \frac{9a^2}{64b}$
non-cooperative	$\frac{9a^2}{64b}, \frac{3a^2}{32b}$	$\frac{a^2}{9b}, \frac{a^2}{9b}$

Simple Example

- In an infinite-horizon setting, suppose the discount factor between periods is $\delta \in (0, 1)$.
- Trigger strategy: Firms start with porducing the collusive quantity $q_1 = q_1^*$ and $q_2 = q_2^*$, and keep producing this quantity if the market price in the previous period is greater or equal to $a/2$. If the price in the previous period is smaller than $a/2$, they switch to q_1^c and q_2^c forever.

Simple Example

- Trigger strategy is a SPE if the profit from collusion is greater than a one-shot deviation, i.e.,

$$\frac{a^2}{8b} \geq (1-\delta)\left[\frac{9a^2}{64b} + \delta\frac{a^2}{9b^2} + \delta^2\frac{a^2}{9b^2}\dots\right]$$

- Inequality satisfied if $\delta \geq 9/15$
- In reality, there is imperfect monotoring.
- In the example, price and total quantity has 1-1 correspondence. Therefore, observing price is enough to infer opponent's quantity.
- What, more realistically, if not?

Green-Porter Model

- Let

$$\widetilde{P} = a - bQ + \widetilde{\varepsilon}$$

- There is a noise to market price.
- For simplicity, assume $\widetilde{\varepsilon} \sim N(0, \sigma^2)$.
- Can't infer the value of Q even if $\widetilde{P}$ is observed.

Green-Porter Model

- Any strategy to sustain collusion?
- Key: market price and total quantity are negatively correlated.
- Since, under collusion, every firm has incentive to cheat by increasing self-quantity, price tends to be lower when a firm cheats.
- Price is then an imperfect signal of cheating.

Green-Porter Model

- A modified trigger strategy:
  Let $\overline{P}$ be the price threshold.
  Firms start with porducing the collusive quantity $q_1 = q_1^*$ and $q_2 = q_2^*$, and keep producing this quantity if the market price in the previous period is greater or equal to $\overline{P}$. If the price in the previous period is smaller than $\overline{P}$, they switch to q_1^c and q_2^c forever.
- Let $V^*(\delta)$ be a firm's payoff if both firms follow the modified trigger strategy.
- Then

$$V^*(\delta) = (1-\delta)\pi^* + \delta F(\overline{P} - \frac{a}{2})\pi^c + \delta(1 - F(\overline{P} - \frac{a}{2}))V^*(\delta).$$

Green-Porter Model

- Sloving for $V^*(\delta)$:

$$V^*(\delta) = \frac{(1-\delta)\pi^* + \delta F(\overline{P} - \frac{a}{2})\pi^c}{1 - \delta(1 - F(\overline{P} - \frac{a}{2}))}.$$

- Note that $V^*(\delta)$ is a linear combination of π^* and π^c and, since $\pi^* > \pi^c$, it must be $V^*(\delta) > \pi^c$.
- Now we want to find the condition under which the modified trigger strategy is a SPE.
- Let $V(q, \delta)$ be a firm's life-time payoff when it switches to q for one period, then follows the modified trigger strategy (i.e., a one-shot deviation to q).

Green-Porter Model

- Easy to see

$$V(q,\delta) = (1-\delta)\left[a - b(\frac{a}{4b} + q)\right] + \delta F(\overline{P} - \frac{3}{4}a + bq)\pi^c +$$
$$\delta(1 - F(\overline{P} - \frac{3}{4}a + bq))V^*(\delta).$$

- $$\frac{\partial V(q,\delta)}{\partial q} = (1-\delta)(\frac{3}{4}a - 2bq) - \delta b f(\overline{P} - \frac{3}{4}a + bq)(V^*(\delta) - \pi^c).$$

- If q_1^* and q_2^* constitute SPE, it must be that

$$\frac{\partial V(q,\delta)}{\partial q}|_{q=\frac{a}{4b}} = 0,$$

i.e, FOC holds at $q_i = q_i^*$.

- $$\frac{1}{4}(1-\delta)a - \delta b f(\overline{P} - \frac{1}{2}a)(V^* - \pi^c) = 0. \quad (1)$$

Green-Porter Model

- Assume $f(0) > a(1-\delta)/4\delta b(V^* - \pi^c)$.
- LHS of (1) is positive when $\overline{P} \to -\infty$, and increases as $\overline{P}$ approaches $a/2$. By assumption, the LHS of (1) is negative when $\overline{P} = a/2$.
- By mean-value theorem, there exists $\overline{P}^* \in (-\infty, \frac{a}{2})$ such that (1) holds.
- $\overline{P}^*$ is the value of the threshold price to be identified.

Green-Porter Model

- Note that, under the modified trigger strategy, the firms eventually go into the non-cooperative stage with probability 1.
- The punishment does not have to be infinitely long. This improves efficiency.
- As a punishment, the firms can engage in a Cournot competition for n period, then go back to collusive state.
- Our previous calculation is for case when $n = \infty$.

Green-Porter Model

- Easy to see that previous result goes through if n is large enough.
- Collusion *á* la Green-Porter will then be characterized by cycles of collusive phrase and competitive phrase.

Rotemberg-Saloner Model

- Suppose the production cost is c for both firms.
- In every period, firms observe the value of $\tilde{\varepsilon}$ before setting price.
- Bertrand competition.
- Bertrand equilibrium: $p_1^B = p_2^B = c$.
- $q_1^B = q_2^B = \frac{a-c+\varepsilon_t}{2}$; where ε_t is the realization of $\tilde{\varepsilon}$ in period t.
- $\pi_1^B = \pi_2^B = 0$.

Rotemberg-Saloner Model

- In any period t, monopolic price solves for

$$Max_P(P-c)\frac{-P+a+\varepsilon_t}{b}.$$

- $p_1^M = p_2^M = \frac{a+c+\varepsilon_t}{2}$.
- $q_1^M = q_2^M = \frac{a-c+\varepsilon_t}{4b}$.
- $\pi_1^M = \pi_2^M = \frac{(a-c+\varepsilon_t)^2}{8b}$.

Rotemberg-Saloner Model

- Consider the following trigger strategy:
- In each period, each firm i sets the price p_i^M. They keep this price as long as the other firm's price is higher or equal to p_i^M in previous period. If in any period, the other firm's price is lower than p_i^M, then switch to p_i^B forever.
- Want to find the condition under which the strategy is SPE.

Rotemberg-Saloner Model

- In every period t,

$$(1-\delta)\frac{(a-c+\varepsilon_t)^2}{8b}+\delta\frac{(a-c)^2+V(\tilde{\varepsilon})}{8b}\geq(1-\delta)\frac{(a-c+\varepsilon_t)^2}{4b}+0.$$

- That is,

$$\frac{\delta}{1-\delta}\frac{(a-c)^c+V(\tilde{\varepsilon})}{8b}\geq\frac{(a-c+\varepsilon_t)^2}{4b}. \tag{2}$$

- As usual, the greater the value of δ, the more likely (2) holds.
- However, the greater the value of ε_t, the less likely (2) holds, implying a boom will more likely trigger the price war.

Rotemberg-Saloner Model

- This contrasts the result in Green-Porter.
- Green-Porter: Recession $\rightarrow$ price war.
- Rotemberg-Saloner: Boom $\rightarrow$ price war.
- The stark difference in conclusions does not come from one being Cournot and the other being Bertrand, but from the fact that ε is not observed before output decision is made in Green-Porter, but observable in Rotemberg-Saloner.