Several Complex Variables and Complex Geometry Conference dedicated to the mathematical work of Yum-Tong Siu

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Talk Abstracts

Damian Brotbek (Université de Lorraine)

Unicity of meromorphic function and hyperbolicity

Nevanlinna's five value theorem states that given five distinct points $a_1, \dots, a_5$ of the projective line, and two meromorphic functions f, g on the complex plane, if $f^{-1}(a_i) = g^{-1}(a_i)$ for all i then $f = g$. Nevanlinna also proved a similar result for four points, if one counts multiplicity. Since the work of Nevanlinna, there has been many results generalizing and refining these unicity statements. In this talk, we will show that these questions can be geometrically interpreted as hyperbolicity problems for some specific log pairs. We will use this to establish some new unicity statements using technics from hyperbolicity. This allows us also to use the Green-Griffiths-Lang conjecture to state some unicity conjectures. This is a joint work with Benoit Cadorel and Yunling Chen.

Tsz on Mario Chan (Pusan National University)

Residue functions, adjoint ideal sheaves and their applications

We introduce in this talk a class of "residue functions", each of which "deforms" holomorphically certain weighted L^2 norm on the ambient complex manifold X to an L^2 norm on the union of certain log-canonical (lc) centres of a given lc pair (X, D). The properties of such residue functions can be encoded into a sequence of analytic adjoint ideal sheaves, which fit into various residue short exact sequences and are useful in facilitating induction on (co)dimension of lc centres in geometric problems involving lc singularities. As an illustration, we will see their use in a solution to Fujino's conjecture, that is, the injectivity theorem for lc pairs on compact Kähler manifolds. The content of this talk is based on the joint work with Young-Jun Choi and Shin-ichi Matsumura.

John Erik Fornæss (Norwagian University of Sciences and Technology)

Newtons method

I report on recent joint work with Hu and Truong. Newtons method is used to find roots of complex polynomials in C. Each root has a basin of attraction. Normally these basins have fractal boundaries. Truong has found a version of Newtons method where computer pictures indicate that the basins have smooth boundaries. We give a rigorous proof of this experimental discovery.

Franc Forstneric (University of Ljubljana)

Runge and Mergelyan theorems on families of open Riemann surfaces

I will present a newly developed Oka theory for maps from families of open Riemann surfaces to any Oka manifold. Particular cases include Runge and Mergelyan approximation theorems and Weierstrass interpolation theorem on such families, with continuous or smooth dependence of the data and the approximating holomorphic functions on the complex structure. I will present applications to the construction of families of directed holomorphic immersions and conformal minimal immersions.

Xianghong Gong (University of Wisconsin-Madison)

Global Newlander-Nirenberg problem on domains with finite smooth boundary in a complex manifold

Let M be a relatively compact C^2 domain in a complex manifold $\mathcal{M}$ of dimension n. Assume that $H^1(M, \Theta) = 0$ where Θ is the sheaf of germs of holomorphic tangent fields of M. Suppose that the Levi-form of the boundary of M has at least 3 negative eigenvalues or at least $n-1$ positive eigenvalues pointwise. We will first construct a homotopy formula for Θ-valued $(0,1)$-forms on $\overline{M}$. We then apply a Nash-Moser iteration scheme to show that if a formally integrable almost complex structure of the Hölder–Zygmund class Λ^r on $\overline{M}$ is sufficiently close to the complex structure on M in the Hölder–Zygmund Λ^{r_0} norm on M for some $r_0 > 5/2$, there is a diffeomorphism F from $\overline{M}$ into $\mathcal{M}$ that transforms the almost complex structure into the complex structure on $F(M)$, where $F \in \Lambda^s(M)$ for all $s < r + 1/2$. This is joint work with Ziming Shi.

Pak-Tung Ho (Tamkang University)

CR Paneitz operator and its application in CR geoemtry

In this talk, I will talk about the CR Paneitz operator and some of its properties. I will then talk about some related results in CR geometry, including the CR positive mass theorem and the convergence of the CR Yamabe flow. Finally, I will mention some of the very recent results about the spectrum of the CR Paneitz operator in the non embeddable case, which is a joint work with Yuya Takeuchi.

Jun-Muk Hwang (Institute for Basic Science)

Convergence of formal equivalences between embeddings

We say that the formal principle with convergence holds for a compact complex submanifold A in a complex manifold X, if a formal isomorphism between the formal neighborhood of A in X and that of another embedding of A in any complex manifold always converges, namely, the formal isomorphism can be extended to a biholomorphic map between suitable Euclidean neighborhoods. Hirschowitz and Commichau-Grauert showed that the formal principle with convergence holds if the normal bundle of the submanifold is sufficiently positive. We discuss the problem when the normal bundle is only weakly positive, but the submanifold satisfies certain geometric conditions. Our main interest is when the submanifold is a general minimal rational curve in a uniruled projective manifold, such as a general line on a rational homogeneous space or a projective hypersurface of low degree. This is a joint work with Jaehyun Hong.

Sung Yeon Kim (Institute for Basic Science))

Proper holomorphic maps between bounded symmetric domains with small rank differences

In this talk, we study the rigidity of proper holomorphic maps $f\colon \Omega \to \Omega'$ between irreducible bounded symmetric domains Ω and Ω' with small rank differences: $2 \leq \operatorname{rank}(\Omega') < 2\operatorname{rank}(\Omega) - 1$. More precisely, if either Ω and Ω' have the same type or Ω is of type III and Ω' is of type I, then up to automorphisms, f is of the form $f = \imath \circ F$, where $F = F_1 \times F_2\colon \Omega \to \Omega'_1 \times \Omega'_2$. Here Ω'_1, Ω'_2 are bounded symmetric domains, the map $F_1\colon \Omega \to \Omega'_1$ is a standard embedding, $F_2 : \Omega \to \Omega'_2$, and $\imath\colon \Omega'_1 \times \Omega'_2 \to \Omega'$ is a totally geodesic holomorphic isometric embedding. As a consequence, $f : \Omega \to \Omega'$ is a holomorphic totally geodesic isometric embedding with respect to Kobayashi metrics. Moreover we show that, under the rank condition above, there exists no proper holomorphic map $f : \Omega \to \Omega'$ if Ω is of type I and Ω' is of type III, or Ω is of type II and Ω' is either of type I or III. This is a joint work with N. Mok and A. Seo.

Takayuki Koike (Osaka Metropolitan University)

Formal principle for line bundles on neighborhoods of an analytic subset of a compact Kähler manifold

We investigate the formal principle for holomorphic line bundles on neighborhoods of an analytic subset of a complex manifold mainly when it can be realized as an open subset of a compact Kähler manifold. In particular, we will explain our recent result mainly when the manifold is of dimension 2.

Song-Ying Li (University of California, Irvine)

Sup-norm-Estimate for Cauchy-Riemann Equations on Bounded Domain in $\mathbf{C}^n$ with Non-smooth Boundary

In this talk, I will present recent developments in the estimation of solutions to the Cauchy-Riemann equation: $\overline{\partial}u = f$ on bounded domains Ω in $\mathbf{C}^n$ with non-smooth boundary $\partial\Omega$.

Specifically, I will focus on the case where $\Omega = D^n$, a product domain, with D being a bounded domain in the complex plane C. The results include my recent work on uniform $L^p(D^n)$-estimate, which establish that

$$\|u\|_{L^p(D^n)} \leq C\|f\|_{L^p(D^n)} \quad \text{with } C \text{ a uniform constant,}$$

for the canonical solution to $\overline{\partial}u = f$ where $1 \leq p \leq \infty$ and the boundary ∂D is $C^{1.\alpha}$ for some $\alpha > 0$.

Additionally, I will discuss my most recent joint work with Long and Luo, where we obtained uniform $L^p(D^n)$ -estimates for $\overline{\partial}$ on on domains D^n with ∂D having a Lipschitz boundary, for all $1 < p \leq \infty$.

Xiaonan Ma (Université de Paris)

Superconnection and family Bergman kernels

We establish an asymptotic version of Bismut's local family index theorem for the Bergman kernel associated with a fiberwise positive line bundle when the power tends to infinity. The key idea is to use the superconnection as in the local family index theorem. In particular, we show the curvature operator of the associated direct image is a Toeplitz operator.

George Marinescu (University of Cologne)

Spectral asymptotics for semipositive line bundles

We present several results concerning semiclassical asymptotics of the spectral function of the Kodaira Laplacian for high powers of a semi-positive line bundle, many of them inspired by the pioneering work of Professor Siu. This talk is based on joint work with Chin-Yu Hsiao, Xiaonan Ma and Nikhil Savale.

Ngaiming Mok (The University of Hong Kong)

Geometric structures and sub-structures arising from varieties of minimal rational tangents and their tangential linear sections

Let X be a uniruled projective manifold and $\mathcal{K}$ be an irreducible component of a moduli space of minimal rational curves on X. Starting in the late 1990s, I have developed with Jun-Muk Hwang the basics of a geometric theory of varieties of minimal rational tangents (VMRTs) $\mathscr{C}_x(X) \subset \mathbb{P}T_x(X)$ on $(X, \mathcal{K})$, captured by the VMRT structure $\pi : \mathscr{C}(X) \to X$, which generalizes $\mathbf{S}$-structures on irreducible Hermitian symmetric spaces $\mathbf{S} = G/P$ of rank ≥ 2. More recently I have been further developing with collaborators a theory of geometric substructures on $(X, \mathcal{K})$, called sub-VMRT structures, supported on a complex submanifold $S \subset W$ of an open subset $W \subset X$ in the complex topology and defined by taking intersections of VMRTs with projectivized tangent subspaces on S, given by $\varpi : \mathscr{C}(S) \to S$, $\mathscr{C}_x(S) := \mathscr{C}_x(X) \cap \mathbb{P}T_x(S)$. Here $\varpi : \mathscr{C}(S) \to S$ is assumed surjective. One source of the study of

geometric substructures on Fano manifolds was the works of Walters (1997), Bryant (2001) and Hong (2007) on Schur rigidity of smooth nonlinear Schubert cycles on irreducible Hermitian symmetric spaces $\mathbf{S}$ of rank ≥ 2 settled by a reduction to the study of certain differential systems. The same problem for singular Schubert varieties $Z \subset \mathbf{S}$, already taken up by the above authors, was inspired by a problem of Hartshorne's on the smoothability of representatives of certain homology classes. The problem of Schur rigidity on M was settled by Robles-The (2012) by Lie-theoretic cohomological methods.

In this lecture, I will focus on characterization problems involving VMRTs and sub-VMRTs. This includes the problem of rigidity under projective deformation of rational homogeneous spaces $X = G/P$ of Picard number 1 (Hwang-Mok 2005), the Recognition Problem for the characterization of $X = G/P$ by its VMRT $\mathscr{C}_x(X) \subset \mathbb{P}T_x(X)$ at a general point (Mok 2008, Hong-Hwang 2008, Hwang-Li 2021), and the problem in algebraic geometry of characterizing certain projective subvarieties of X such as Schubert cycles (Hong-Mok 2013, 2021; Mok-Zhang 2019). In addition, we give applications of our study of sub-VMRT structures to rigidity problems concerning proper holomorphic maps on the interface of several complex variables and Cauchy-Riemann geometry (Kim-Mok-Seo 2023) and to the (yet unpublished) characterization of certain holomorphic isometric embeddings of the complex unit ball in Kähler geometry (Mok-Yang).

Duong H. Phong (Columbia University)

Non-linear heat flows in symplectic geometry

The equations for the Type IIA string theory stand apart from the equations for the other four string theories by having symplectic manifolds as their setting. Remarkably, they have led to a confluence of symplectic geometry, special holonomy, and non-integrable complex structures. They have also led to new problems for the theory of non-linear partial differential equations, some of which have been solved, but many are wide open. We describe these developments, with emphasis on the open problems. This is joint work of the speaker with Teng Fei, Sebastien Picard, and Xiangwen Zhang.

Erwan Rousseau (Université de Brest)

A generalization of Bloch-Ochiai's theorem

The classical Bloch-Ochiai theorem states that a complex projective manifold with irregularity larger than its dimension has no Zariski dense entire curve. I will present a generalization of this theorem in the setting of pairs. (Joint work with S. Kebekus)

Min Ru (University of Houston)

Vojta's abc conjecture for entire curves in toric varieties highly ramified over the boundary

In this talk, I will discuss my joint work with Julie Tzu-Yueh Wang on Vojta's abc conjecture for projective space $\mathbb{P}^n(\mathbb{C})$, assuming that the entire curves in $\mathbb{P}^n(\mathbb{C})$ are highly ramified over the coordinate hyperplanes. We also explore the corresponding results for projective toric varieties. Consequently, we establish a version of Campana's orbifold conjecture for finite coverings of projective toric varieties.

Yum-Tong Siu (Harvard University)

Overview of recent results and problems in several complex variables

This overview seeks to introduce, to a general mathematical audience, recent results and problems in several complex variables by surveying currently active topics such as the following.

(i) Insolvable partial differential equation of Hans Lewy. Hörmander's hypoellipticity of sums of squares of vector fields. Kohn's subelliptic multipliers for weakly pseudoconvex domains with D'Angelo's finite type condition.

(ii) Roles of flat directions in global non-deformability of irreducible compact Hermitian symmetric manifolds and in strong and super rigidity problems.
(iii) Effective theorems such as the Fujita conjecture, especially its very ampleness part.
(iv) Skoda's division and finite generation of canonical ring for any general compact complex manifold.
(v) Hyperbolicity problem to lower degree required for hyperbolicity of generic complex hypersurface. Nevanlinna theory with counting function for nonsingular complex hypersurface in complex projective space.
(vi) Gelfond-Schneider's technique for Hilbert's 7th problem. Lang-Bombieri's theory of algebraic values of meromorphic maps. Bombieri-Pila's counting of integral points on arcs. Pila-Wilkie's o-minimal geometry, together with further later developments by many others, to treat the André-Oort conjecture.

Laurent Stolovitch (Universite Cote dÁzur)

Local rigidity of actions of isometries on compact Riemannian manifolds

In this joint work with Zhiyan Zhao (Nice), we consider perturbations of isometries of a compact Riemannian manifold M. We prove that, under some conditions, a finitely presented group of such small enough perturbations is analytically or smoothly conjugate on (analytic or smooth) M to the same group of isometry it is a perturbation of. The result generalizes the rigidity theorems of Arnold, Herman, Yoccoz, Moser, etc. about circle diffeomorphisms which are small perturbations of rotations. It also generalizes Fisher-Margulis's theorem on group actions satisfying Kazhdan's property (T). The proof relies on a "Diophantine-like" condition, relating the actions of the isometry group and the eigenvalues of the Laplace-Beltrami operator.

Kuang-Ru Wu (National Tsing Hua University)

A potential theory for the Wess–Zumino–Witten equation in the space of Kähler potentials

We develop a potential theory for the Wess–Zumino–Witten (WZW) equation in the space of Kähler potentials which is parallel to the potential theory for the Hermitian–Yang–Mills equation. A concept called ω-harmonicity on graphs is introduced which characterizes the WZW equation. The harmonic map into the space of Kähler potentials, as a special case of the WZW equation, is also investigated. In particular, we show the solvability of the Dirichlet problem for the harmonic map, and the approximation/quantization by its finite dimensional counterparts.

Ming Xiao (University of California, San Diego)

A new Poincaré type rigidity phenomenon with applications

In this talk, we discuss a new Poincaré type phenomenon. More precisely, we will present an optimal rigidity theorem for local CR mappings between circle bundles that are defined in a canonical way over (possibly reducible) bounded symmetric domains. We prove such a local CR map, if nonconstant, must extend to a rational biholomorphism between the corresponding disk bundles. The result includes as a special case the classical Poincaré-Tanaka-Alexander theorem. Among other applications, we construct, for any $n \geq 2$, a countably infinite family of compact locally homogeneous strongly pseudoconvex CR hypersurfaces (in complex manifolds) of real dimension $2n+1$ with transverse symmetry such that they are all obstruction flat and Bergman logarithmically flat. Moreover, their local CR structures are mutually inequivalent. Such a family cannot exist in dimension three by known results: A Bergman logarithmically flat CR hypersurface must be spherical, and so is a compact obstruction flat CR hypersurface with transverse symmetry.

Dmitri Zaitsev (Trinity College Dublin)

Global regularity in the d-bar-Neumann problem and finite type conditions.

The celebrated work of Catlin on the global regularity of the $\bar{\partial}$-Neumann operator for pseudoconvex domains of finite type links local algebraic- and analytic geometric invariants through potential theory with estimates for $\bar{\partial}$-equation. Yet despite their importance, there seems to be a major lack of understanding of Catlin's techniques, resulting in a notable absence of an alternative proof, exposition or simplification.

The goal of my talk will be to present an alternative proof based on a new notion of a "tower multi-type". The finiteness condition of the tower multi-type is an intrinsic geometric condition that is more general than the finiteness of the regular type, which in turn is more general than the finite type. Under the latter condition, we obtain a generalized stratification of the boundary into countably many level sets of the tower multi-type with certain convexity properties. The existence of such stratification implies Catlin's potential-theoretic "Property (P)", which, in turn, is known to imply global regularity via compactness estimate. Notable applications of global regularity include Condition R by Bell and Ligocka and its applications to boundary smoothness of proper holomorphic maps generalizing a celebrated theorem by Fefferman.

Xiangyu Zhou (Beijing Academy of Sciences)

Siu's lemma, L^2 estimates, and curvature positivity

In this talk, we'll first recall some basic properties of multiplier ideal sheaves associated to plurisubharmonic functions (or pseudoeffective line bundles) including Siu's lemma on semi-continuity of multiplier ideal sheaves, and then introduce some recent results including 1) a solution of Demailly's strong openness conjecture (Guan- Zhou); 2) variants of Siu's lemma (Zhou-Zhu); 3) strong openness of the multiplier submodule sheaves which are the vector bundle version of multiplier ideal sheaves (Liu-Xiao-Yang-Zhou); 4) some applications. We'll also present our result on characterizing Nakano positivity via solving $\bar{\partial}$- equations with L^2 estimates (Deng-Ning-Wang-Zhou), which is a converse proposition of Hörmander-Demailly's L^2 existence theorems. As an application of the criterion, we give an affirmative answer to Lempert's problem (Liu-Yang-Zhou), which asks whether the limit metric of an increasing sequence of hermitian metrics with Nakano semi- positive curvature on holomorphic vector bundles is still Nakano semi-positive. Our work has been inspired by Siu's achievements on L^2 theory and multiplier ideal sheaves.