Thm: M is compact, Riem 3-mfld

$M \times SL(2;\mathbb{C})$. Sequence $\{\mathbb{A}_k = A_k + i\alpha_k\}$

A_k: SU(2) connection

α_k: 1-form w/ values in Lie alg SU(2)

such that

$$\int_M \left(|F_{A_k} - \alpha_k \wedge \alpha_k|^2 + |d_{A_k}\alpha_k|^2 + |d_{A_k} * \alpha_k|^2 \right) < E < \infty$$

1) $\exists$ a subsequence with

$$\int_M |\alpha_k|^2 < R < \infty$$

$\Rightarrow$ There exists $h_k : M \to SU(2)$ such that a subseq. of

$$\{ (h_k^* A_k = h_k A_k h_k^{-1} + h_k d h_k^{-1},\ h_k \alpha_k h_k^{-1}) \}$$

converges weakly in L_1^2-top.

2) $\nexists$ no subsequence with

$$\int |\alpha_k|^2 < R < \infty$$

2) No subsequence with

$$\int_M |\sigma_n|^2 < R < \infty$$

a) a Holder $C^{0,1/2}$ and L^2_1 function $f: M \to [0,\infty)$

b) $f^{-1}(0) \equiv Z$ is contained in a countable union of Lipshitz curves & has Haussdorf dimension ≤ 1

c) A real line bundle $\mathcal{I} \to M \setminus Z$

d) A 1-form on $M \setminus Z$ with values in $\mathcal{I}$ that is v

i) Harmonic: $dv = 0$ $d * v = 0$

ii) $|v| = f$

iii) $|\nabla v|$ is an $L^2(M)$ function

e) An $SU(2)$ connection, $\hat{A}$ on $M \setminus Z$

f) An $\hat{A}$-covariantly constant $\mathcal{I}$-valued map Lie-alg. $SU(2)$ on $M \setminus Z$

σ $|\sigma| = 1$

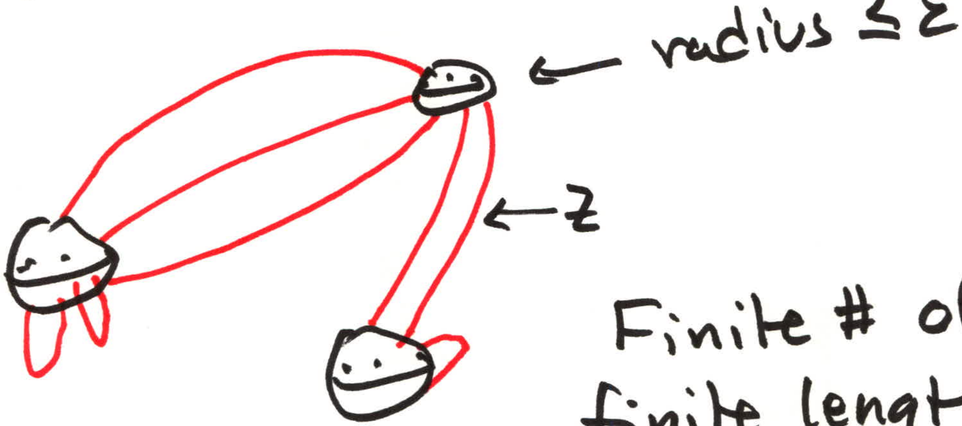
$\exists$ a sequence $h_n : M \setminus Z \to SU(2)$
with

i) $h_n^* A_n$ converges on $M \setminus Z$ in weak, $L^2_{1;loc}$ topology to $\hat{A}$

ii) $\Omega_n^2 = \int_M |\sigma_n|^2$ then

$\frac{1}{\Omega_n} \sigma_n$ converges in $L^2_{1\,loc}$ topology to $\nu \sigma$

iii) Fix $\varepsilon > 0$



Finite # of finite length components.

2 - dim version:

$M = S^1 \times \Sigma$ Σ is a Riemann surface

$\{\mathbb{A}_n = A_n + i\mathfrak{a}_n\}$ is S^1 invariant.

- Z is a $S^1 \times \underbrace{\{\text{finite set}\}}_{= \Theta}$ $\leq 4g-4$ points $g = \text{genus}(\Sigma)$

- ν is S^1 invariant, $\mathfrak{g}$-valued, harmonic 1-form on $\Sigma \setminus \Theta$ $d\nu = 0$, $d{*}\nu = 0$

$T^*\Sigma \otimes \mathbb{C} = T^{1,0} \oplus T^{0,1}$

$\nu = \nu_{1,0} + \nu_{0,1}$ $\nu_{1,0} \in T^{1,0} \otimes \mathfrak{g}$

$\nu_{1,0}$ is a holomorphic section

$\nu_{1,0}^2$ is a section $T^{2,0}$

$\nu_{1,0}^2$ is a holom. quadratic differential

μ = holomorphic, quadratic differential

$$\mu \sim z^p$$

$$v_{1,0} \sim z^{p/2}$$

p = odd

v changes sign

$\mathcal{F}$ is nontrivial if μ has simple zeros for example.

Second example

M , $\{ \mathbb{A}_n = A_n + i\alpha_n \}_{n=1}^{\infty}$

these are <u>flat</u> connections

$\mathbb{A} = A + i\alpha$ is flat

$$F_A - \alpha \wedge \alpha = 0 \qquad d_A \alpha = 0$$

$$d_A * \alpha = 0$$

Equivalence classes under $SL(2;\mathbb{C})$ automorphism $\Longleftrightarrow$

$$\left\{ \begin{array}{l} \text{Homomorphisms from } \pi_1(M) \\ \quad \text{to } SL(2;\mathbb{C}) \end{array} \right\} \Big/ \text{conjugation}$$

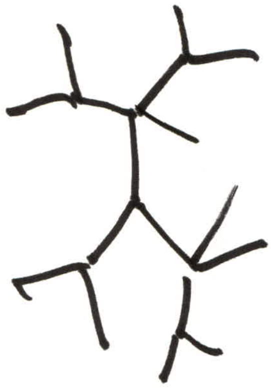
$$\rho : \pi_1(M) \longrightarrow SL(2;\mathbb{C})$$

Morgan-Shalen compactification

$$\{ \mathrm{Hom}(\pi_1(M)); SL(2;\mathbb{C}) \}$$

$\pi_1(M)$-equivariant maps ~~for~~ from $\hat{M}$'s universal cover an $\mathbb{R}$-tree.

$\mathbb{R}$-tree, Y, is a contractible metric space such that any 2-pts have a unique path between them

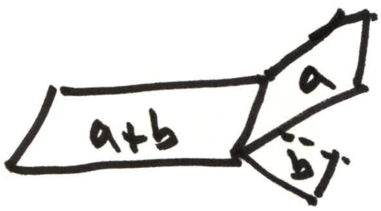


Maps from $\pi_1(M)$ to some $\mathbb{R}$-tree

- Morgan-Shalen, Gabai, Thurston, Oertel, Haken, . .

1.) Branched surface

incompressible



2.) Singular, transversally measured foliation

3.) transversally measured lamination

4.) Equiv. maps to $\mathbb{R}$-trees

- Korevaar–Schoen: MS map to an $\mathbb{R}$-tree
+ Daskalopoulos–Dostoglu–Wentworth: can be realized by a harmonic map

$$u : \hat{M}^n \to Y$$

- X.–Sun: Set of singular values $(du = 0)$ is closed, & has Hauss. dim ≤ 1

$v \qquad dv = 0 \qquad$ on $M \setminus Z$

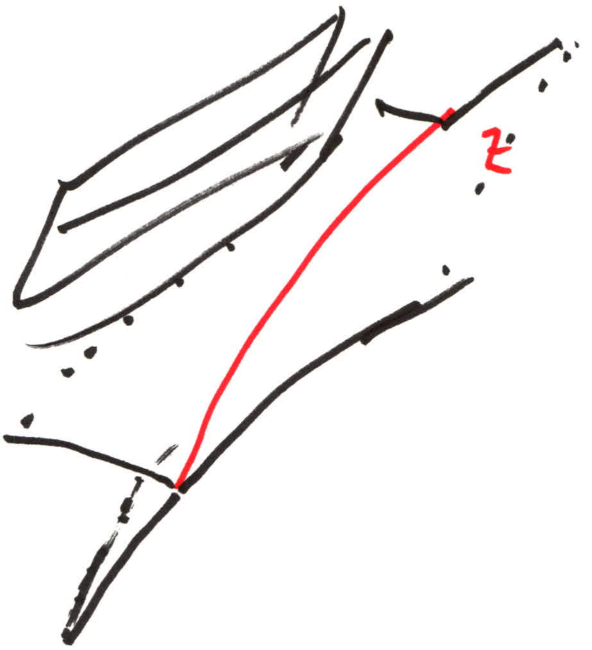
$\qquad d{*}v = 0$

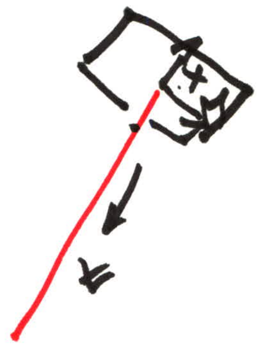
$$\begin{array}{c} \hat{M}^n \\ \downarrow \pi \\ M \end{array} \qquad \pi^* v = du$$

General case $M \setminus Z$

$\ker(v)$ is a 2-plane bundle in $T(M \setminus Z)$

integrable $\Leftrightarrow$ $dv = 0 \quad d{*}v = 0$





$v \sim \mathrm{Re}$(square root of holom. differential)

$v \qquad z = x+iy$

$v \sim z^{p/2}$ $\quad$ p an integer

$\mathrm{Re}\, v \sim dw \qquad w \sim \mathrm{real}(z^{\frac{p+2}{2}})$

$w^{-1}(c)$ these are leaves of foliation