2019 Taipei Conference on Geometric Invariance and Partial Differential Equations
Institute of Mathematics, Academia Sinica, Taipei, Taiwan
January 11 - 14, 2019

Einstein asymptotically complex hyperbolic metrics and almost complex structures

Yoshihiko Matsumoto

Osaka University
E-mail:**matsumoto@math.sci.osaka-u.ac.jp**

Abstract

I will discuss two problems on asymptotically complex hyperbolic spaces (ACH spaces): Given a domain Ω whose boundary $\partial\Omega$ is equipped with a contact distribution H and a compatible (i.e., partially integrable) almost CR structure $\gamma \in \Gamma(\mathrm{End}(H))$ that is strictly pseudoconvex,

(I) construct an Einstein ACH metric on Ω with conformal infinity γ;

(II) extend the conformal infinity γ to an almost complex structure of Ω in a preferable way.

Our models are bounded strictly pseudoconvex domains Ω in $\mathbb{C}^n$ (then $\partial\Omega$ has the induced CR structure γ_{ind}, which is strictly pseudoconvex). In this case, Cheng–Yau's complete Kähler-Einstein metric is certainly an Einstein ACH metric with conformal infinity γ_{ind}, and the complex structure on Ω inherited from $\mathbb{C}^n$ should undoubtedly be considered "preferable."

Problem (I) is taken up by Roth and Biquard; they proved that any compatible almost CR structure γ on S^{2n-1} sufficiently close to the standard one can be filled with an Einstein ACH metric on B^{2n} constructed by deforming the complex hyperbolic metric (the "complex" counterpart of the Graham–Lee theorem). We shall give its generalization to deformations of the Cheng–Yau metric on an arbitrary strictly pseudoconvex domain in $\mathbb{C}^n$—under the technical assumption $n \geq 3$.

Problem (II) is completely new. Our approach is to consider, for a fixed Einstein ACH metric g, critical points of a certain torsion functional defined in the space of almost complex structures with respect to which g is Hermitian. Then such a critical almost complex structure is shown to exist for metrics constructed in problem (I).