

A construction of Poincaré-Einstein metrics of cohomogeneity one on the ball

Yoshihiko Matsumoto

Osaka University

E-mail: matsumoto@math.sci.osaka-u.ac.jp

Abstract

Poincaré-Einstein metrics and their relationship with conformal structures on the boundary at infinity have attracted much interest since 1980s, but the knowledge on the existence of such metrics is still quite limited. After sketching the current state of the matter, I will discuss yet another explicit construction of a one-parameter family of Poincaré-Einstein metrics given by a method of Page and Pope, which takes place on the unit ball B^{2n} of even dimension, $n \geq 2$. This is a generalization of Pedersen's construction in B^4 , and the associated conformal infinities are given by the Berger metrics on the sphere.

The limiting behavior of this family is of particular interest. As the fibers of the boundary S^{2n-1} become large, the Einstein metric tends to the complex hyperbolic metric on the ball; at the other end of the family, the boundary collapses to the complex projective space $\mathbb{C}P^{n-1}$ and the interior also collapses to a $(2n - 1)$ -dimensional Einstein space that has one singular point.