2022 Workshop on Algebraic Combinatorics Institute of Mathematics, Academia Sinica, Taipei, Taiwan Jan. 24-26, 2022

Covering by polynomial planks

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Abstract

In 1932, Tarski conjectured that a convex body of width 1 can be covered by planks, regions between two parallel hyperplanes, only if the total width of planks is at least 1. In 1951, Bang proved the conjecture of Tarski. In this work we study the polynomial version of Tarski's plank problem.

We note that the recent polynomial proofs of (particular cases of) the spherical and complex plank covering problems by Zhao and Ortega-Moreno give some general information on zeros of real and complex polynomials restricted to the unit sphere. We establish polynomial analogs of the Bang theorem by explaining how to find a point in the unit ball sufficiently far from the zero set of a given polynomial. As a corollary of these results, we establish a conjecture about covering a sphere by spherical segments, thus generalizing the zone conjecture of Fejes Toth.

This is a joint work with Roman Karasev and Alexandr Polyanskii.