2022 Workshop on Algebraic Combinatorics
Institute of Mathematics, Academia Sinica, Taipei, Taiwan
Jan. 24-26, 2022

Covering cubes by hyperplanes

Hao Huang
National University of Singapore
huanghao@nus.edu.sg

Abstract

The vertices of the n-dimensional cube 0, 1^n can be covered by two affine hyperplanes x_1=1 and x_1=0. However if there is a vertex not allowed to cover, then suddenly at least n affine hyperplanes are needed. This is a classical result of Alon and Furedi, followed from the Combinatorial Nullstellensatz.

In this talk, we consider the following natural generalization of the Alon-Furedi theorem: what is the minimum number of affine hyperplanes such that the vertices in 0, 1^n-0 are covered at least k times, and 0 is uncovered? We conjecture that for large n, the answer is n + $\binom{k}{2}$ and verify it for k≤3, using a punctured version of the Combinatorial Nullstellensatz. We also completely solve the fractional version of this problem, by developing an analogue of the Lubell-Yamamoto-Meshalkin inequality for subset sums.

Joint work with Alexander Clifton (Emory University).