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Semibiplanes and an inequality for incidence structures

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Abstract

A semibiplane is a connected point-block incidence structure such that any two points are in either 0 or 2 common blocks, and any two blocks have either 0 or 2 common points. If a semibiplane has n points and every block contains k points, then the number of pairs of points which are in 2 common blocks is at least $nk(k-1)/4$. This inequality can be proved by a purely combinatorial method. In this talk, however, we present a nearly algebraic proof using the representation theory of the (non-commutative) coherent algebra of the coherent configuration which is the union of the two Johnson schemes $J(n,k)$ and $J(n,2)$. The method generalizes Delsarte's linear programming bound to a non-commutative setting, and it can also be regarded as a semidefinite programming method in $H(n,2)$.

This is based on joint work with Jon Xu.