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The Terwilliger algebra of the halved cube

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ABSTRACT. Let $D \geq 3$ denote an integer. For any $x \in \mathbb{F}_2^D$ let $w(x)$ denote the Hamming weight of x . Let X denote the subspace of \mathbb{F}_2^D consisting of all $x \in \mathbb{F}_2^D$ with even $w(x)$. The D -dimensional halved cube $\frac{1}{2}H(D, 2)$ is a finite simple connected graph with vertex set X and $x, y \in X$ are adjacent if and only if $w(x - y) = 2$. Fix a vertex $x \in X$. The Terwilliger algebra $\mathcal{T} = \mathcal{T}(x)$ of $\frac{1}{2}H(D, 2)$ with respect to x is the subalgebra of $\text{Mat}_X(\mathbb{C})$ generated by the adjacency matrix A and the dual adjacency matrix $A^* = A^*(x)$ where A^* is a diagonal matrix with

$$A^*_{yy} = D - 2w(x - y) \quad \text{for all } y \in X.$$

In this paper we decompose the standard \mathcal{T} -module into a direct sum of irreducible \mathcal{T} -modules.