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Complete conjugate invariants of Clifford groups

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Abstract

Roughly speaking, unitary design is a finite subset of the unitary group which approximates the whole unitary group concerning integrals. It rises from the need in experiments and engineering related to quantum physics. Formally a unitary t-design is a subset X of the unitary group $U(d)$ such that

$$\frac{1}{|X|}\sum_{U\in X} U^{\otimes t}\otimes (U^{\dagger})^{\otimes t} = \int_{U(d)} U^{\otimes t}\otimes (U^{\dagger})^{\otimes t}\,\mathrm{d}U.$$

The complex Clifford group, which is relatively easy to implement in quantum physics, outstands as an infinite family of unitary 3-designs. It is known that the complex Clifford group fails gracefully to be a unitary 4-design, and conjectured that projective 5-designs may come for free from projective 4-designs by the complex Clifford group.

The invariants of complex Clifford group of genus m were characterized by Runge's theorem. It says that the space of polynomial invariants is spanned by the genus-m complete weight enumerators of binary doubly even self-dual codes. We generalize Runge's theorem from polynomial invariants to complex conjugate polynomial invariants. Together with generalized MacWilliams identity and classification of certain type of self-dual codes, we confirm that projective 5-designs come for free from projective 4-designs by the complex Clifford group.