Proceedings of the Workshop on the 16th Conference in Differential Equations

第十六屆微分方程研討會

摘要集

主辦單位：國立交通大學應用數學系

贊助單位：國科會數學研究推動中心

主 辦 人：

協 辦 人：

時間：中華民國九十七年元月四、五、六日

第十六屆微分方程研討會

時間：九十七年元月四日至六日(星期五、星期六、星期日)

地點：國立交通大學應用數學系

元月四日(星期五)

1:50 PM-2:00 PM **開幕致詞**

主持人：賴明治 教授

2:00 PM-2:50 PM 郭鴻基 教授(國立台灣大學)

Two dimensional turbulence and vortex dynamics

3:00 PM-3:30 PM Yung-Sze Choi 教授(Department of Mathematics, University of Connecticut)

Nonlinear Biharmonic Equations With Negative Exponents

3:40 PM-4:10 PM 陳國璋 教授(國立清華大學)

A variational proof for Moulton's theorem on the existence of some satellite orbits for the restricted thee-body problem

4:10 PM-4:30 **茶會**

主持人：羅春光 教授

4:30 PM-5:00 PM 林太家 教授(國立台灣大學)

Stability of spikes in Bose-Einstein condensates

5:10 PM-5:40 PM 郭岳承 教授(國立高雄大學)

Numerical Study for Rotating Bose-Einstein Condensate

5:50 PM-6:20 PM 林惠莉 教授(長庚大學)

Multiple positive solutions of semilinear elliptic equations in exterior domains

6:30 PM **晚宴**

元月五日(星期六)

主持人：_____教授

9:00 AM-9:30 AM 蔡東和 教授(國立清華大學)

Center of Expansion and Isoperimetric Difference for Plane Curves Evolution

9:40 AM-10:10 AM 彭振昌 教授(國立嘉義大學)

Transversal homoclinic orbits for perturbations of singular difference equations

10:10 AM-10:30 AM **茶會**

主持人：劉晉良 教授

10:30 AM-11:00 AM 謝英恆 教授(國立中興大學)

Modeling the Impact of Travel Between Patches for Spatial Spread of Disease

11:10 AM-11:40 AM 蔡志強 教授(國立中正大學)

Front propagation in a class of chemical models

主持人：_____教授

1:30 PM-2:20 PM 劉太平 教授(中央研究院)

Future of PDE, Perils and Opportunities

2:30 PM-3:00 PM Jiahong Wu 教授(Oklahoma State University)

The 2D surface quasi-geostrophic equation

3:00 PM-3:20 PM **茶會**

主持人：郭忠勝 教授

3:20 PM-3:50 PM 羅春光 教授(國立中山大學)

Inverse nodal problems for nonlinear Sturm-Liouville equations

4:10 PM-4:40 PM 謝忠村 教授(淡江大學)

Isospectral sets and inverse problems for vector-valued Sturm-Liouville equations

4:50 PM-5:20 PM 張乃珩教授(國立台東大學)

Spin liquid model in higher space dimensions

5:30 PM-6:00 PM 李國明 教授(國立中正大學)

Nonlinear integral equations method in inverse scattering problem

元月六日(星期日)

主持人：簡澄陞 教授

9:00 AM-9:30 AM 劉晉良 教授(國立高雄大學)

A brief review on quantum hydrodynamic models

9:40 AM-10:10 AM 謝世峰 教授(國立台灣師範大學)

Eigenvalue Estimates Using the Kolmogorov-Sinai Entropy

主持人：王懷權 教授

10:30 AM-11:00 AM 簡澄陞 教授(國立中興大學)

Numerical continuation for computing energy levels of Bose-Einstein condensation

11:10 AM-11:40 AM 陳怡全 教授(中央研究院)

FAMILY OF INVARIANT CANTOR SETS AS ORBITS OF DIFFERENTIAL EQUATIONS

11:50 AM-12:20 AM 吳菁菁 教授(國立交通大學)

Traveling waves for periodic lattice dynamical system

~ The End ~

摘要索引

Two dimensional turbulence and vortex dynamics

HUNG-CHI KUO

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Abstract

The two-dimensional turbulence, which may be stemmed either from the fluid rotation or from the fluid internal stratification, has been a paradigm for geophysical fluid dynamics for many years. It is a striking fact that for any type of random initial state or external forcing, a two-dimensional fluid will rapidly organize itself into a system of coherent, interacting vortices swimming through a sea of passive filamentary structure produced from earlier vortex interactions. This discipline has also been instrumental in the development of single charge plasma physics, galaxy spiral, the Great Red spot, the ozone-hole physics and the typhoon dynamics. A brief discussion of 2D and 3D turbulence based on the dimensional analysis will be given. The simplest partial differential equation model to represent the vortex dynamics will be presented. The dynamics of selective decay, vortex filamentation, binary vortex interactions and the concentric vortex structure will be discussed.

The space-times estimates of some famous partial differential equations

Chi-Kun Lin

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Abstract

In this talk we will apply the dimensional analysis to guess the famous inequalties including decay, Strichartz and space-time estimates used in PDEs. It is also interesting to inteprete the div, curl and gradient in terms of dimensional analysis.

Nonlinear Biharmonic Equations With Negative Exponents

Y.S. Choi[1] **and Xingwang Xu**

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Abstract

We study positive C^4 solutions of the geometrically interesting equation: $\Delta^2 u + u^{-q} = 0$ with $q > 0$ in $\mathbf{R}^3$. We will establish several existence and non-existence theorems. In particular, if $1 < q \leq 7$ and u has exactly linear growth uniformly at infinity, i.e.,

$$\lim_{|x|\to\infty} \frac{u(x)}{|x|} = \alpha_1 \quad \text{uniformly} \tag{1}$$

for some non-negative finite constant α_1, then $q = 7$ and u is given by

$$u(x) = (1 + |x|^2)^{1/2}, \tag{2}$$

up to constant multiple, translation and dilation.

[1] the speaker

Stability of spikes in Bose-Einstein condensates

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Abstract

We consider the orbital stability of single-spike bound states of semi-classical nonlinear Schrodinger equations with critical nonlinearity and a trap potential. Due to the effect of the trap potential, we derive the asymptotic expansion formulas and obtain the necessary conditions for orbital stability and instability of single-spike bound states. Our argument is applied to two-component systems of nonlinear Schrodinger equations with a common trap potential, cubic nonlinearity in two spatial dimensions. The orbital stability of bound states with spikes of these systems is investigated. Our results show the existence of stable spikes in two-dimensional Bose-Einstein condensates.

Numerical Study for Rotating Bose-Einstein Condensate

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Abstract

We study the discrete nonlinear Schrödinger equation (DNLSE) that models rotating Bose-Einstein Condensates (BEC) both analytically and numerically. Due to the difficulties arising in the nature of transformation invariant solutions, standard continuation methods may not be used to compute the bound states of the DNLSE. We propose a hyperplane-constrained continuation method to circumvent the obstacle and then use the angular velocity as the continuation parameter to follow the solution curves. Analytically we analyze the solution and bifurcation properties of the primal stalk solution curve corresponding to the DNLSE for an isotropic trap. On the other hand, our numerical results assert the existence of bistable region corresponding to the bound states with 0 or 1 vortex. This finding not only nicely matches the physics experimental phenomena, but further provides an explanation why either 0 or 1 vortex may be observed in a certain region of stirring angular velocity. Furthermore, we discuss how trap potentials and intra-component lengths may affect the bistable region by numerical experiments.

Multiple positive solutions of semilinear elliptic equations in exterior domains

Huei-li Lin

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Abstract

In this talk, we consider the following semilinear ellptic equation

$$\begin{cases} -\triangle u + u = q(z)\,|u|^{p-2}\,u \text{ in } \mathbb{R}^N \backslash \overline{D}; \\ u \in H_0^1(\mathbb{R}^N \backslash \overline{D}), \end{cases} \tag{3}$$

where D is a $C^{1,1}$ bounded domain in $\mathbb{R}^N$, $2 < p < 2^* = \frac{2N}{N-2}$ for $N \geq 3$. Assume that q satisfies the suitable conditions, we prove that there are multiple positive solutions of Equation (3) in an exterior domain.

A brief review on quantum hydrodynamic models

Jinn-Liang Liu

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Abstract

A brief review on QHD models in semiconductor physics will be given in respect to analytical and numerical results. The model hierarchy will be presented from quantum Boltzmann equation to quantum hydrodynamic, quantum energy transport, and quantum drift diffusion models. In particular, a quantum energy transport model will be presented in details with known results in the literature and open problems for future research.

Numerical continuation for computing energy levels of Bose-Einstein condensation

C.-S. Chien

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Abstract

We study some continuation algorithms for computing energy levels of Bose-Einstein condensation (BEC). First we transform the nonlinear Schrodinger equation (NLS) to a nonlinear eigenvalue problem by using a well-known formula of separation of variables. Then we compute the first few energy levels of the associated Schrodinger eigenvalue problem (SEP). Various discretization methods are described to discretize the Laplacian. The proposed algorithm has the advantage that it is unnecessary to discretize or integrate the partial derivatives of wave functions. Moreover, the wave functions can be computed for any time scale. Numerical results on the ground state solutions of the BEC, rotating BEC and BEC with periodic potentials are reported.

Modeling the Impact of Travel Between Patches for Spatial Spread of Disease

Ying-Hen Hsieh

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Abstract

A multi-patch model is proposed to study the impact of travel on the spatial spread of disease between patches with different level of disease prevalence. The basic reproduction number for the ith patch in isolation is obtained along with the basic reproduction number of the system of patches, R_0. For a two-patch model with one high prevalence patch and one low prevalence patch, results pertaining to the dependence of R_0 on the travel rates between the two patches are obtained. For parameter values relevant for influenza, these results show that, while banning travel of infectives from the low to the high prevalence patch always contributes to disease control, banning travel of symptomatic travelers only from the high to the low prevalence patch could adversely affect the containment of the outbreak under certain ranges of parameter values. Moreover, banning all travel of infected individuals from the high to the low prevalence patch could result in the low prevalence patch becoming disease free, while the high prevalence patch becomes even more disease-prevalent, with the resulting number of infectives in this patch alone exceeding the combined number of infectives in both patches without border control. Under the set of parameter values used, our results demonstrate that if border control is properly implemented, it could contribute to stopping the spatial spread of disease between patches. An open problem on extension of this model will be discussed.

Front propagation in a class of chemical models.

Je-Chiang Tsai

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Abstract

We will discuss the existence and stability of traveling waves in a class of chemical models.

Spin liquid model in higher space dimensions

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Abstract

In this talk, we investigate the global well-posedness of the Cauchy problem for the spin liquid model in three space dimensions. Basically, we perform a certain transformation such that the equations for the spin liquid model can be written as a system of coupled Schrödinger equations for which the a priori estimates can be obtained.

Future of PDE, Perils and Opportunities

Tai-Ping Liu

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Abstract

The study of partial differential equations, PDE, has been so fruiteful for over a century. This remarkable fact naturally raises questions as to what are the possible directions of future PDE research. Of course, the answer varies from person to person. There is the point of view that so many open problems remain to be solved. On the other hand, there is also the concern that many of the active fields are maturing. The purpose of this talk is to survey a corner of the vast PDE landscape and to discuss some future opportunities. Also, we hope to use this as an example to open the discussions among the audience asto how to nurture the next generation of researchers.

The 2D surface quasi-geostrophic equation

Jiahong Wu

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Abstract

This talk focuses on the global existence and uniqueness of solutions to the 2D dissipative surface quasi-geostrophic (QG) equation. We first present some of the recent progress made by Kiselev, Nazarov and Volberg and by Caffarelli and Vasseur for the critical case. We then detail the major results of two recent manuscripts by Constantin and Wu on the supercritical case. If time permits, we will also summarize the major existing results on the inviscid QG equation and show some recent numerical results of Lai and Tseng.

Inverse nodal problems for nonlinear Sturm-Liouville equations

Chun-Kong Law

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Abstract

Consider the nonlinear Sturm-Liouville problem

$$\begin{aligned} -u'' + w(x)|u|^p u &= \lambda u \ , \\ u(0) = u(1) &= 0 \ , \end{aligned}$$

where $p > 0$. We shall discuss the recovery of the potential w using the zeros of some solutions u_n associated with a sequence of parameters λ_n. In particular, under certain conditions, uniqueness and reconstruction of w are solved using the nodal set together with the norming constants $\alpha_n = \|u_n\|_2$ where u_n is the nth 'eigenfunction'. The stability issue is also solved.

This is joint work with Chao-Nien Chen of National Changhua University of Education.

Isospectral sets and inverse problems for vector-valued Sturm-Liouville equations

Chung-Tsun Shieh

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Abstract

In this paper, we investigate inverse spectral problems for vectorial Sturm-Liouville equations via the matrix-valued Gelfand-Levitan equation. With this approach, we prove some uniqueness theorems for the even problem, mixed data problem and interior spectral data problem for vectorial Sturm-Liouville equations.

Nonlinear integral equations method in inverse scattering problem

Kuo-Ming Lee

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Abstract

The inverse problem of recovering the geometry and the physical properties of a scatterer from the knowledge of the far field pattern of a scattered field is of fundamental importance for example in non-destructive testing or in medical imaging.

In this talk we consider a time-harmonic scattering problem which is modelled by the exterior boundary value problem governed by the Helmholtz equation. For the reconstruction of the unknown boundary, we use a system of two nonlinear integral equations which are proven to be equal to the original inverse problem instead of the traditional far field equation. Some numerical examples are given at the end of the talk to demostrate the feasibility of the method.

Transversal homoclinic orbits for perturbations of singular difference equations

Chen-Chang Peng

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Abstract

In this talk, there are two main subjects. First, we study difference equations $x_{k+2} = F_B(x_{k+1}, x_k, Bx_{k-1})$ and consider the case when F_B has a singular limit depending on two variables as $B \to 0$ i.e. $x_{k+2} = F_0(x_{k+1}, x_k, 0) \equiv f(x_{k+1}, x_k)$. We prove that if the difference equation f has a strongly snap-back repeller then the difference equation F_B has a transversal homoclinic orbit for B close enough to 0. Second, we study a class of two-dimensional maps (or called the Mira map) and prove that there exist strongly snap-back repellers for the map in some parameter regions including near its anti-integrable limit and far away its anti-integrable limit. Finally, combining the above two results, we establish the existence of transversal homoclinic orbits in family of the Arneodo-Coullet-Tresser map near singularities.

Eigenvalue Estimates Using the Kolmogorov-Sinai Entropy

Shih-Feng Shieh

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Abstract

This talk is threefold. Firstly, we generalize Parry's theorem to general $N \times N$ nonnegative matrices and use this generation to estimate lower bounds for dominant eigenvalues of nonnegative matrices. The lower bound is better than the Rayleigh quotient. Secondly, we use the generalized Parry's Theorem to give a nontrivial lower bounds for dominant eigenvalues of $\underline{\underline{A}}$ and $\underline{\underline{A}}+\underline{\underline{V}}$. Thirdly, we use the previous results to give an explicit lower bound for the gap between the first two eigenvalues of a symmetric tridiagonal matrix.

A variational proof for Moulton's theorem on the existence of some satellite orbits for the restricted three-body problem.

Kuo-Chang Chen

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Abstract

The n-body problem concerns the motion of n celestial bodies moving in space in accordance with Newton's law of universal gravitation. When one of the celestial bodies has zero mass, the problem is called a restricted problem. The orbit of an infinitesimal point mass for the restricted n-body problem is called a satellite orbit. In Moulton's classical treatise in 1912, he proved the existence of periodic satellite orbits for the restricted thee-body problem near infinity by the continuation method. In this talk I will present a variational proof for Moulton's theorem.

Family Of Invariant Cantor Sets As Orbits Of Differential Equations

Yi-Chiuan Chen

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Abstract

It is well-known that the invariant Cantor sets of the logistic map $g_\mu(x) = \mu x(1-x)$ for $\mu > 4$ form a continuous family. In this talk, I shall show that this family can be obtained explicitly through solutions of initial value problems for a system of infinitely coupled differential equations.

Traveling waves for periodic lattice dynamical system

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Abstract

We study the traveling waves for a system of ordinary differential equations in discrete periodic media. We shall consider two different nonlinearities, namely, monostable and bistable cases. The main concerns are about the existence, uniqueness and stability of the traveling wave for the bistable nonlinearity, and the uniqueness and stability for the monostable nonlinearity.