

# 第二十四屆微分方程研討會

The 24<sup>th</sup> Annual Workshop on Differential Equations

## 議 程 表

105 年 1 月 22 日 (星期五)

地點：國研大樓一樓

09：00～09：50	報 到	
09：50～10：00	開 幕	
10：00～10：50	林長壽院士 (台大&中山)	
10：50～11：10	茶 會	
11：10～12：00	鄺文錦教授 (香港理工大學)	
12：00～13：15	午 餐	
13：15～13：45	郭忠勝 (淡江)	吳德琪 (中研院)
13：50～14：20	班榮超 (東華)	詹傳宗 (東海)
14：30～15：00	曾睿彬 (政大)	陳志杰 (台大)
15：05～15：35	王峰彬 (長庚)	郭庭榕 (台大)
15：35～16：05	茶 會	
16：05～16：35	林景隆 (成大)	鄭經數 (中央)
16：40～17：10	陳隆暉 (中正)	江金城 (清大)
17：15～17：45	關汝琳 (NCTS)	陳子軒 (交大)
18：30～	大會晚宴	

105 年 1 月 23 日 (星期六)

地點：國研大樓一樓

09：00～09：50	劉太平院士 (中研院)	
09：50～10：20	茶 會	
10：20～10：50	吳恭儉 (成大)	陳俊全 (台大)
10：55～11：25	郭鴻文 (成大)	陳志有 (彰師)
11：30～12：00	王海濤 (中研院)	黃信元 (中山)
12：00～13：15	午 餐	
13：15～14：05	尤釋賢教授 (新加坡大學)	
14：05～14：30	茶 會	
14：30～15：00	方永富 (成大)	黃杰森 (中山)
15：05～15：35	黃博峙 (中央)	吳金典 (交大)
15：35～15：55	茶 會	
15：55～16：25	謝天長 (NCTS)	葉宗鑫 (南大)
16：30～17：00	高 琦 (NCTS)	劉筱凡 (16：30～16：50) 何怡慧 (16：50～17：10)

# Plenary Talks

## Wave motion around 2D viscous shock profile

Shih-Hsien Yu (尤釋賢), National University of Singapore

### Abstract

In this talk we will introduce a qualitative and quantitative approach to study the perturbations of a viscous shock profile global in the space-time domain.

## Finding real roots of Eisenstein series of weight one by Painlevé equation VI

Chang-Shou Lin (林長壽),  
National Taiwan University & National Sun Yat-sen University

### Abstract

For any  $(r, s) \in \mathbb{C}^2 \setminus \frac{1}{2}\mathbb{Z}^2$ , we define  $p_{r,s}(\tau) \in \mathbb{C}$  by

$$\wp(p_{r,s}(\tau)|\tau) = \wp(r + s\tau|\tau) + \frac{\wp'(r + s\tau|\tau)}{2[\zeta(r + s\tau|\tau) - (r\eta_1(\tau) + s\eta_2(\tau))]}, \quad \tau \in \mathbb{H}$$

The Hitchin theorem says that  $p_{r,s}(\tau)$  is a solution of a Painlevé VI equation. Remarkably, the denominator  $\zeta(r + s\tau|\tau) - r\eta_1(\tau) - s\eta_2(\tau)$  is the Eisenstein series of weight one if  $(r, s)$  is an N-torsion point.

Set  $Z_{r,s}(\tau) = \zeta(r + s\tau|\tau) - r\eta_1(\tau) - s\eta_2(\tau)$ . Then  $Z_{r,s}(\tau)$  has another link with geometric and PDE aspects. Any zero  $\tau$  of  $Z_{r,s}(\tau)$  for some real pair  $(r, s) \in \mathbb{C}^2 \setminus \frac{1}{2}\mathbb{Z}^2$ , the PDE (mean field equation)

$$\Delta u + e^u = 8\pi\delta(0) \quad \text{in } E_\tau,$$

has a solution, where  $E_\tau$  is the torus  $\mathbb{C}/\Lambda_\tau$ ,  $\Lambda_\tau$  is the lattice generated by 1 and  $\tau$ , and  $\delta(0)$  is the Dirac measure at lattice points. In this talk, I will give a survey about the story about the connection among such subjects: mean field equations, Painlevé VI, and modular forms. The center of these connections are Green functions and Lamé equations.

## **Some thoughts on partial differential equations**

Tai-Ping Liu (劉太平), Academia Sinica

### **Abstract**

After Hadamard, the standard research goal in the study of partial differential equations has been to study the well-posedness problem. There are also the issue of large time behavior and regularity of solutions. However, it becomes increasingly clear that these should not be the only issues, in fact may not even be the main issues. For one thing, many of the main equations seem to be not well-posed. Besides, the questions in science where these equations originated often address other concerns. In many cases, it seems more relevant to seek for particular solutions or to interpret the equations in approximate sense. The goal can be to understand in which way the given equations can offer for the natural phenomena in hand. We plan to give these issues a thought and offer some examples for illustration.

## **Some boundary value problems arising from electro-diffusion theory**

Man-kam Kwong (鄭文錦), Hong Kong Polytechnic University

### **Abstract**

A Neumann two-point boundary value problem arising in an electro-diffusion model is studied. Existence criteria, in terms of the various physical parameters involved in the equation, are sought. The problem is unconventional because the differential equation itself involves a parameter that depends on the not-yet-determined boundary values of the solution. Partial existence results have previously been obtained using the technique of upper and lower solutions. The results are improved by first transforming the problem into an equivalent one and then by using a two-dimensional shooting argument and topological index technique. In another paper, a related nonlinear boundary value problem is studied, this time with a Robin boundary condition. Existence and multiplicity results are obtained using a variational approach. These are joint work with P. Amster and C. Rogers.

# Invited Talks

## Well-posedness and adiabatic limit for quantum Zakharov system

Yung-Fu Fang (方永富), National Cheng Kung University

### Abstract

For a Quantum Zakharov system, we investigate the LWP and GWP in 1D, adiabatic limit, soliton waves, and *least energy solution*.

For multilinear estimates of  $(QZ)$ , we characterize the dependence of the quantum parameter within the constants.

We obtain the *adiabatic limit* for  $(QZ)$  to a quantum modified NLS. We also prove the existence of homoclinic solutions with the least energy.

Future study for  $(QZ)$ : LWP in 2D and 3D, Ill-posedness, classical limit, subsonic limit, ground state, soliton waves, asymptotic behavior, and *scattering problem*.

(joint work with Tsai-Jung Chen, Hung-Wen Kuo Jun-Ichi Segata, Hsi-Wei Shih, Kuan-Hsiang Wang, Tsung-fang Wu)

## A PDE system modeling the dengue transmission with nonlocal infections and crowding effects

Feng-Bin Wang (王峰彬), Chang Gung University

### Abstract

Dengue fever is a disease endemic in the subtropical and tropical regions of the world. The control of the spread of dengue fever is an important issue in the public health due to its high infection rate and high death rate of its severe form dengue hemorrhagic fever. In order to understand the influences of the spatial heterogeneity, crowding effect and non-local infection caused by the movements of the latent mosquitoes on the dynamics of dengue transmission, we modify an existing model to obtain a nonlocal and time-delayed reaction-diffusion system with the Neumann condition on the boundary. Then we define the basic reproduction number  $\mathbf{R}_0$  for the model system, and show that the global threshold dynamics of the model system can be determined by  $\mathbf{R}_0$ .

## **Time-asymptotic behavior of scalar viscous rarefaction wave**

Hai-Tao Wang (王海濤), Academia Sinica

### **Abstract**

The stability of viscous rarefaction wave is a classical problem and the standard approach for it is energy method. We are more interested in the quantitative behavior. In this talk, I will show how to obtain the point-wise structure of the solution through Green's function approach. Specifically, we use viscous Burgers solution as an approximation and construct the Green's function by linear Hopf-Cole transformation. Then by proposing appropriate Ansatz and delicate analysis, we can obtain the desired result.

## **Boltzmann collision operator for the infinite range potential**

Jin-Cheng Jiang (江金城), National Tsing Hua University

### **Abstract**

The conventional Boltzmann collision operator for the infinite range inverse power law model was derived by Maxwell by adopting a kernel which is a limit of that for the finite range model under the assumption that glancing angles can be ignored. The purpose of this paper is to clarify the physical meaning of this collision operator. We first prove that it is legitimate to ignore glancing angles under suitable conditions. Furthermore we prove that taking limit to collision operator with finite range potential directly will lead to the conventional one. The analysis depends on a new method to estimate the upper bound of collision operator. This new method is required since the analysis must show how the upper bound varies with the range of potential. The core of analysis turns out relying on the study of a specific kind of Radon transforms, which are of independent interest. The rich structure of these transforms are explored; each of it can be expressed as the summation of a pseudo differential operator, Fourier integral operators, their transforms and degenerates. Each Radon transform as a whole satisfies the diffusion estimates in Sobolev space. The structural description of these transforms suggests the norm spaces to discuss the problem and enables us to see how the upper bound varies from finite range to infinite range in the suitable Sobolev spaces.

## **A pipeline for 3D image morphing based on geodesic spline landmark matching**

Chin-Tien Wu (吳金典), National Chiao Tung University

### **Abstract**

In this talk, we will introduce some of our recent works on 3D images obtained from 3D scanner. Brief reviews on global parametrization using conformal mapping, object extraction and model merging will be shown. One of the major challenge on 3D animation is to efficiently manage huge amount of data generated by scanner. To overcome this difficulty, recently, Gu and Yau propose to compress the data using mean curvature  $H$  and conformal factor  $\lambda$ . In this work, we follow Gu and Yau's idea and further apply it to 3D surface morphing where matching landmark points on parametric domains, interpolating  $(H, \lambda)$  for generating remaining frames, and reconstructing 3D geometry and texture are essential. The corner stones in our frame works include (i) a robust two phase quasi-implicit Euler method recently developed by Lin and Huang for building the Riemann map (ii) Finite element method combined with multigrid method for multi-resolution surface reconstruction and (iii) Geodesic spline landmark matching using discrete Green function. We shall briefly address the above key steps and show some of our numerical results.

## **Nonlinear stability of the 1D-Boltzmann Equation in a periodic box**

Kung-Chien Wu (吳恭儉),  
National Cheng Kung University & National Sun Yat-sen University

### **Abstract**

We study the nonlinear stability of the Boltzmann equation in the 1D periodic box with size  $1/\varepsilon$ , where  $0 < \varepsilon \ll 1$  is the Knudsen number. The convergence rate is algebraic for small time region and exponential for large time region. The algebraic rate is optimal and the exponential rate depends on the size of the domain (Knudsen number). This problem is highly nonlinear and hence one can not treat the nonlinear term as a perturbation of the linear problem.

## **The inverse scattering problem of integrable systems**

Der-Chyi Wu (吴德琪), Academia Sinica

### **Abstract**

The talk will start with an introduction on the inverse scattering theory of the KdV equation. Then I will discuss the progress of the inverse scattering theory of the multi-dimensional integrable systems, in particular, the Kadomtsev-Petviashvili-II (KP-II) equation which describes shallow water wave and is a  $2 + 1$  analogue of the KdV equation.

## **Unique continuation property for anomalous slow diffusion equation**

Ching-Lung Lin (林景隆), National Cheng Kung University

### **Abstract**

A Carleman estimate and the unique continuation of solutions for an anomalous diffusion equation with fractional time derivative of order  $0 < \alpha < 1$  are given. The estimate is derived via some subelliptic estimate for an operator associated to the anomalous diffusion equation using calculus of pseudo-differential operators.

## **Entropy of free shifts of finite type**

Jung-Chao Ban (班榮超), National Dong Hwa University

### **Abstract**

This talk studies the entropy of tree shifts of  $\tilde{O}$ nite type with and without boundary conditions. We demonstrate that computing the entropy of a tree shift of  $\tilde{O}$ nite type is equivalent to solving a system of nonlinear recurrence equations. Furthermore, the entropy of binary Markov tree shifts defined on two symbols is either 0 or  $\ln 2$ . Meanwhile, the realization of entropy of one-dimensional shifts of  $\tilde{O}$ nite type is elaborated, which indicates that tree shifts are capable of rich phenomena. Considering the influence of three different types of boundary conditions, say, the periodic, Dirichlet, and Neumann boundary conditions, the necessary and sufficient condition for the coincidence of entropy with and without boundary condition are addressed.

## **A survey on a two-component Ginzburg Landau model**

Qi Gao (高琦), National Taiwan University

### **Abstract**

In this talk, firstly I will give a brief introduction to Ginzburg-Landau equations, then I will focus on a two-component Ginzburg-Landau model in the following two settings: 1. For the model without magnetic field, existence, uniqueness, asymptotic behavior of solutions at infinity, monotonicity and stability will be presented. 2. For the model with magnetic field, we recently prove the breakdown of superconductivity in the presence of large magnetic field. This is a joint work with Tien-Tsan Shieh.



## Mean field equation, isomonodromic deformation and Painlevé VI equation

Ting-Jung Guo (郭庭榕), National Taiwan University

### Abstract

In literature, it is known that any solution of Painlevé VI equation governs the isomonodromic deformation of a second order linear Fuchsian ODE. A solution is called *real* if its associated monodromy representation to the corresponding generalized Lamé equation is *unitary*. In this talk, we will talk about the surprising connection of real solutions to Painlevé VI with bubbling solutions of the mean field equation. This connection provides us a geometric interpretation of real solutions to Painlevé VI. By employing the PDE result in the theory of the mean field equation, we are able to prove that any real solution  $\lambda(t)$  of  $\text{PVI}(\frac{1}{8}, \frac{-1}{8}, \frac{1}{8}, \frac{3}{8})$  and its Bäcklund transformations  $\lambda^{-1}(t)$ ,  $(1 - \lambda(t))^{-1}$  and  $(t - \lambda(t))^{-1}$  are smooth for  $t \in \mathbb{R} \setminus \{0, 1\}$  which seems not easy to be proved from Painlevé VI point of view.

## Traveling wave solutions for a discrete diffusive epidemic model

Jong-Shenq Guo (郭忠勝), Tamkang University

### Abstract

We study the traveling wave solutions for a discrete diffusive epidemic model. The traveling wave is a mixed of front and pulse types. We derive the existence and non-existence of traveling wave solutions of this model. The proof of existence is based on constructing a suitable pair of upper and lower solutions and the application of Schauder's fixed point theorem. By passing to the limit for a sequence of truncated problems, we are able to derive the existence of traveling waves by a delicate analysis of wave tails. Some open problems are also addressed.

This is a joint work with S.-C. Fu and C.-C. Wu.

## **Asymptotic expansion for Rayleigh's problem according to kinetic theory**

Hung-Wen Kuo (郭鴻文), National Cheng Kung University

### **Abstract**

We investigate Rayleigh's problem at small Mach numbers in rarefied gas dynamics. The linearized BGK model equation is used to describe intermolecular collisions. The asymptotic expansions are given for the microscopic density function and the macroscopic velocity. In particular, the long time behavior is investigated.

## **Asymptotic properties of stationary Navier-Stokes flows in the setting of hyperbolic spaces**

Chi-Hin Chan (陳子軒), National Chiao Tung University

### **Abstract**

In this talk, I will present a piece of recent joint work with Che-Kai Chen and Magdalena Czubak. In this work, we consider a given stationary Navier-Stokes flow which passes a circular-shape obstacle in a 2-dimensional hyperbolic space. We suppose that the stationary Navier-Stokes flow under our consideration satisfies the finite Dirichlet norm property. Under this assumption alone, we show that the velocity field itself will decay to zero at infinity, and that we can obtain exponential decay of the associated vorticity of the flow in the far range. The material which will be reported in this talk is a preliminary version of a piece of unpublished recent joint work with Che-Kai Chen and Magdalena Czubak. During the talk, I will also mention the relations between this piece of work with the standard working knowledge in the classical theory of stationary Navier-Stokes flows passing an obstacle in the 2D Euclidean setting.

## **On the uniqueness and structure of solutions to the system arising from Maxwell-Chern-Simons $U(1)$ model**

Zhi-You Chen (陳志有), National Changhua University of Education

### **Abstract**

In this talk, we prove the uniqueness of topological multivortex solutions for the self-dual Maxwell-Chern-Simons  $U(1)$  model if the Chern-Simons coupling parameter is sufficiently large and the charge of electron is sufficiently small or large. On the other hand, we also establish the sharp region of the flux for non-topological solutions and provide the classification of radial solutions of all types in the case of one vortex point.

## **A generalization of the Hitchin theorem on Painlevé VI equation and applications**

Zhijie Chen (陳志杰), National Taiwan University

### **Abstract**

In 1995, Hitchin obtained his famous formula to express generic solutions of Painlevé VI equation with parameter  $(\frac{1}{8}, \frac{-1}{8}, \frac{1}{8}, \frac{3}{8})$ . In this talk, I will introduce our generalization of Hitchin's formula to Painlevé VI with parameters  $(\frac{1}{2}(n + \frac{1}{2})^2, \frac{-1}{8}, \frac{1}{8}, \frac{3}{8})$  for all  $n \geq 1$ . As applications, we prove that any pre-modular form obtained by Chai, Lin and Wang has simple zeros only. Furthermore, we prove Dahmen Conjecture concerning integral Lamé equations. This is a joint work with Professors Ting-Jung Kuo and Chang-Shou Lin.

## A new maximum principle for diffusive Lotka-Volterra systems of two competing species

Chiun-Chuan Chen (陳俊全), National Taiwan University

### Abstract

Using an elementary approach, we establish a new maximum principle for the diffusive Lotka-Volterra system of two competing species. This maximum principle gives apriori estimates for the total mass of the two species. Moreover, applying it to the system of three competing species leads to a nonexistence theorem of traveling wave solutions.

## On the inverse interior-exterior transmission problem in scattering theory

Lung-Hui Chen (陳隆暉), National Chung Cheng University

### Abstract

In this talk, we study the inverse scattering problem in the following interior-exterior transmission problem.

$$\left\{ \begin{array}{ll} \Delta w + k^2 n(x)w = 0, & \text{in } D; \\ \Delta v + k^2 v = 0, & \text{in } D; \\ w = v, & \text{on } \partial D; \\ \frac{\partial w}{\partial \nu} = \frac{\partial v}{\partial \nu}, & \text{on } \partial D; \\ \lim_{r \rightarrow \infty} r \left[ \frac{\partial w}{\partial r} - ikw \right] = 0; \\ \lim_{r \rightarrow \infty} r \left[ \frac{\partial v}{\partial r} - ikv \right] = 0, \end{array} \right. \quad (1)$$

where  $\nu$  is the unit outer normal and  $D$  is a starlike domain in  $\mathbb{R}^3$  containing the origin. The index of refraction is a bounded perturbation to the constant background, which we assume to be 1. However, the support of  $1 - n$  may not be contained in  $D$ . The inhomogeneity  $n \in \mathcal{C}^2(\mathbb{R}^3)$  and  $n(x) > 0$  for all  $x \in \mathbb{R}^3$ . The equation(1) is called the interior-exterior transmission eigenvalue problem. The interior-exterior transmission problem happens naturally when the plane waves are perturbed in the exterior of the cavity  $D$  surrounded by certain inhomogeneity defined by the index of refraction. The signal is placed in the cavity and we study the inverse scattering problem by the measurements in the far-fields.

## **Multistability for delayed neural networks via sequential contracting**

Jui-Pin Tseng (曾睿彬), National Chengchi University

### **Abstract**

In this talk, we explore a variety of new multistability scenarios in the general delayed neural network system. Geometric structure embedded in equations is exploited and incorporated into the analysis to elucidate the underlying dynamics. Criteria derived from different geometric configurations lead to disparate numbers of equilibria. A new approach named sequential contracting is applied to conclude the global convergence to multiple equilibrium points of the system. The formulation accommodates both smooth sigmoidal and piecewise-linear activation functions. Several numerical examples illustrate the present analytic theory. This is a joint work with Chang-Yuan Cheng (NPTU), Kuang-Hui Lin (NCTU), and Chih-Wen Shih (NCTU).

## **The semi-Lagrangian finite difference WENO method for hyperbolic conservation laws and its application to Vlasov equation**

Chieh-Sen Huang (黄杰森), National Sun Yat-sen University

### **Abstract**

In this talk, we propose a semi-Lagrangian Finite difference formulation for approximating conservative form of advection equations with general variable coefficients. In 2011, Qiu and Shu developed the first semi-Lagrangian finite difference WENO methods. However, in their schemes “linear weights” fail to exist when advection is not constant. For variable advection, Qiu and Shu used “empirical” linear weights to circumvent this problem. Our goal is to develop a semi-Lagrangian finite difference scheme with exact “linear weights” for variable advection problems. We also extend this SL-WENO finite difference scheme to the scalar hyperbolic conservation laws and also apply the new scheme to Vlasov equation.

# The domain geometry and the bubbling phenomenon of rank two Gauge theory

Hsin-Yuan Huang (黃信元), National Sun Yat-sen University

## Abstract

Let  $\Omega$  be a flat torus and  $G$  be the green's function of  $-\Delta$  on  $\Omega$ . One intriguing mystery of  $G$  is how the number of its critical points is related to blowup solutions of certain PDEs. In this article we prove that for the following equation that describes a Chern-Simons model in Gauge theory:

$$\begin{cases} \Delta u_1 + \frac{1}{\varepsilon^2} e^{u_2} (1 - e^{u_1}) = 8\pi \delta_{p_1} \\ \Delta u_2 + \frac{1}{\varepsilon^2} e^{u_1} (1 - e^{u_2}) = 8\pi \delta_{p_2} \end{cases} \quad \text{in } \Omega, \quad p_1 - p_2 \text{ is a half period,} \quad (2)$$

if fully bubbling solutions of Liouville type exist,  $G$  has exactly three critical points. In addition we establish necessary conditions for the existence of fully bubbling solutions with multiple bubbles. This is a joint work with Lei Zhang.

# Global transonic solutions of planetary atmospheric escape model in hydrodynamic region

Bo-Chih Huang (黃博峙), National Central University

## Abstract

Spacecraft exploration of the planets in our solar system and the discovery of exoplanets has been raised a great interest in atmospheric escape from planetary objects. The hydrodynamic escape problem (HEP) is significant on the study of the evolution of planetary atmospheres. In this talk, we establish the global transonic solutions for hydrodynamic escape problem (HEP), which is characterized by a free boundary value problem of Euler equation with gravity and heat. The global existence of transonic solutions to HEP is established by the generalized Glimm's method. The range of the hydrodynamic escape region in planetary atmospheres will also be discussed.

# **Classification of bifurcation curves for a multiparameter diffusive logistic problem with Holling type-III functional response**

Tzung-Shin Yeh (葉宗鑫), National University of Tainan

## **Abstract**

We study exact multiplicity and bifurcation curves of positive solutions for a multiparameter diffusive logistic problem with Holling type-III functional response. Under some hypotheses, we give a classification of three qualitatively different bifurcation curves: an S-shaped curve, a broken S-shaped curve, and a monotone increasing curve.

# **Connection preserving approach to q-difference equations**

Chuan-Tsung Chan (詹傳宗), Tunghai University

## **Abstract**

In a seminal work, Jimbo and Sakai derived the q-Difference Painleve VI equation based on the connection preserving deformation (CDP) approach. Such an approach is closely related to the isomonodromy deformation approach in the study of continuous Painleve equations and leads to algebraic expressions which allows geometric analysis and classifications. In this talk, I shall introduce some basic ideas behind this approach and study the generalized cases of the Lax pairs with non-diagonalizable leading matrices.

## Diffeomorphisms with prescribed Jacobian and boundary data

Ching-hsiao Cheng (鄭經戡), National Central University

### Abstract

In this talk we consider the problem of finding diffeomorphisms whose Jacobian and boundary data are of Sobolev class and the domain under consideration is also of Sobolev class; that is, finding a diffeomorphism  $\psi$  between two Sobolev class domains  $\Omega_1$  and  $\Omega_2$  satisfying

$$\begin{aligned}\det(\nabla\psi) &= f && \text{in } \Omega_1, \\ \psi &= g && \text{on } \partial\Omega_1,\end{aligned}$$

where  $f$  and  $g$  are given functions possessing Sobolev class regularity. The result for the case that  $f$ ,  $g$  and  $\Omega_1$  possess classical regularity is well-known; however, such kind of results appear to undergo loss of regularity in the sense that when  $\Omega_1$  is of class  $C^{k+3,\alpha}$  and the forcing are of class  $C^{k,\alpha}$ , the solution is of class  $C^{k+1,\alpha}$ . Our result show that when studying this problem on Sobolev class domain with Sobolev class forcing, there exists a solution to the equation above satisfying

$$\|\nabla\psi\|_{H^{k+1}(\Omega_1)} \leq C \left[ \|f\|_{H^k(\Omega_1)} + \|g\|_{H^{k+0.5}(\partial\Omega_1)} \right]$$

for some generic constant  $C$  depending on the  $H^{k+1}$ -regularity of  $\Omega_1$ . Therefore, no loss of regularity is encountered.

## Ground state patterns and phase transitions of spin-1 Bose-Einstein condensates via $\Gamma$ -convergence theory

Tien-Tsan Shieh (謝天長), National Center for Theoretical Sciences

### Abstract

We develop an analytic theory for the ground state patterns and their phase transitions for spin-1 Bose-Einstein condensates on a bounded domain in the presence of a uniform magnetic field. Within the Thomas-Fermi approximation, these ground state patterns are composed of four basic states: magnetic state, nematic state, two-component state and three-component state, separated by interfaces. A complete phase diagram of the ground state patterns are found analytically with different quadratic Zeeman energy  $q$  and total magnetization  $M$  for both ferromagnetic and antiferromagnetic systems. Using the  $\Gamma$ -convergence technique, it is found that the semi-classical limits of these ground states minimize an energy functional which consists of interior interface energy plus a boundary contact energy. As a consequence, the interface between two different basic states has constant mean curvature, and the contact angle between the interface and the boundary obeys Young's relation.



# **Strong unique continuation property for the Lamé system in two dimension**

Ru-Lin Guan (關汝琳), National Center for Theoretical Sciences

## **Abstract**

There are many researches on strong unique continuation property for the Lamé system. So far the best result of them is that the Lamé coefficients  $\mu$  and  $\lambda$  satisfy  $\mu \in C^{0,1}$  and  $\lambda \in L^\infty$ . In our recent work, we reduce the regularity assumption to  $\mu \in W^{1,p}$  and  $\lambda \in L^\infty$ , for  $p$  sufficiently large. However, our method only can work in two dimension.

# Contributed Talks

## The star mean curvature flow on a 3-manifold

Hsiao-Fan Liu (劉筱凡), Academia Sinica

### Abstract

Let  $g$  be a Riemannian or Lorentzian metric on a 3-dimension manifold  $N$ . We consider the following curve flow on the space of immersed curves in  $N$ ,

$$\gamma_t = *_{\gamma}(H(\gamma(\Delta, t))),$$

where  $*_{\gamma(x)}$  is the Hodge star operator on the normal plane  $\nu(\gamma)_x$  and  $H(\gamma(\Delta, t))$  is the mean curvature vector for  $\gamma(\Delta, t)$ . For example, the  $*$ -MCF on  $\mathbb{R}^3$  is the vortex filament equation (VFE), which is related to the nonlinear Schrödinger equation (NLS). In this talk, we will explain the connections between the  $*$ -MCF on  $N$  and the NLS.

## Competition and coexistence for one nutrient with internal storage and predation

Yi-hui Ho (何怡慧), National Tsing Hua University

### Abstract

In this talk, we investigate a mathematical model of two species competing in a chemostat for one resource that is stored internally, where one of the species can act as an intraguild predator that also feeds on the other species. We utilize theory of uniform persistence to prove that coexistence is possible under some suitable conditions, and our numerical simulations also confirm theoretical results. It is worth noting that Smith and Waltman proved that competitive exclusion holds for the classical model without predation, that is, the species that can grow at the lowest nutrient concentration will win the competition. From our study, intraguild predation may promote the possibility of coexistence.

This is a joint work with Drs. Sze-Bi Hsu and Feng-Bin Wang.