International Workshop on Several Complex Variables, Complex Geometry & Diophantine Geometry

in honor of Min Ru's 60-th Birthday

Aug. 14 - 18, Auditorium, Institute of Mathematics, Academia Sinica (NTU campus)

SPEAKERS

Pietro Corvaja (University of Udine)
Dinh Tien Cuong (National University of Singapore)
Ya Deng (Université de Lorraine)
John Erik Fornaess (Norwegian University of Sciences and Technology)
Gordon Heier (University of Houston)
Yunping Jiang (Queens College, City University of New York)
Aaron Levin (Michgan State University)
Song-Ying Li (University of California, Irvine)
Hsueh-Yung Lin (National Taiwan University)
Steven Lu (Université du Québec à Montréal)
Junjiro Noguchi (University of Tokyo)
Keiji Oguiso (University of Tokyo)
Tran Van Tan (Hanoi National University of Education)
Amos Turchet (Roma Tre University)
Paul Vojta (University of California, Berkeley)
Yu Yasufuku (Nihon University)

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中央研究院 ACADEMIA SINICA

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Talk Abstracts

Pietro Corvaja (University of Udine)

gcd estimates in algebraic geometry

Twenty years ago, Bugead, Zannier and myself proved, for multiplicatively independent positive integers a, b, the upper bound on the greatest common divisors

$$\gcd(a^n - 1, b^n - 1) \ll \exp(\epsilon n)$$

A link between such a result and a special case of the celebrated Vojta's Conjecture was noticed by Silverman, while an analogue in Nevanlinna theory was proved by Noguchi, Winkelmann and Yamanoi. Generalizations to higher dimensions, over number fields, have been obtained by A. Levin.

The object of this talk is the discussion of extensions of these bounds to function fields, obtained in the last decades by Corvaja-Zannier, A. Turchet, J. Wang and her collaborators. These results can be interpreted as upper bounds for the singularity of an algebraic curve on a semi-abelian variety. A recent result, obtained in collaboration with U. Zannier and F. Zucconi, concerns abelian surfaces.

Ya Deng (Université Lorraine)

Green-Griffiths-Lang Conjecture for Algebraic Varieties with Big Fundamental Groups

The Green-Griffiths-Lang (GGL) conjecture asserts that any entire curve in a complex projective variety of general type cannot be Zariski dense. This conjecture fascinates many complex geometers, in part due to its arithmetic analogy with the Bombieri-Lang conjecture for rational points in algebraic varieties over number fields. In this talk I will report a recent work with Cadorel and Yamanoi, focusing on the proof of the generalized GGL conjecture for quasi-projective varieties whose topological fundamental groups possess a big and reductive representation into a complex general linear group.

Dinh Tien Cuong (National University of Singapore)

Every complex Henon map is exponentially mixing of all orders and satisfies the CLT

We show that the measure of maximal entropy of every complex Henon map is exponentially mixing of all orders for Holder observables. As a consequence, the Central Limit Theorem holds for all Holder observables. The talk is based on a joint work with Fabrizio Bianchi.

John Erik Fornaess (Norwagian University of Sciences and Technology)

Transcendental Henon maps on $\mathbb{C}^2$

This is joint work with Leandro Arosio, Anna Miriam Benini and Han Peters. I will discuss some of our previous work and work in progress. A Henon map is a map of the form $H(z, w) = (f(z) + aw, z)$. We are studying the case when f is a transcendental function. Most of the previous work such as Bedford-Smillie, Fornaess-Sibony and many others have dealt with the case when f is a polynomial.

Gordon Heier (University of Houston)

A Schmidt-Nochka theorem for closed subschemes in subgeneral position

The seminal Subspace Theorem due to Schmidt has been generalized in many ways over the years. One such generalization was our formulation in terms of Seshadri constants for closed subschemes in general position. In this talk, we will present a further refinement in terms of Seshadri constants and weighted sums involving closed subschemes in subgeneral position. The use of weights in this context is motivated by Nochka's proof of a conjecture of Cartan on defects of holomorphic curves in projective space relative to a possibly degenerate set of hyperplanes. Our refinement yields new Diophantine inequalities. This is joint work with Aaron Levin.

Yunping Jiang (The City University of New York-Graduate Center and Queens College)

Holomorphic Liftings in Teichmüller Theory as a Problem in Several Complex Variables

Consider a subset E of the Riemann sphere containing $n+3$ points and let $\widetilde{E} = E \cup \{q\}$ for $q \notin E$. The Teichmüller spaces $T(E)$ and $T(\widetilde{E})$ are bounded simply connected domains in $\mathbb{C}^n$ and $\mathbb{C}^{n+1}$, respectively. There is a natural holomorphic split submersion $\mathcal{P} : T(\widetilde{E}) \to T(E)$ called the forgetful map. Suppose W is another connected bounded domain in $\mathbb{C}^k$ with a basepoint 0 and $f : W \to T(E)$ is a basepoint-preserving holomorphic map. The lifting problem in Teichmüller theory is that can we find a basepoint-preserving holomorphic map $\widetilde{f} : W \to T(\widetilde{E})$ such that $f = \mathcal{P} \circ \widetilde{f}$? In this talk, I will explain this problem and show that when W is the unit disk Δ in the complex plane $\mathbb{C}$, we can use some method in PDE to give an affirmative answer. Furthermore, I will explain that when W is a higher-dimensional polydisk Δ^k or a higher-dimensional unit ball $B_k(0,1)$, it is still a difficult problem to be solved.

Aaron Levin (Michigan State University)

Diophantine Approximation to Closed Subschemes

We discuss recent results developing Diophantine approximation in the setting of closed subschemes, including a new inequality on surfaces (joint work with Keping Huang and Zheng Xiao).

Song-Ying Li (University of California, Irvine)

Bergman metrics as pull-backs of the Fubini-Study metric

In this talk, we study domains in $\mathbb{C}^n$ or Stein manifolds M such that their Bergman metrics have constant holomorphic sectional curvature. We prove a uniformization theorem when $\kappa < 0$ through the Calabi rigidity theorem and holomorphic extension theorems. We also discuss the case when $\kappa \geq 0$. We provide several interesting examples of existence of such M. Under certain conditions on M, we prove that the Bergman metric of M can not have non-negative constant holomorphic sectional curvatures. This is a joint work with Xiaojun Huang at Rutgers University.

Hsueh-Yung Lin (National Taiwan University)

On the cone conjectures for Schoen varieties

The cone conjectures were formulated by Kawamata, Morrison, and Totaro for Calabi-Yau varieties, and more generally for Calabi-Yau pairs. We will explain what they are and survey some known results and predictions. We will then study the cone conjectures for Schoen varieties, based on joint work with C. Gachet, I. Stenger, and L. Wang.

Steven Lu (Université du Québec à Montréal)

Rigidity of maps and the Shafarevich conjecture

The classical Shafarevich conjecture solved by Faltings claims in the complex geometric setting the finiteness of isomorphic one parameter varying families of genus $g > 1$ curves with fixed degeneration locus on the parameter curve C. This is equivalent to the finiteness of the space of maps from a fixed Zariski open subset C_0 of C to the moduli space of curves whose strategy of proof proceeds in two steps: boundedness of the space of such maps and rigidity, i.e. that this space is zero dimensional.

I will report on a joint work with Ariyan Javanpeykar, Ruiran Sun and Kang Zuo in the one pointed setting, specifically, on the finiteness of the space of pointed maps from C_0 to the moduli space of canonically polarized varieties and its generalizations.

Junjiro Noguchi (University of Tokyo)

Some transcendence problems in arithmetic and analytic functions

Goal: Explore applications and related subjects of the Nevanlinna theory.

Let A be a semi-abelian variety with an exponential map $\exp : \mathrm{Lie}(A) \to A$. In this talk we discuss the Nevanlinna theory and the applications of the entire curve $\widehat{\exp} f := (\exp f, f) : \mathbf{C} \to A \times \mathrm{Lie}(A)$ associated with an entire curve $f : \mathbf{C} \to \mathrm{Lie}(A)$.

After recalling an application of Big Picard Theorem (N. '81) to Manin–Mumford Conjecture (Raynaud's Theorem) (N. '18), we give a Nevanlinna theoretic proof to the *analytic Ax-Schanuel Theorem* for semi-abelian varieties, which was proved by J. Ax '72 in the case of formal power series by Kolchin's differential algebra.

We assume some non-degeneracy condition for f such that for $A = (\mathbf{C}^*)^n$, the elements of the vector-valued function $f(z) - f(0) \in \mathrm{Lie}(A) \cong \mathbf{C}^n$ are $\mathbf{Q}$-linearly independent. Our proof is based on the Log Bloch-Ochiai Theorem and a key estimate which we show.

Our next aim is to establish a *2nd Main Theorem* for $\widehat{\exp} f$ and its k-jet lifts with truncated counting functions at level one. We give some applications to a problem of a type raised by S. Lang and the unicity. The results, I hope, clarify the relationship between the problems of Ax-Schanuel type and Nevanlinna theory.

Keiji Oguiso (University of Tokyo)

Kawaguchi-Silverman Conjecture for endomorphisms of Ueno-type varieties

Ueno-type varieties are the blow-up of the cyclic quotient varieties $E^n/\langle \zeta_n I_n \rangle$ of the self-product E^n of an elliptic curve E at the singular points, where ζ_n is a primitive n-th root of 1 with $n = 2$, 3, 4 or 6 according to the types of E. Ueno-type varieties provide interesting examples in algebraic dynamics as observed by several authors.

In this talk, I would like to show Kawaguchi-Silverman Conjecture (KSC) for endomorphisms of Ueno-type varieties defined over an algebraic closure of the field of rational numbers. This in particular provides a first non-trivial and non-vacuous example of rationally connected varieties of dimension 3, 4, 5, satisfying KSC for all endomorphisms without extra conditions and a better understanding of a result of Lesieutre and Satriano on Calabi-Yau threefolds with non-trivial c_2-contraction.

Tran Van Tan (Hanoi National University of Education)

Holomorphic mappings having bounded spherical derivative on the preimage of hypersurfaces

We show that an entire curve in $P^n(\mathbb{C})$ which is Brody on the preimage of sufficiently many hypersurfaces should be Brody. We also investigate the minimal surfaces whose Gauss curvature vanishes or is bounded on the preimage of sufficiently many hypersurfaces.

Amos Turchet (Università degli Studi Roma Tre)

Applications of the Ru-Vojta method on hyperbolicity

We discuss various hyperbolicity results for algebraic varieties both in the arithmetic setting (i.e. for varieties defined over number fields), in the function field setting, and in the Nevanlinna setting. One of the main tools used is a version of the Main Theorem of Ru and Vojta with truncation. This is joint work with Erwan Rousseau and Julie Wang.

Paul Vojta (University of California, Berkeley)

Schmidt's Subspace Theorem over (Geometric) Function Fields

Currently, my long-term goal is to prove Schmidt's Subspace Theorem over arithmetic function fields (finitely generated field extensions of Q, with the formalisms of heights, etc., as defined by Moriwaki in the early 2000s). As a first step in this effort, I have been trying to find a proof of the Subspace Theorem over geometric (i.e., classical) function fields in one or more variables over a constant field F of characteristic zero, that could serve as a model for a proof over arithmetic function fields. (This theorem has been proved by Julie Wang, but her proof uses derivatives in ways that would not carry over to arithmetic function fields.)

In this talk I will describe work in progress in finding such a proof, based on an early proof by Lang of Roth's theorem over function fields in one or more variables.

Yu Yasufuku (Nihon University)

Applying Ru–Vojta method to GCD inequalities

Bugeuad–Corvaja–Zannier and Corvaja–Zannier obtained GCD inequalities for certain two-variable expressions evaluated at units, and Silverman observed that these are implied by Vojta's conjecture on certain blowups. Levin has generalized these results to higher-dimensions, which was further generalized by Wang–Yasufuku. In this talk, I will talk about GCD inequalities related to these results, and compare them with what are implied by Vojta's conjecture. The main ingredient of the proof is the Ru–Vojta invariant which enables an efficient usage of the Schmidt subspace theorem.

Agenda

	Aug. 14 (Mon)	Aug. 15 (Tue)	Aug. 16 (Wed)	Aug. 17 (Thu)	Aug. 18 (Fri)
09:30 – 10:20	Pietro Corvaja	Keiji Oguiso	John Erik Fornaess	Paul Vojta	Steven Lu
10:20 – 10:50	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
10:50 – 11:40	Yu Yasufuku	Hsueh-Yung Lin	Dinh Tien Cuong	Gordon Heier	Junjiro Noguchi
11:40 – 14:00	Lunch	Lunch	Lunch	Lunch	Lunch
14:00 – 14:50	Ya Deng	Aaron Levin	Free discussions	Tran Van Tan	
15:10 – 16:00	Yunping Jiang	Amos Turchet		Song-Ying Li	
16:00 – 16:30	Reception	Tea Time		Tea Time	
16:30 – 17:20		TBA		TBA	

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