2 Bertrand Paradox.

In real world usually we use price as strategy rather than quantity. We like Cournot's result, but we don't like its approach.

Bertrand Paradox: 2 firms produce identical goods.

$$Di(P_i, P_j) = \begin{cases} D(p_i) & \text{if } P_i < P_j \\ \frac{1}{2}D(P_j) & \text{if } P_i = P_j \\ 0 & \text{if } P_i > P_j \end{cases}$$

Nash Equilibrium: $P_1 = P_2 =$MC, that is, the product is selling at zero economic profit. Even a duopoly would suffice to restore competition.

Bertrand's result is paradoxical because if the number of firms goes from one to two, the price decreases from the monopoly price to the competitive price and stays at the same level as the number of firms increases further. This is not very realistic, as in reality, markets featuring a small number of firms with market power typically charge a price in excess of marginal cost. The empirical analysis shows that in the most industries with two competitors, positive profits are made. Solutions to the Paradox attempt to derive solutions that are more in line with solutions from the Cournot model of competition, where two firms in a market earn positive profits that lie somewhere between the perfectly competitive and monopoly levels.

Some reasons the Bertrand paradox does not strictly apply:

(1) Edgeworth solution: capacity constraints and/or increasing MC.

(2) Product Differentiation:

spatial location game; Monopolistic competition; vertical differentiation

(3) Repeated game: Implicit Collusion

2.1 Capacity constraints.

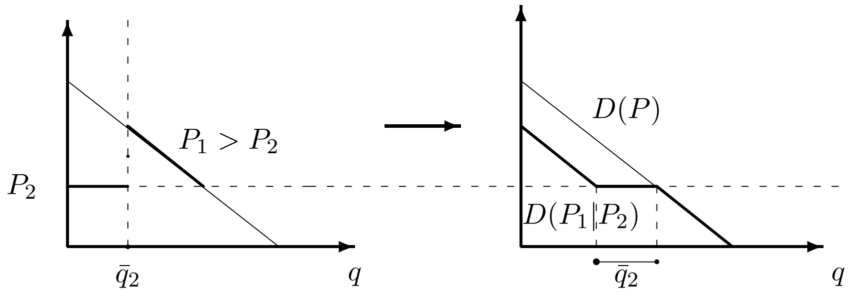
$\overline{q}_2 < D(P_2)$

Rationing Rules:

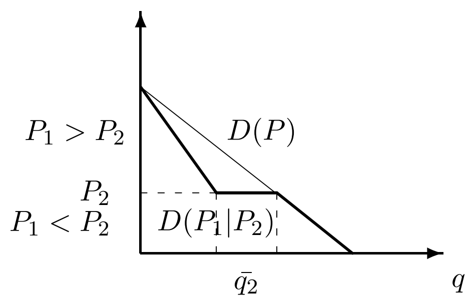
(1)Efficient Rationing: It is easier for higher value consumers to access the lower price

$$\widetilde{D}_1(P_1, P_2) = \begin{cases} \max[D(P_2) - \overline{q}_2\ , 0] & \text{if } P_2 < P_1 \\ D(P_1) & \text{if } P_2 \geq P_1 \end{cases}$$

(2)Random Rationing:



(1) Efficient Rationing Rule



(2) Random Rationing Rule

$$\widetilde{D}_1(P_1, P_2) = \begin{cases} D(P_1)(\frac{D(P_2)-\overline{q}_2}{D(P_2)}) & \text{if } P_2 < P_1 \\ D(P_1) & \text{if } P_2 \geq P_1 \end{cases}$$

some high value people will buy at P_1, sales more.

others: ex: queueing and waiting may yield the eager consumers buy at first

Capacity-Constrained Price Game: Kreps & Scheinkman(1983) and Davidson and Deneckere (1986)

$D(P) = 1 - P$

-assume $c_0 \in [\frac{3}{4}, 1]$,

$T = 1$:given c_0, firm i chooses his capacity $\bar{q}_i$ with cost $c_0\bar{q}_i$

$T = 2$:firms choose prices P_1, P_2 given $c_1 = 0$

(1)- Efficient Rationing Rule:

without loss generality suppose $\overline{q}_i < \frac{1}{3}$

$\because \pi_i \leq P(1-P) - c_0\overline{q}_i \leq \frac{1}{4} - c_0\overline{q}_i \leq 0$ if $\overline{q}_i \geq \frac{1}{3}$

Lemma 1 $T = 2, \;\; P^* = 1 - (\overline{q}_1 + \overline{q}_2)$

Proof: lower price? can not sell more, Hence, revenue↓

Raise price?

Suppose firm 1 chooses $P > P^*$

Let $q \equiv 1 - P - \overline{q}_2 \Leftrightarrow P = 1 - \overline{q}_2 - q)$. Thus, we have firm 1's profit equals

$P(1 - P - \overline{q}_2) = (1 - \overline{q}_2 - q)q \equiv \pi_1(q)$

First order condition implies

$\pi_1'(q) = 1 - \overline{q}_2 - 2q > 0 \ \forall q, \overline{q}_2 < \frac{1}{3}$

$\rightarrow q \uparrow \Leftrightarrow P \downarrow$ *Q.E.D.*

At $T = 1$,firm 1 $\max_{\overline{q}_1 \geq 0} \overline{q}_1(1 - \overline{q}_1 - \overline{q}_2 - c_0)$ (like cournot game)

1.No fixed cost.

2.have capacity cost

Firms choose $\bar{q}_1 = \bar{q}_2 = \frac{1-c_0}{3}$, the same equilibrium quantity as in the cournot game.

Implication: The Cournot equilibrium can be viewed as the result of price competition among firms, as long as they choose the scale of operation before they set prices. However, this statement is not robust. Under different rationing rule, we can show that the counot outcome cannot be an equlibrium outcome for this game.

(2)Random Rationing Rule:

$T = 2 \ \max_{P \geq 0} P \cdot \widetilde{D}_1(P_1, P_2) = (\frac{\overline{q}_1}{\overline{q}_1 + \overline{q}_2}) \max_{P \geq 0} P \cdot D(P)$

Hence, optimal $P = P^M$.

Firms have incentive to raise the price higher than the market clearing price.

because in this model higher price has more chance to sell to the high value consumers than the efficient rationing rule.

$P = P^M > P^*$ if $\bar{q}_1 + \bar{q}_2$ sufficiently high ($c_0 < \frac{1}{4}$). Hence, the cournot outcome result of Kreps & Scheinkman fails.

2.2 Product Differentiation

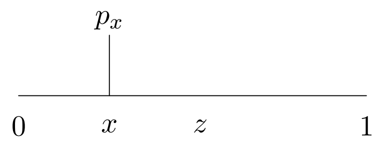
- **Horizontal differentiation** refers to differences between brands based on different product characteristics but not on different overal quality. Example : McDonalds's Quarter Pounder v.s. Burger King Whopper; Toyota Camry v.s. Ford Taurus

- **Vertical Differentiation** refers to differences in the actual quality of two brands. Example: Lexus v.s. Taurus.

1. Horizontal differentiation (Spatial Competition):

1.1. the linear city: minimal vs maximal product differentiation

Model: Consumers are uniformly distributed along an interval, $[0, 1]$. A consumer z pays a mill price (p_x) + transportation cost ($f(x, z)$) for purchasing from store x. (Assume each consumer's valuation for one unit of product is sufficiently high.)



1.1.1. When firms' prices are fixed , we have minimum product differentiation (Simple Location Game)

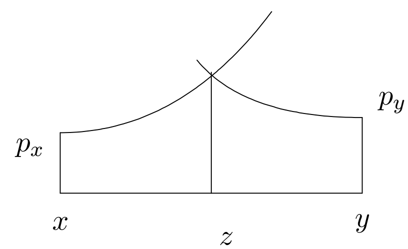
Two firms simultaneously choose locations. Prices are fixed at 1. No cost.

Same location results in the equal sharing of the entire market.

* The only N.E.: Each firm will chooses $x = \frac{1}{2}$. There is a Welfare loss.

Three firms simultaneously choose locations. No pure strategy Nash equilibrium exists.

1.1.2. Price Game without locational decisions.



Consumer z's utility

$U_z(x, p_x) = V - f(x, z) - p_x$

Consumers' valuation V is sufficiently high

$f(x, z)$: travel cost, an increasing function of the traveling distance $|z - x|$.

ex. $f(x, z) = t(x - z)^2$ quadratic model or $f(x, z) = t|x - z|$ linear model.

Suppose $f(x, z) = t(x - z)^2$ and there is no production cost. Given (p_x, p_y), find demand:

Consumer z purchases goods from x iff $p_x + t(z)^2 \leq p_y + t(1 - z)^2$

Find $\hat{z}$, the marginal consumer's location:

$$p_x + t(z)^2 = p_y + t(1 - z)^2.$$

$$D_x(p_x, p_y) = \hat{z} = \frac{p_y - p_x + t}{2t}$$

Hence, D_x is increasing in p_y and decreasing in p_x.

$$D_y(p_x, p_y) = 1 - D_x$$

Firm x chooses p_x to maximize profits: $\max_{p_x} p_x D_x(p_x, p_y)$

$$\max_{p_x} p_x \frac{p_y - p_x + t}{2t}$$

Take derivative with respect to p_x and then set it equal to 0.

$$\frac{p_y}{2t} - \frac{p_x}{t} + \frac{1}{2} = 0$$

$$p_x = \frac{1}{2}(p_y + t) \tag{1}$$

Firm y chooses p_y to maximize profits: $\max_{p_y} p_y D_y(p_x, p_y)$

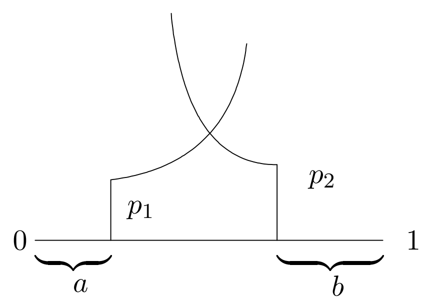
Similarly, we have

$$p_y = \frac{1}{2}(p_x + t) \tag{2}$$

From (1) and (2), we can solve $p_x = p_y = t$. Hence, $\pi_x^* = \pi_y^* = \frac{1}{2}t$

1.1.3. First, firms choose location (a, b). After observe (a, b), firms choose prices (p_1, p_2) simultaneously.

(Maximal product differentiations)



Marginal consumer $\hat{z}$:

$p_1 + t(z-a)^2 = p_2 + t(1-b-z)^2$

$D_1(p_1, p_2) = \hat{z} = \frac{p_2 - p_1}{2(1-a-b)t} + \frac{1-b+a}{2}$ increasing in a

Given a,b:

$\pi_1(p_1, p_2) = p_1 D_1(p_1, p_2)$

Solve F.O.C.

$p_1^*(a, b) = t(1-a-b)(1 + \frac{a-b}{3})$

$p_2^*(a, b) = t(1-a-b)(1 + \frac{b-a}{3})$

$D_1\left(p_{1,}^* p_2^*\right) = \frac{1}{2} - \frac{1}{6}(b-a)$

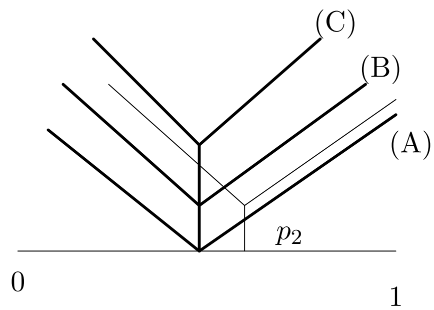
We can show that $\frac{\partial \pi_1}{\partial a} < 0$. Hence, $a^* = 0$, $b^* = 0$. We have Maximal product differentiation.

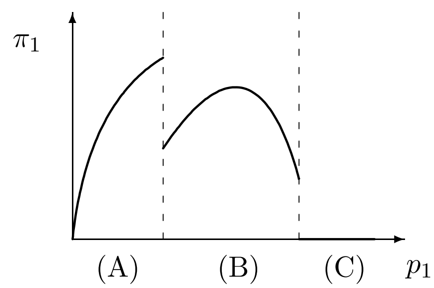
Social Optimal Location: Minimize total travel cost: $a = \frac{1}{4}$, $b = \frac{1}{4}$

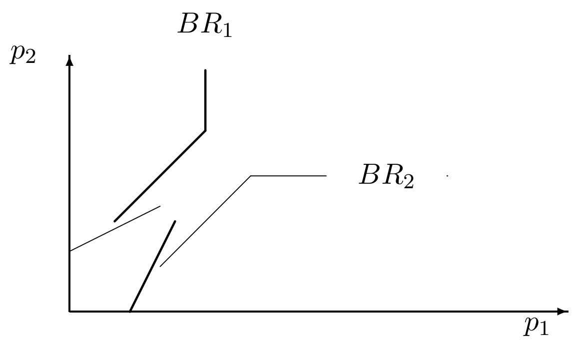
Total travel cost in Competition is bigger. Social welfare loss.

Note that for linear travel cost $f(x, z) = t|x - z|$, the profit function $\pi_1(p_1, p_2)$ is discoutinuous in p_1 and the pure strategy Nash equilibrium does not exist.

(A) Take the entire market (B) Share Market (normal case) (C) No market share







the best response functions

1.2 Circular Model (Salop 1979)

T=1: entry decision n firms enter.

T=2: pricing decision

Assume linear travel cost $t|x - z|$, and fixed entry cost $f > 0$

Suppose all other firms set price $= p$

$p_0 + tz = p + t(\frac{1}{n} - z)$

$z = \frac{p - p_0 + \frac{t}{n}}{2t}$

$D_0(p_0, p) = 2z$, $\pi_0 = 2p_0 \frac{p - p_0 + \frac{t}{n}}{2t}$
$p^* = \frac{t}{n}$ and $\pi^* = \frac{t}{n^2}$
Free entry implies zero profits, $\frac{t}{n^2} = f$. Hence, $n^c = \sqrt{\frac{t}{f}}$

What is social optimal number of firms

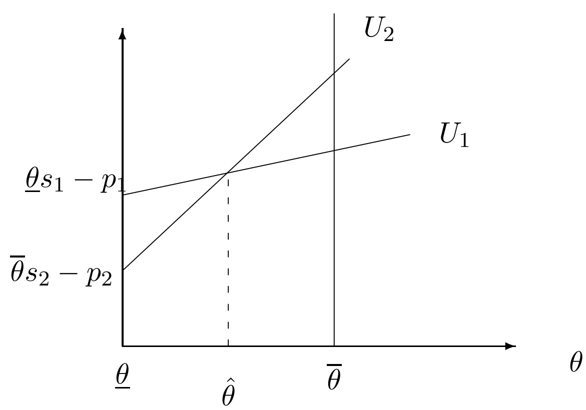
$$\min_n nf + 2n \int_0^{\frac{1}{2n}} tx \mathrm{d}x$$

$n^s = \frac{1}{2}\sqrt{\frac{t}{f}} = \frac{1}{2}n^c$
Too many firms in the market
Social optimal : balance fixed cost and transportation cost
competition: business stealing incentive

2. Vertical Product Differentiation (Maximal quality differentiation)
Demand:

$$U = \begin{cases} \theta s - p & \text{if buy at } p \text{ with quality } s \\ 0 & \text{if not buy} \end{cases}$$

$MC = 0$
w.l.o.g $s_2 > s_1$
$\theta \in [\underline{\theta}, \overline{\theta}]$ where $\overline{\theta} = 1 + \underline{\theta} > 1$
Assumptions:
1. $\overline{\theta} \geq 2\underline{\theta}$ 2 firms can survive.
2. $\frac{\overline{\theta} - 2\underline{\theta}}{3}(s_2 - s_1) \leq \underline{\theta} s_1$ market is covered



$\hat{\theta}s_1 - p_1 = \hat{\theta}s_2 - p_2$

$\hat{\theta} = \frac{p_2 - p_1}{\Delta s}$, where $\Delta s = s_2 - s_1$.

Hence, $D_1(p_1, p_2) = \hat{\theta} - \underline{\theta}$ and $D_2 = \overline{\theta} - \hat{\theta}$.

Each firm chooses price to maximize profit: $\max_{p_i} p_i D_i(p_1, p_2)$

F.O.C. $\Rightarrow$ Best response function: $p_1(p_2) = (p_2 - \underline{\Delta})/2$, where $\underline{\Delta} = \underline{\theta}\Delta s$.

$p_2(p_1) = (p_1 + \overline{\Delta})/2$, where $\overline{\Delta} = \overline{\theta}\Delta s$.

$p_1^* = \frac{\overline{\theta} - 2\underline{\theta}}{3}\Delta s$

$p_2^* = \frac{2\overline{\theta} - \underline{\theta}}{3}\Delta s > p_1^*$

$\pi_1^* = \frac{(\overline{\theta} - 2\underline{\theta})^2 \Delta s}{9}$

$\pi_2^* = \frac{(2\overline{\theta} - \underline{\theta})^2 \Delta s}{9} > \pi_1^*$

$\Delta s \uparrow \Rightarrow \pi_1, \pi_2 \uparrow$

Hence, we have Maximal quality differentiation.

3. The Multinomial Logit : (See Discrete Choice Theory of Product Differentiation by Anderson, de Palma and Thisse) The probability that a consumer choosing a from S is

$$P_S(a) = \frac{\exp U(a)}{\sum_{b \in S} \exp U(b)}$$

The first justification :

The choice axiom proposed by Luce is a relation between choice probabilities defined on different subsets of A. For any $S \subset A$ and $T \subset A$ such that $S \subset T$, and

1. if, for given $a \in S$, $P(a, b) \neq 0, 1$ for all $b \in T$, then

$$P_T(a) = P_T(S) \cdot P_S(a)$$

2. if $P(a, b) = 0$ for some a, $b \in T$, then for all $S \subset T$

$$P_T(S) = P_{T - \{a\}}(S - \{a\}).$$

Part 1 is a path independence property. Part 2 means that if $a \in T$ is never chosen in a pairwise comparison with some other $b \in T$, then a can be deleted from T.

Theorem (Luce, 1959) Assume that $P(a, b) \neq 0, 1$ for all a, $b \in A$. Part 1 of the choice axiom is satisfied if and only if there exists a positive real-valued function u defined on A such that

$$P_s(a) = \frac{u(a)}{\sum_{b \in S} u(b)}$$

Let $U(a) = \ln u(a)$, then we have the multinomial logit model.

The second justification :

Consider a population of individuals facing the same choice set A. What is the fraction of the population choosing a given alternative ? The utility from purchasing good $i \in A$ is $U_i = u_i + e_i$, where u_i is observable and e_i is not. Hence, $\tilde{U}_i = u_i + \varepsilon_i$, where ε_i takes into account the idiosyncratic taste differences of consumers. Suppose there are two stores and let $\varepsilon = \varepsilon_1 - \varepsilon_2$. If the distribution function for ε is given by $\frac{1}{1+\exp(-(x/\mu))}$ then the probability of choosing 1 is

$$P_A(1) = \frac{\exp(u_1/\mu)}{\exp(u_1/\mu) + \exp(u_2/\mu)}$$

2.3 Dynamic Model of Oligopoly

Fluctuating Demand $D(P, \varepsilon)$, $\varepsilon \sim$i.i.d.

- Green and Poter (1984 Econometrica):
  - ε unobservable, incomplete information (Bayesian EQ),
  - "Price war" during recession.
  - Punishment phase $P = c < P^M$
  - prevent secret price cuts, switch to punishment phase once for a while
- Rotemberg and Saloner (1986 AER):
  - ε observale, complete information (Subgame perfect),
  - "Price war" during boom.
  - cooperation phase $P^M > P > c$
  - lower the benifit from deviation in boom, punishment phase never happen in EQ path

Green and Poter (1984 Econometrica)
Cournot game
Cannot see quantity$\rightarrow$price signal
but. some noise in demand side. demand shock.
$\rightarrow$price information can not tell how much the other firm produce. (deviate or not)
Public signal is an imperfect signal.
N firms
$q \in [0, \overline{q}]^N$
P: price(public signal)
$r_i(q_i, P)$:realized payoff in the stage game

$h^t \in (P^0, ..., P^{t-1})$

$g_i(q) = E_P\{r_i(q_i, P)\} = \int r_i(q_i, P)\ \mathrm{d}F(P \mid q_i)$

where $F(P \mid q) = \Pr ob\{\widetilde{P} \leq P \mid q\}$

ex:$P = h(Q) + \varepsilon.$ or $P = \theta h(Q)$ $\varepsilon, \theta \sim iid$

satisfies MLRP (Monotone Likelihood Ratio Property, i.e., $\frac{f(P|q)}{f(P|q')}$ is increasing in P for $q > q'$)

Let q^*(symmetric) be the stage game Nash quantity vector

$\lim_{N\to\infty} g_i(q^*) \to 0$

Trigger Price Strategy

Specifies $\widehat{q}$, $\widehat{P}$, $\widehat{T}$

Two phases $\begin{cases} \text{cooperate} & \text{Play } \widehat{q} \\ \text{punish} & \text{Play } q^* \end{cases}$

Firms play cooperate phase as long as $P \geq \widehat{P}$ and switch to punish phase for $\widehat{T}$ periods if $P < \widehat{P}$

Let $\lambda(q) = 1 - F(\widehat{P} \mid q)$ be the probability of $P \geq \widehat{P}$. Thus λ is decreasing in q.

conform $\widehat{\pi}$: present discounted value of profit at t in the collusive phase

$\widehat{\pi} = g(\widehat{q}) + \delta\lambda(\widehat{q})\widehat{\pi} + \delta[1 - \lambda(\widehat{q})]\delta^{\widehat{T}}\widehat{\pi}$

$\Rightarrow \widehat{\pi} = \frac{g(\widehat{q})}{[1-\delta\lambda(\widehat{q})-\delta^{\widehat{T}+1}(1-\lambda(\widehat{q}))]} < \frac{g(\widehat{q})}{1-\delta}$

For $\lambda = 1$ or $\widehat{T} = 0$ we have $\widehat{\pi} = \frac{g(\widehat{q})}{1-\delta}$

$$\max_{\widehat{P},\ \widehat{T}\ \widehat{q}} \widehat{\pi}$$

s.t. $g(\widehat{q}) + \delta\lambda(\widehat{q})\widehat{\pi} + \delta[1 - \lambda(\widehat{q})]\delta^{\widehat{T}}\widehat{\pi}$

$\geq g(q_i, \widehat{q}_{-i}) + \delta\lambda(q_i, \widehat{q}_{-i})\widehat{\pi} + \delta[1 - \lambda(q_i, \widehat{q}_{-i})]\delta^{\widehat{T}}\widehat{\pi}$

$\forall q_i$

$[g(q_i, \widehat{q}_{-i}) - g(\widehat{q})]$ one period gain from increasing production

$\leq \frac{\delta[1-\delta^{\widehat{T}}][\lambda(\widehat{q})-\lambda(q_i,\widehat{q}_{-i})]g(\widehat{q})}{1-\delta\lambda(\widehat{q})-\delta^{\widehat{T}+1}(1-\lambda(\widehat{q}))}$

$q_i \uparrow$ then Prob. remaining in coorperation $\downarrow$

Two ways to set : $\widehat{T} \uparrow, \widehat{P} \downarrow$ or $\widehat{T} \downarrow, \widehat{P} \uparrow$

(See Tirole textbook section 6.7 and ex 6.8, ex 6.9)

Rotemberg & Saloner(1986 AER)

- Main Result: Colluding Oligopolists behave more competitively when demand is high.

N: identical Firm

Homogeneous (Perishable) good.

MC=AC=c

Market Inverse Demand $P(Q_t, \varepsilon_t)$

$Q_t = \sum_{j=1}^{N} q_{jt}$

$\varepsilon_t \in [\underline{\varepsilon}, \overline{\varepsilon}]\tilde{\ } F(.)$ observed before q_{jt} is set, ε_t iid

$P(Q, \varepsilon_0) \geq P(Q, \varepsilon_1)\ \forall \varepsilon_0 > \varepsilon_1 \forall Q$

$\pi(Q, \varepsilon_t) \equiv [P(Q, \varepsilon_t) - c]\frac{Q}{N}$

maximized at $\pi^m(Q_t, \varepsilon_t) = \pi^m(\varepsilon_t)$

$\pi(Q, \varepsilon_t) \uparrow$ in $\varepsilon_t\ \forall Q$

$\Rightarrow \pi^m(\varepsilon_t) \uparrow$ in ε_t

Suppose strategic variable is price.

Single Period Result $P_1^* = c, \quad i = 1, ..., N$

Infinite Horizon-Implicit Collusion.

K$\equiv$ punishment cost

$\pi^m(\varepsilon_t)$ sustainable

$$\Leftrightarrow \begin{array}{ccc} N\pi^m(\varepsilon_t) - k & \leq & \pi^m(\varepsilon_t) \\ (P_j < P^c) & & (P_j = P^c) \end{array} \tag{3}$$

Note: π (cheating) grows more quickly in ε_t

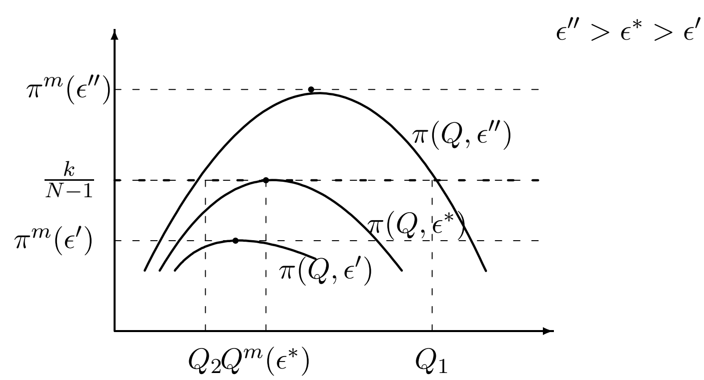
(1) $N\pi^m(\varepsilon_t) - k \leq \pi^m(\varepsilon_t)$

(2) $\varepsilon_t^* = \varepsilon \quad s.t.\ \ \varepsilon_t > \varepsilon_t^* \Rightarrow (1)$ fails

(3) $\pi^c(\varepsilon_t, \varepsilon_t^*) = \begin{cases} \pi^m(\varepsilon_t), & \varepsilon_t \leq \varepsilon_t^* \\ \pi^m(\varepsilon_t^*) = \frac{k}{N-1}, & \varepsilon_t > \varepsilon_t^* \end{cases}$

at $\varepsilon_t^*(1)$ holds

use $P_i = c$ as subgame punishment strategy



$k(\varepsilon_t, \varepsilon_t^*) = E\{\sum_{j=1}^{\infty} \delta^j \pi^c(\varepsilon_{t+j}, \varepsilon_t^*) \mid \varepsilon_t\}$

(4) $k(\varepsilon_t^*) = \frac{\delta}{1-\delta}[\int_{\underline{\varepsilon}}^{\varepsilon^*} \pi^m(\varepsilon)\ \mathrm{d}F(\varepsilon) + (1 - F(\varepsilon^*)\pi^m(\varepsilon^*)]$

$\varepsilon_t \in [\underline{\varepsilon}, \overline{\varepsilon}]\tilde{\ } iid$

$k \leftrightarrows \varepsilon_t^*$

seek largest ε^* s.t (1)˜(4) hold.

(5) $g(\varepsilon') \equiv \pi^m(\varepsilon') - \frac{k(\varepsilon')}{N-1}$

$g(\varepsilon^*) = 0 \Rightarrow$(1)˜(4) hold.

$g(\underline{\varepsilon}) = \pi^m(\underline{\varepsilon})[1 - \frac{\delta}{(1-\delta)(N-1)}] < 0 \Leftrightarrow N < \frac{1}{1-\delta}$ (6a)

$g(\overline{\varepsilon}) = \pi^m(\overline{\varepsilon}) - \frac{\delta}{(1-\delta)(N-1)}] \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} \pi^m(\varepsilon) \, \mathrm{d}F(\varepsilon) > 0 \Leftrightarrow \frac{\pi^m(\varepsilon)}{E(\pi^m(\varepsilon))} > \frac{\delta}{(1-\delta)(N-1)}$ (6b)

$g(\underline{\varepsilon}) < 0, g(\overline{\varepsilon}) > 0, g(\varepsilon')$ continuous.

$\Longrightarrow$exist an interior ε^* satisfying (1)-(4)

Intuition:$\frac{b}{c}$ ε_t is iid. penalty to cheating is constant. but the gain from cheating $\uparrow$ in ε_t.

Cartel picks Q_1 over Q_2 because Q_1 $\Rightarrow$lower gain from cheating.

$\varepsilon_t > \varepsilon^* \Rightarrow Q^c(\varepsilon_t)[P(Q^c(\varepsilon_t), \varepsilon_t) - c]$ constant.

Then $Q_T^c \uparrow$ in $\varepsilon_T \Rightarrow P \downarrow$ in ε_t ,$\varepsilon_t > \varepsilon^*$

Firms approach competitive behavior when demand is high.