Peng-Hsiang Wang, Taipei (Taiwan)

Principles as Ideal Ought. Semantic Considerations on the Logical Structure of Principles*

The aim of this paper is to employ possible worlds semantics to deal with some problems in the logical analysis of the structure of principles. Firstly, I examine Jan-Reinard Sieckmann's logical construction of principles as normative arguments and point out that this approach lacks a precise semantic foundation. I then give an outline of the Kripke-Hintikka semantics for deontic logic. Finally, this semantic framework, mutatis mutandis, will be applied to explicate the notion of ideal ought and real ought, optimization requirements, and weighing and balancing.

I. Introduction

Since the appearance of Ronald Dworkin's celebrated article 'The Model of Rules' in 1967,[1] the logical structure of principles and their application, i.e., weighing and balancing, has become a widely discussed issue in legal theory. Although Dworkin claims that the difference between rules and principles is a *logical* distinction,[2] he has never given a precise account of the logical structure of principles. Robert Alexy, who enthusiastically adopts Dworkin's distinction between rules and principles and develops a more sophisticated theory of principles, has defined principles as '*optimization requirements*'. According to Alexy's standard definition, 'principles are norms which require that something be realized to the greatest extent possible given the legal and factual possibilities'; in contrast, rules are definitive requirements that 'contain fixed points in the field of the factually and legally possible'.[3] In an earlier work, Alexy characterized principles as 'ideal ought' and rules as 'real ought', but denied the need to introduce two different deontic operators to represent these two different kinds of ought.[4] More recently, he slightly modified his standard definition of principles

* I am grateful to Prof. Dr. Jan-Reinard Sieckmann for accepting this paper in this volume and for his encouragement in past years. I am also indebted to Dr. Carsten Heidemann for some ideas inspired by his unpublished manuscript 'The theory of principles and the best of all possible worlds' (presented at the 20th IVR World Congress in Amsterdam, 2001).

1 Ronad Dworkin, 'The Model of Rules' (1967) 35 *University of Chicago Law Review* 15–46, reprinted as 'The Model of Rules I' in Dworkin, *Taking Rights Seriously* (Harvard University Press: Cambridge Mass., 1978), 14–45.

2 Dworkin, *Taking Rights Seriously* (n. 1), 24–8.

3 Robert Alexy, *A Theory of Constitutional Rights*, trans. J. Rivers (Oxford University Press: Oxford, 2002), 47–8.

4 Robert Alexy, 'Zum Begriff des Rechtsprinzips' in Alexy, *Recht, Vernunft, Diskurs* (Suhrkamp Verlag: Frankfurt a.M., 1995), 204. Recently, Alexy has withdrawn this thesis. Now he intro-

as optimization requirements. This modified definition draws a distinction between 'requirements to be optimized' ('commands to be optimized') and 'optimizing requirements' ('commands to optimize'). Principles as the objects of weighing and balancing are requirements to be optimized and can be termed 'ideal "ought"' or 'ideals'. An ideal "ought", according to Alexy, is something that is to be optimized and thereby transformed into a real "ought". The optimizing requirements are now placed at the meta-level; they prescribe what is to be done in applying principles, namely, to realize the ideal "ought" to the greatest extent possible.[5]

Alexy's modified definition is criticized by Sieckmann because this modification does not take principles as reasons for a particular result of weighing and balancing. In Sieckmann's view, principles are not only objects of balancing but also normative arguments for a definitively valid norm, namely, a real "ought" or a rule, which is the outcome of the procedure of balancing.[6] In a series of works Sieckmann has developed a complex theory, which he calls 'principles as normative arguments', to elaborate this idea.[7] To a large extent, his theory can be viewed as a refinement of Alexy's theory of principles. In contrast to Alexy, however, Sieckmann is probably the first to attempt to make use of deontic logic to explicate the logical structure of principles. In the following section I will summarize the main points of Sieckmann's theory and point out some possible drawbacks to it.

duces an indexed deontic operator 'O_i' for ideal ought and formalizes the optimization requirements as '*O*Opt*Op*', where 'Opt' stands for 'to optimize'. Alexy claims that '*O*Opt*Op*' ('*Op*' ought to be optimized) is equivalent to 'O_ip'. See Robert Alexy, 'Ideales Sollen' in L. Clérico and J.-R. Sieckmann (eds.), *Grundrechte, Prinzipien und Argumentation. Studien zur Rechtstheorie Robert Alexys* (Nomos: Baden-Baden, 2009), 24–6. An assessment of Alexy's formalism is beyond the scope of this paper and shall be discussed elsewhere.

5 Robert Alexy, 'On the Structure of Legal Principles' (2000) 13 *Ratio Juris* 300–1.

6 Jan-R. Sieckmann, 'Principles as Normative Arguments' in Ch. Dahlmann and W. Krawietz (eds.), *Values, Rights and Duties in Legal and Philosophical Discourse. Rechtstheorie Beiheft 21* (Duncker & Humblot: Berlin, 2005), 197–9.

7 See, e.g., Jan-R. Sieckmann, 'Semantischer Normbegriff und Normbegründung' (1994) 80 *ARSP* 227–45; 'Logische Eigenschaften von Prinzipien' (1994) 25 *Rechtstheorie* 163–89; 'Zur Analyse von Normkonflikten und Normabwägungen' in G. Meggle (ed.), *Analyomen 2 Vol. III* (De Gruyter: Berlin / New York, 1997), 349–56; 'Begriff und Struktur von Regeln, Prinzipien und Elemente im Recht' in B. Schlicher, P. Koller and B.-C. Funk (eds.), *Regeln, Prinzipien und Elemente im System des Rechts* (Verlag Östereich: Wien, 2000), 69–82; 'Principles as Normative Arguments' (n. 6), 197–209; *Recht als normatives System* (Nomos: Baden-Baden, 2009), 21–65.

II. Sieckmann's Theory of Principles as Normative Arguments

Sieckmann's point of departure is a distinction among (1) normative sentences or norm formulations, (2) normative statements, and (3) normative arguments.[8] The underlying reason for this distinction is that Sieckmann, following Alexy, adopts the semantic concept of norms. According to this conception, a norm is the meaning of normative sentences and the relation between normative sentences and norms are analogous to that between assertive sentences and propositions. One norm can be expressed by several normative sentences which have the same meaning.[9] The semantic concept of norms is not beyond controversy, of course, but I will not take up this question here. For the sake of simplicity I will sometimes use 'norm' and 'normative sentence' interchangeably in this paper. With help of the formal language of deontic logic, the basic form of normative sentences can be represented as

(1) *Op*,

where '*O*' is a deontic operator 'it is obligatory that...' and '*p*' stands for the content of obligation. For example, if '*p*' stands for 'the insulting speech is protected', then '*Op*' expresses the norm 'it is obligatory that the insulting speech is protected', or more naturally, 'the insulting speech ought to be protected'.

According to Sieckmann, a normative sentence can be used to make a normative statement which asserts that a norm *Op* exists, in the sense of being definitively valid; it can also be used to put forward a normative argument which requires that a certain norm shall be definitively valid.[10] The distinction between normative statements and normative arguments is the cornerstone of Sieckmann's whole theoretical construction.[11] One of Sieckmann's main theoretical concerns is to build this distinction into the formal language of deontic logic, which he intends to employ to analyze the structure of principles and balancing. A difficulty, however, arises from the semantic concept of norms. The semantic conception assumes that the concept of norms and the question of validity are strictly separated, such that norms are to be defined without including the element of validity.[12] A norm in a purely semantic sense is merely the meaning content of normative sentences, in other words, a normative sentence is the linguistic formulation of norms. A norm formulation by itself does not assert that the norm which it expresses is valid, but a normative statement is an assertion about the definitive validity of a norm. Sieckmann has made an interesting observation about the semantic relation between the truth of normative sentences

8 Sieckmann, 'Semantischer Normbegriff und Normbegründung' (n. 7), 228–38; 'Zur Analyse von Normkonflikten und Normabwägungen' (n. 7), 352.

9 Alexy, *A Theory of Constitutional Rights* (n. 3), 21–5; Sieckmann, 'Semantischer Normbegriff und Normbegründung' (n. 7), 228–35.

10 Sieckmann, 'Logische Eigenschaften von Prinzipien' (n. 7), 165; 'Principles as Normative Arguments' (n. 6), 199.

11 Sieckmann, *Recht als normatives System* (n. 7), 41–64.

12 Alexy, *A Theory of Constitutional Rights* (n. 3), 25; Sieckmann, 'Semantischer Normbegriff und Normbegründung' (n. 7), 228.

and the truth of normative statements. In this, he states that a normative statement which asserts that a norm is valid is true, if this norm is valid, and that a normative sentence can be defined as 'true' if and only if the corresponding normative statement is true, i.e., the norm formulated by it is valid.[13] For example, the normative statement 'it is definitively valid that the insulting speech ought to be protected' is true, if the norm that the insulting speech ought to be protected is valid, and if this norm is valid, the normative sentence 'the insulting speech ought to be protected' is also true. If normative statements are assertions about the validity of norms and valid norms are expressed by true normative sentences, there are two alternatives for Sieckmann to deal with the notion of validity in his logical construction. The first one is to define the semantic notion of truth of normative sentences at the level of meta-language, as the possible worlds semantics for deontic logic does, and regard valid norms as the facts that true normative sentences state. In this way, validity is the existence of norms. Sieckmann does not choose this alternative, but rather favors the second one, which incorporates the expression 'validity' into the object-language of deontic logic. In order to be able to express the valid norms and thereby to represent normative statements, Sieckmann introduces a symbol, '*G*', which predicates the definitive validity of a norm.[14] A normative statement asserting that a certain norm *Op* is definitively valid can be formalized as

(2) *GOp*.

Furthermore, Sieckmann uses the term '*n*' as an individual constant (name) denoting a norm such as *Op*. Thus, (2) can be reformulated as '*Gn*'.[15] '*GOp*' or '*Gn*' is true if and only if *n*, the norm expressed by '*Op*', is definitively valid.

In Sieckmann's view, neither normative sentences nor normative statements can adequately capture the logical features of principles in legal reasoning. On the one hand, normative sentences are too weak to represent the normative content of principles, because principles always contain a claim of validity.[16] For example, in the case of insulting speech the principle of the freedom of speech claims that what it requires, i.e., 'the insulting speech ought to be protected', shall be definitively valid. However, the normative sentence '*Op*' expresses only a norm in a purely semantic sense, without saying whether or not it is valid. On the other hand, normative statements are too strong to be the logical structure of principles.[17] If a principle were a definitively valid norm, then what it requires would be a definitive obligation without taking into account other countervail-

13 Sieckmann, 'Semantischer Normbegriff und Normbegründung' (n. 7), 235.

14 Recently, Sieckmann uses 'VAL_{DEF}' to denote the validity predicate, see Sieckmann, 'Principles as Normative Arguments' (n. 6), 199; *Recht als normatives System* (n. 7), 27. But the difference is merely a verbal one.

15 Sieckmann, 'Semantischer Normbegriff und Normbegründung' (n. 7), 233–4; *Recht als normatives System* (n. 7), 51–2.

16 Sieckmann, 'Logische Eigenschaften von Prinzipien' (n. 7), 168; 'Principles as Normative Arguments' (n. 6), 199.

17 Sieckmann, 'Logische Eigenschaften von Prinzipien' (n. 7), 168; 'Principles as Normative Arguments' (n. 6), 198–9; *Recht als normatives System* (n. 7), 25, 57.

ing principles. Yet this is not the way in which principles are applied. Principles must be balanced against one another in order to determine what is definitively required in a concrete case. For example, in the above case the principle of the freedom of speech (P_1) and the principle of the right to personal honor (P_2) collide with each other. While P_1 requires that the insulting speech ought to be protected (Op), P_2 requires that it ought not to be ($O\neg p$). Which norm is definitively valid is to be decided and justified by balancing P_1 and P_2. Following Alexy, Sieckmann also holds that the task of balancing is to establish a (conditional) priority relation between the colliding principles.[18] So, if P_1 prevails over P_2, then Op is definitively valid (GOp). In contrast, if P_2 prevails over P_1, then $O\neg p$ is definitively valid ($GO\neg p$). This example illustrates Sieckmann's idea of the justification of valid norms in his 'model of principles'. The central claim of this model is that the definitive validity of a norm is established by weighing and balancing, and that principles figure as reasons for or against a definitively valid norm in weighing and balancing.[19] In Sieckmann's model, principles are normative arguments which require that a certain norm Op ought to be definitively valid. Hence, normative arguments are validity obligations or requirements of validity. The rudimentary structure of normative arguments is

(3) $OGOp$ (or 'OGn', where 'n' denotes Op).[20]

In Sieckmann's theory, the difference between normative statements and normative arguments is intended to be an explication of Alexy's distinction between ideal ought and real ought. According to Alexy,

> 'An ideal ought is an ought which does not presuppose that what ought to be is both factually and legally possible to the full extent, but which demands to be fulfilled approximately or as far as possible. By contrast, the command character of directives which can be either fulfilled or not fulfilled can be characterized as "real ought."'[21]

However, the notion of ideal ought is characterized by Sieckmann in a somewhat odd way. If a principle requires p in a given case and this requirement is formulated as 'Op', then, in Sieckmann's analysis, the ideal situation which P demands to realize is not that p is the case, i.e., Op is fulfilled, but rather that Op is definitively valid.[22] This is the very reason why Sieckmann thinks that principles are normative arguments having the structure of requirements of validity ($OGOp$). In Sieckmann's view, if the demand of a principle P_1, say, 'the insulting speech ought to be protected', is expressed by the normative sentence 'Op', then, as said above, the optimal result from the viewpoint of P_1 is that the norm that the insulting speech ought to be protected is definitively valid, i.e., GOp. However, P_1

18 Sieckmann, 'Zur Begründung von Abwägungsurteilen' (1995) 26 *Rechtstheorie* 46–7; *Recht als normatives System* (n. 7), 66–7. See also Alexy, *A Theory of Constitutional Rights* (n. 3), 50–4; 'On the Structure of Legal Principles' (n. 5), 296–7.

19 Jan-R. Sieckmann, 'Legal System and Practical Reason. On the Structure of a Normative Theory of Law' (1992) 5 *Ratio Juris* 291–3; *Recht als normatives System* (n. 7), 19–21.

20 Sieckmann, 'Logische Eigenschaften von Prinzipien' (n. 7), 170; 'Principles as Normative Arguments' (n. 6), 170; *Recht als normatives System* (n. 7), 27.

21 Alexy, 'Zum Begriff des Rechtsprinzips' (n. 4), 204.

22 Sieckmann, 'Logische Eigenschaften von Prinzipien' (n. 7), 170.

is merely a normative argument which claims that *Op ought to be* definitively valid; it alone cannot guarantee that *Op is really* definitively valid. In Alexy's words, a principle contains only an ideal ought that is not yet relativized to the factual and legal possibilities.[23] Since the legal possibilities for realizing a principle are essentially determined by competing principles, whether an ideal ought $OGOp$ can be realized and transformed to a real ought asserted by the normative statement GOp depends upon the result of balancing it against opposing normative arguments (e.g., $OGO\neg p$ or $OG\neg Op$). If the normative argument constituted by P_1 is defeated by a stronger counter-argument constituted by another principle P_2, then what P_1 demands will not be definitively valid, and *Op* will not become a real ought. This explains the prima-facie character of principles: an ideal ought is only a prima-facie ought.

However, characterizing principles as normative arguments which require that a certain norm shall be definitively valid is not the end of Sieckmann's construction. The semantic concept of norms impels him to complicate the construction of the logical structure of principles. The formulation of normative arguments '$OGOp$' is still a normative sentence. One might ask whether it represents a norm in a purely semantic sense or states a definitively valid norm. The first possibility is excluded because '$OGOp$' also stands for the requirement of principles (e.g., the principle of the freedom of speech requires that the norm 'the insulting speech ought to be protected' shall be valid). As an ideal ought, every principle demands that what it requires is to be definitively valid. So one might add the predicate 'G' before '$OGOp$', but this will turn a normative argument into a normative statement '$GOGOp$', which is too strong to be the logical form of principles, because what a principle actually says is that its requirement *ought to be* definitively valid. Hence one must immediately insert another 'O' before '$GOGOp$', thereby constructing a new normative argument '$OGOGOp$'. But then the same problem emerges once again, and the same operation has to be carried out endlessly. Therefore, Sieckmann maintains that the structure of principles must be a normative argument containing an *infinite reiteration of requirements of validity*:

(4) $\ldots OGOGOp$.[24]

Let 'n_0' denote the norm *Op* which a principle requires to be definitive valid in a given case, and 'n_1' be the first-order requirement of validity OGn_0. According to Sieckmann, the complete structure of principles as normative arguments should be an infinite set of requirements of validity of ever higher order supporting the first-order requirement of validity: $\{n_1\text{: } OGn_0,\ n_2\text{: } OGn_1,\ \ldots,\ n_{i+1}\text{: } OGn_i, \ldots\}$.[25]

Although Sieckmann's approach aims at capturing the structural features of principles, his construction is not wholly satisfactory. Why should we not say that the ideal situation which a set of principles demands to realize is simply that the obligations arising from the principles $Op_1, \ldots, Op_n$ are all fulfilled, namely, all

23 Alexy, 'On the Structure of Legal Principles' (n. 5), 300.

24 Sieckmann, 'Logische Eigenschaften von Prinzipien' (n. 7), 170–2; 'Principles as Normative Arguments' (n. 6), 200; *Recht als normatives System* (n. 7), 27, 51.

25 Sieckmann, 'Principles as Normative Arguments' (n. 6), 203, 208.

the demanded actions are simultaneously performed or every obligatory state of affairs $p_1,\ldots, p_n$ is the case? The optimizing character of principles or ideal ought may be explained in the way that such an ideal situation, when the principles collide with each other in a given case, is impossible to bring about and can only be realized approximately. It is therefore questionable whether principles must have the complex structure of normative arguments, as Sieckmann maintains. Moreover, the difference between normative arguments and normative statements does not seem to make the distinction between the ideal and real ought more comprehensible. A normative statement *GOp* is true, as mentioned above, if the norm expressed by the normative sentence '*Op*' is definitively valid, in other words, if *Op* is a real ought. One might ask: Does *Op* not also envisage an ideal situation in which *p* is the case? Does not every valid norm demand that the ideal situation it envisages shall be realized as much as possible? Sieckmann might object that there is a difference between '*OGOp*' and '*GOp*', for the ideal situation envisaged by the former is that *Op* is valid rather than that *p* is the case. However, if an ideal situation is a situation in which all obligations are fulfilled, does not the fact that *Op* is valid in an ideal situation imply that *p* is the case in this situation?

The problems sketched above indicate that the main drawback of Sieckmann's theory lies in the lack of a solid semantic foundation. This deficiency leads to a bewildering 'logic of normative arguments'. Although Sieckmann makes use of the language of deontic logic to work out his ideas about the logical structure of principles, he does not provide a formal semantics to give a precise interpretation of the deontic formulae he uses. In addition to the deontic operator, Sieckmann also introduces '*G*' to formalize normative statements such as '*Op* is definitively valid' (*GOp*), as well as normative arguments like '*Op* should be definitively valid' (*OGOp*), and claims that principles have the structure of reiterated requirements of validity, e.g. 'it should be valid that *Op* should be valid' (*OGOGp*). But Sieckmann does not specify the truth conditions for the simple deontic sentence '*Op*', nor for the complex ones such as '*GOp*' and '*OGOp*'. In fact, '*G*' is a predicate which is true of norm-individuals. Therefore, in Sieckmann's approach norms are not only the meaning of normative sentences, but also abstract entities, and the predicate '*G*' denotes a distinct subset of the norm-individuals in a world. What Sieckmann relies upon is *ipso facto* a higher-order deontic predicate logic, which is much more complex than the standard deontic propositional logic and gives rise to serious ontological and semantic problems, which Sieckmann seems to be unaware of.[26]

Without a precise formal semantics it is hard to define the central logical concepts such as 'satisfiability', 'logical validity' or 'logical consequence' in Sieckmann's logic of normative arguments. For example, Sieckmann holds that

26 For the complex semantics of higher-order modal logics, see, e.g., Nino B. Cocchiarella and Max Freund, *Modal Logic: An Introduction to Its Syntax and Semantics* (Oxford University Press: Oxford, 2008), 215–52.

(5) $GOGn \rightarrow Gn$,
(6) $OGn \rightarrow OOGn$,
(7) $OGn \rightarrow OGOGn$

are all logically valid. But he does not rely on a semantic theory of deontic modalities (e.g., the possible worlds semantics which is usually employed to define the truth conditions for modal sentences, as well as the concept of logical validity and consequences in modal logics) to prove the logical validity of these sentences. Rather, he seems to be content with an intuitive interpretation of them and thinks that their translations in ordinary language are plausible in the context of his theory of normative system and normative argumentation.[27] For example, (5) is valid because 'if requirements, on the contrary, refer to the validity of norms ($GOGn$) and the validity of a norm depends only on the subject's own judgment, as it corresponds to an autonomous moral, then it must be reasonable to recognize the validity of this norm by a requirement of the validity of a norm'.[28] According to Sieckmann, (6) and (7) are valid in the framework of 'interest-based norm justification'.[29] It is therefore not clear whether these sentences are really logically valid or merely represent the substantive claims of Sieckmann's theory of normative argumentation.

The lack of a formal semantics also gives rise to problems concerning normative inconsistency in Sieckmann's logic of normative arguments. Sieckmann distinguishes two kinds of normative conflict: that between normative statements and that between normative arguments. According to Sieckmann, the conflict between normative statements is a logical contradiction. If two normative statements are in conflict with each other, they cannot both be true, and at least one of them must be false.[30] Of course, there is a logical contradiction between Gn ('the norm n is definitely valid') and $\neg Gn$ ('the norm n is not definitively valid'), but this is not the intended type of the conflict between normative statements in Sieckmann's sense. Rather, the contradiction should be understood in the following manner: While one normative statement asserts that Op (n_1) is definitively valid, the other asserts that $O\neg p$ (n_2) is definitively valid. However, if they are formalized as first-order sentences 'Gn_1' and 'Gn_2', as Sieckmann does, it is hard

27 This can be clearly seen in Sieckmann's interpretation of the deontic counterparts of some axioms of alethic modal logics, such as S4 '$\Box p \rightarrow \Box\Box p$' and S5 '$\Diamond p \rightarrow \Diamond\Box p$' (For instance, (6) is the counterpart of the S4 axiom.), see Sieckmann, 'Logische Eigenschaften von Prinzipien' (n. 7), 175–6. Sieckmann seems to ignore the fact that the validity and the semantic plausibility of these axioms depend on certain requirements imposed on the accessibility relation on the set of possible worlds in a model structure. Whether the deontic counterparts of these axioms are plausible as well also relies upon how the relation of deontic alternativeness between possible worlds is to be determined in the semantics for deontic logic. See Section III, below.

28 Sieckmann, 'Zur Analyse von Normkonflikten und Normabwägungen' (n. 7), 354.

29 See Sieckmann, 'Logische Eigenschaften von Prinzipien' (n. 7), 175; 'Zur Analyse von Normkonflikten und Normabwägungen' (n. 7), 354.

30 Jan-R. Sieckmann, 'Zur Abwägungsfähigkeit von Prinzipien' in H.-J. Koch and U. Neumann (eds.), *Praktische Vernunft und Rechtsanwendung. ARSP Beiheft 53* (Steiner Verlag: Stuttgart, 1994), 206; *Recht als normatives System* (n. 7), 42–3.

to see where the logical contradiction lies. Why cannot they both be true? Perhaps Sieckmann assumes that two norms with mutually contradictory contents cannot both be definitively valid.[31] But this is a normative requirement rather than a requirement of logical consistency. Furthermore, this normative requirement still presupposes a precise definition of the inconsistency of a set of normative sentences, otherwise it is impossible to know whether two norms are 'contradictory' to each other. Such a definition, however, is not found in Sieckmann's theory. Now consider the conflict between normative arguments, which Sieckmann claims cannot be regarded as a logical contradiction. This claim is a prerequisite for weighing and balancing normative arguments.[32] Accordingly, two colliding normative arguments such as $OGOp$ and $OGO\neg p$ can be simultaneously valid or true. But exactly what does this mean? Surely, it does not mean that the corresponding normative statements $GOGOp$ and $GOGO\neg p$ can both be true, because this will violate the normative requirement that two incompatible norms cannot both be valid, unless a different notion of validity designed especially for normative arguments is introduced.[33] Regarding the character of ideal ought, perhaps the non-contradiction between two normative arguments $OGOp$ and $OGO\neg p$ should be interpreted in this way: In an ideal situation both Op and $O\neg p$ are valid. If Op and $O\neg p$ are simultaneously valid in an ideal situation, then there must be another ideal situation, whether they are the same or not, in which p as well as $\neg p$ is the case. Yet such a situation is impossible, because $p \wedge \neg p$ is a logical contradiction. If Sieckmann insists that the conflict between normative statements GOp and $GO\neg p$ is a logical contradiction, then there is no reason to think that the colliding normative arguments do not lead to a logical contradiction. Hence it seems that the conflict between normative arguments cannot be adequately defined without resort to the notion of contradiction or inconsistency. To put it more generally, without a sound semantic notion of normative inconsistency it is hard to see why and when normative arguments or statements come into conflict.

To avoid the difficulties provoked by Sieckmann's approach, I will apply the possible worlds semantics (the Kripke-Hintikka semantics) for deontic logic, *mutatis mutandis*, to explicate the notion of ideal and real ought, the collision of principles, and weighing and balancing. Within this semantic framework the following considerations will be elaborated:

First, although the 'ideal ought' may be regarded as obligations valid in an 'ideal' world and represented as reiterated obligations such as 'OOp', it can be proved that under certain conditions the iteration of the deontic operator 'O' is superfluous.

31 Sieckmann, *Recht als normatives System* (n. 7), 25.

32 Sieckmann, 'Zur Abwägungsfähigkeit von Prinzipien' (n. 30), 206; 'Logische Eigenschaften von Prinzipien' (n. 7), 165.

33 This is the strategy that Sieckmann recently used. He introduces another symbol 'VAL_{ARG}' to denote the predicate of the validity of normative arguments. See Sieckmann, 'Principles as Normative Arguments' (n. 6), 203; *Recht als normatives System* (n. 7), 52–3.

Second, if a set of principles is inconsistent in a given situation (i.e., they come into conflict), this situation cannot be transformed into an ideal world in which all obligations contained in this set are fulfilled. Under these circumstances, we have to look to the sub-ideal situations close to the ideal worlds as much as possible, and determine which of them are the 'best' or 'optimal'. This process may be called 'weighing and balancing'.

Third, instead of incorporating the predicate '*G*' into the object-language, 'validity' will be regarded as a semantic notion. The idea of real ought as the result of weighing and balancing is explicated in the way that something is definitively obligatory if and only if it is the case in all of the best 'almost ideal' worlds relative to a given situation.

III. An Outline of the Kripke-Hintikka Semantics for Deontic Logic

In this section I will give a brief outline of the possible worlds semantics of Hintikka and Kripke.[34] The underlying idea of the Kripke-Hintikka semantics for deontic logic can be understood in the following manner, as illustrated by Georg Henrik von Wright. The norm-contents of a given set of norms, in von Wright's view, constitute a description of an alternative, 'ideal' world. Compared with the actual world, this description might not be true, even almost false, because it is not always the case that all obligations are fulfilled in the actual world. This means that the actual world is not 'perfect', the ideal not realized. However, the ideal world must be a realizable possible world. Thus, von Wright notes: 'the function of norms, one could say, is to urge people to realize the ideal, to make them act in such a way that the description of the real approximates the description of the ideal'.[35]

Based on this idea, the truth conditions of normative sentences and the satisfiability (consistency) of a set of sentences can be defined by introducing a set of possible worlds. Let $p, q, r\ldots$ be sentential variables. '$\neg$' (not), '$\wedge$' (and), '$\rightarrow$' (if..., then...) are the familiar sentential connectives. A set of sentences W is called a 'partial description of a possible world' if and only if the following conditions are satisfied:

34 Saul A. Kripke, 'Semantical Considerations on Modal Logics' in L. Linsky (ed.), *Reference and Modality* (Oxford: Oxford University Press, 1971), 63–72; Jakko Hintikka, 'Some Main Problems of Deontic Logic' in R. Hilpinen (ed.), *Deontic Logic: Introductory and Systematic Readings* (D. Reidel: Dordrecht, 1981), 59–104. The outline of semantics for deontic logic presented here follows more closely Hintikka's model set and model system, though with a minor modification in the manner of Kripke. For an illuminating introduction to possible worlds semantics, see James W. Garson, *Modal Logic for Philosophers* (Cambridge University Press: Cambridge, 2006), 57–115.

35 Georg Henrik von Wright, 'Is and Ought' in S. L. Paulson and B. Litschewski Paulson (eds.), *Normativity and Norms. Critical Perspectives on Kelsenian Themes* (Oxford University Press: Oxford, 1998), 374–5.

(C. $\neg$) If $p \in W$, then not $\neg p \in W$.

(C. $\wedge$) $p \wedge q \in W$ if and only if $p \in W$ and $q \in W$.

(C. $\rightarrow$) $p \rightarrow q \in W$ if and only if it is not the case that $p \in W$ and $\neg q \in W$.

If no misunderstanding is provoked, I will call a set of sentences W satisfying these conditions a 'possible world'.[36] Intuitively, '$p \in W$' ('$\in$' is read as 'is a member of') can be understood as 'p is true in the possible world W'. A sentence can be true in some possible world but false in another. (C.$\neg$), (C.$\wedge$) and (C.$\rightarrow$) together specify the truth conditions of compounded sentences built from sentential variables and truth-functional connectives. A set of sentences $\{p_1, \ldots, p_n\}$ is satisfiable (or consistent) if and only if there is a world W such that $p_i \in W$ for every p_i ($i = 1, \ldots$, n), shortly, $\{p_1, \ldots, p_n\} \subseteq W$ ('$\subseteq$' is read as 'is a subset of'). q is a logical consequence of $\{p_1, \ldots, p_n\}$ if and only if $\{p_1, \ldots, p_n, \neg q\}$ is not satisfiable. p is contradictory if and only if $\{p\}$ is not satisfiable. p is logically valid if and only if $\neg p$ is contradictory. I will use '$\perp$' to indicate a contradiction (such as '$p \wedge \neg p$'), and (C.$\neg$) can be formulated in another manner:

(C.$\perp$) There is no possible world W such that $\perp \in W$.

(C.$\perp$) says that $\perp$ cannot be true in any possible world. Thus, a possible world is what is described by a set of consistent sentences. Furthermore, the symbol '$\vdash$' will stand for the consequence relation. Let S be a set of sentences, '$S \vdash p$' means that p is a logical consequence of S.

Let us now consider the truth condition of normative sentences with the form 'Op'. According to the underlying idea illustrated above, 'p is obligatory in the actual world' means that p is the case in every possible world that we can bring about and in which all obligations in the actual world are fulfilled. Such possible worlds are called 'deontically perfect worlds' or 'ideal worlds'. To be more precise, a binary relation '$\boldsymbol{R}$' among possible worlds will be introduced. '...$\boldsymbol{R}$...' is read as '...*is a deontic alternative to* ...'. For any two possible worlds W and W^+, $W\boldsymbol{R}W^+$ holds (W^+ is a deontic alternative to W) if and only if W^+ is an ideal world relative to W. Intuitively, we may think of W^+ as a deontic alternative to W in the sense that what is obligatory in W is the case in W^+. Thus, the truth condition for 'Op' can be defined as follows:

(C.O) $Op \in W$ if and only if $p \in W^+$ for every W^+ such that $W\boldsymbol{R}W^+$.

(C.O.) says that Op is true in a world W if and only if p is true in every deontic alternative to W (p is the case in all ideal worlds). In the standard deontic logic, another deontic operator 'P' for 'it is permitted that...' is defined as '$\neg O \neg$' ('Pp' $=_{\mathrm{df}}$ '$\neg O \neg p$'). Therefore, 'p is permitted in a world W' means that there is at least one ideal world in which p is the case without violating any obligation. The truth condition of 'Pp' is as follows:

(C.P) $Pp \in W$ if and only there is a possible world W^+ such that $W\boldsymbol{R}W^+$ and $p \in W^+$.[37]

36 Precisely speaking, W is only a 'partial' description of a possible world because it is not required that for every sentence p, either $p \in W$ or $\neg p \in W$.

37 'p is forbidden' can be defined as '$O \neg p$' or '$\neg Pp$'. Accordingly, 'p is forbidden' is true if and only if $\neg p$ is true in every ideal world.

In the semantics for deontic logic the relation $\boldsymbol{R}$ is not reflexive, i.e., it is not acceptable that $W\boldsymbol{R}W$ holds for every possible world W. Some possible world, such as our actual one, cannot be a deontic alternative to itself. This is due to the fact that obligations are often violated in the actual world. In other words, the real world is not ideal, and what is obligatory is not always actually the case. Since $\boldsymbol{R}$ cannot be reflexive,

(8) $Op \rightarrow p$

is not logically valid. In the standard deontic logic, another condition required of the relation $\boldsymbol{R}$ is seriality, which says that there is always a deontic alternative to any possible world. This condition can be formulated as

(C.O*) For every possible world W, if $Op \in W$, then there is at least one possible world W^{+} such that $W\boldsymbol{R}W^{+}$ and $p \in W^{+}$.

The assumption behind seriality and the corresponding condition (C.O*) is that the ideal world described by the norm-contents must be a 'genuine' possible world which can be brought about through our action. If something is obligatory, it must be possible that it is the case in some world, though not necessarily in the actual one. In short, an ideal world must be realizable. Accordingly, a norm having contradictory content such as $O\bot$ (or $O(p \wedge \neg p)$) cannot hold in any possible world, because, according to (C.$\bot$), there is no possible world in which $\bot$ is true. What a set of sentences containing $\bot$ describes is an 'impossible world'. (C.O*) and (C.P) together make

(9) $Op \rightarrow \neg O \neg p$.

logically valid. (9) says that what is obligatory is also permitted.

Although it seems plausible that nothing impossible is obligatory, (C.O*) and (9) are not beyond question, especially when the possibility of normative conflicts is recognized. It is not uncommon for a normative system to have valid norms whose contents are mutually contradictory, such as Op and $O\neg p$. Above all, if a legal system contains principles, then in many situations it gives rise to norms that cannot be jointly fulfilled. If our actual world W contains conflicting norms, e.g., $Op \in W$ as well as $O\neg p \in W$, then there is no ideal world relative to W. For this reason, it seems that (C.O*) and (9) have to be given up. However, I do not think that (C.O*) is an unreasonable requirement, because the normative conflict between principles is normally a kind of *conditional inconsistency*, i.e., inconsistency *modulo* certain facts.[38] If a set of principles is consistent in itself, then only under certain circumstances will it give rise to conflicting obligations, but this does not amount to saying that the contents of principles cannot be described by a consistent set of sentences. If the ideal situation envisaged by a set of principles is still a possible world, there seems no reason to reject (C.O*). I will return to the collision of principles and the conditional normative inconsistency in the next section.

38 For the normative conflict and conditional normative inconsistency, see Carlos. E. Alchourrón, 'Conflicts of Norms and the Revision of Normative Systems' (1991) 10 *Law and Philosophy* 413–25.

There are some further requirements imposed on the relation of deontic alternativeness R. These requirements are of significance to the problems concerning iterated deontic operators. In order to avoid unnecessary complexities, Sieckmann's validity predicate 'G' will be dropped in the following discussion about the semantics of iterated obligations. Since the truth conditions for normative sentences have been defined in the Kripke-Hintikka semantics, I will assume that a norm is valid in a world if and only if the corresponding normative sentence 'Op' is true in this world. According to Sieckmann, principles contain 'requirements of a particular normative state and, hence, a reiteration of normative modalities'.[39] For example, the principle of the freedom speech requires that insulting speech be permitted, and the principle of the right to personal honor requires that insulting speech not be permitted; both requirements can be represented as 'OPp' and '$OO\neg p$', respectively. In the following only the iteration of the obligation operator 'O' will be considered. One possible way to characterize principles as ideal ought is to think that principles have the structure of iterated obligations, such as 'OOp', which says that it ought to be the case that p is obligatory, e.g., 'it ought to be the case that insulting speech ought to be protected'. How shall we interpret a normative sentence containing iterated deontic operators such as 'OOp' in the semantics for deontic logic? Applying (C.O) we get the truth condition for 'OOp':

(C.OO) $OOp \in W$ if and only if $Op \in W^{+}$ for every W^{+} such that WRW^{+}.

(C.OO) says that OOp is true in a possible world W if and only if Op is true in all deontic alternatives to W. If 'OOp' is the logical form of ideal ought, we may say that an ideal ought is an 'obligation in the ideal worlds' or an 'obligation we should adopt'. Furthermore, if $Op \in W^{+}$, then, applying (C.O) again, p is true in every deontic alternative to W^{+} (the condition (C.O*) guarantees that there must be such an alternative world to the ideal world W^{+}). Hence, (C.OO) can be revised to

(C.OO*) $OOp \in W$ if and only if $p \in W^{++}$ for every W^{+} and W^{++} such that WRW^{+} and $W^{+}RW^{++}$.

In other words, OOp is true in a possible world W if and only if p is true in every deontic alternative to the ideal worlds relative to W. Surely, W^{++} is an ideal world relative to the ideal world W^{+} because $W^{+}RW^{++}$ holds. Is W^{++} also an ideal world relative to W, i.e., WRW^{++}? If the relation of deontic alternativeness R is transitive, then WRW^{++} follows from WRW^{+} and $W^{+}RW^{++}$. This means that every deontic alternative to a deontic alternative to some possible world is also an ideal world relative to this world. The requirement of transitivity seems plausible in so far as R can be interpreted as '…better than…'. A deontic alternative is a better world than the actual world, and any possible world which is better than a deontic alternative to the actual world is also better than the actual world. The transitivity of R can be formulated in the following condition:

(C.OO^{+}) If $Op \in W$, then $Op \in W^{+}$ for every W^{+} such that WRW^{+}.

39 Sieckmann, 'Principles as Normative Arguments' (n. 6), 198.

The reason for accepting (C.OO$^+$) under the constraint of transitivity is obvious: If $Op \in W$, then p must be true in every deontic alternative to W. If W^{++} is a deontic alternative to some deontic alternative to W, say, W^+, then, because of transitivity, W^{++} is also a deontic alternative to W. It follows that $p \in W^{++}$. Since W^{++} is also a deontic alternative to W^+, according to (C.O), $p \in W^{++}$ implies that $Op \in W^+$. Intuitively, (C.OO$^+$) says that every obligation obtaining in the actual world also obtains in the ideal worlds. It is easy to see that (C.OO$^+$) validates

(10) $Op \rightarrow OOp$.[40]

From (10) we can infer '$OOp \rightarrow OOOp$', '$OOOp \rightarrow OOOOp$'...and so on. This means that if an obligation holds in the actual world, this obligation can be reiterated infinitely. (C.OO$^+$) and (10) appear to correspond the reiterated requirement of validity in Sieckmann's theory.[41] Nevertheless, I think that iterating deontic operators is not an adequate way to represent the logical structure of ideal ought if another property of R is considered.

Although reflexivity is unacceptable because our actual world is not 'perfect' or 'ideal', it seems reasonable to adopt a weaker assumption that when a world is an ideal one relative to ours, it is also an ideal world relative to itself, more precisely, if WRW^+, then W^+RW^+. This 'weak' reflexivity is called 'secondary reflexivity' or 'shift reflexivity'. If R is secondarily reflexive, then every deontic alternative is also a deontic alternative to itself. Secondary reflexivity is plausible for the following reason: An ideal world, by definition, is a world in which *all* obligations are fulfilled. These naturally include not only 'old' obligations (the obligations in the world to which the ideal world is a deontic alternative) but also 'new' obligations obtaining in the ideal world itself. The constraint of secondary reflexivity can be formulated in the following condition:

(C.O)$_{rest}$ If $Op \in W^+$ and W^+ is a deontic alternative to some possible world W (WRW^+), then $p \in W^+$.

(C.O)$_{rest}$ validates

(11) $O(Op \rightarrow p)$,

and (11) implies

(12) $OOp \rightarrow Op$.[42]

From (10) and (12) we can infer

(13) $Op \leftrightarrow OOp$.

40 It is to be noticed that '$Pp \rightarrow OPp$' is logically valid if R is euclidean, i.e., if WRW^+ and WRW^{++}, then W^+RW^{++}.

41 One might accept a weaker condition that the transitive relation holds only among the ideal worlds: For every W^+ that is a deontic alternative to the actual world W, if W^+RW^{++} and $W^{++}RW^{+++}$, then $W^{++}RW^{+++}$. The corresponding condition will be modified into the following:

(C.OO^{++}) If $Op \in W^+$ for every W^+ such that WRW^+, then $Op \in W^{++}$ for every W^{++} such that W^+RW^{++}.

Under this weaker condition '$OOp \rightarrow OOOp$' is valid, but (10) is not. This seems more close to Sieckmann's original idea of reiteration of requirements of validity.

42 (12) is derivable from (11) together with the axiom K: $O(p \rightarrow q) \rightarrow (Op \rightarrow Oq)$. It is obvious that (12) is valid under (C.O)$_{rest}$: If $OOp \in W$, then $Op \in W^+$ for every W^+ such that WRW^+. According to (C.O)$_{rest}$, if $Op \in W^+$, then $p \in W^+$, therefore $Op \in W$.

(13) amounts to the idea that reiteration of the deontic operator '*O*' is superfluous, that is to say, every iterated obligation can be reduced to a non-iterated one.[43] This is the very reason why it does not make sense to regard the reiterated obligations as the characteristic structure of the ideal ought or principles. On account of the reduction theorem (13), there is no genuine difference between the obligations in the ideal world (the obligations we should adopt) and the obligations in the actual world (the obligations we actually have). It is therefore inadequate to think that the structural distinction between the ideal and real ought exists in the formal difference between '*OOp*' and '*Op*'.

In my view, the demands of principles can still be represented simply as '*Op*', for it is redundant to reiterate the deontic operator '*O*' in order to represent the ideal ought. If this is correct, the question arises: How shall we understand the ideal or optimizing character of principles and the structural difference between the ideal and real ought? According to Alexy, principles 'comprehend an ideal "ought" that is not relativized to the actual and legal possibilities',[44] but 'demands to be realized as far as possible', and 'a statement about their real demand-content therefore always presupposes a statement about the actual and legal possibilities'.[45] This rather vague contention may be interpreted in this way: The ideal situation described by the contents of principles is a realizable possible world in the abstract. But this ideal world cannot be fully realized in some cases, because there are certain possible worlds which cannot be transformed into 'perfect' or 'ideal' ones. If our actual world is one of these, the best we can do is to try and make it as ideal as possible. The content of the real ought is thus constituted by what is the case in all of the best worlds which approximate the ideal worlds as much as possible. This idea will be explained more precisely in the following section.

IV. Ideal Ought and Real Ought

The ideal ought contained in a set of principles cannot be fully realized if the principles in this set come into conflict in a given situation. I presuppose that the conflict of principles is a situation-dependent normative inconsistency. Since it is assumed that the requirements of principles can be represented as '*Op*', I will consider only the normative inconsistency concerning obligatory norms.

Let N:{$Op_1,\ldots, Op_n$} be a set of principles in a possible world W, and I_N= {$p_1,\ldots, p_n$} be the set of the norm-contents of N. N is consistent if and only if I_N is satisfiable, in other words, there is a possible world W^+ such that every sentence in I_N is true in W^+ (i.e., $p_i \in W^+$ for every p_i ($1 \leq i \leq n$)). The concept of *normative consistency* is defined as follows:

43 It is to be noticed that if R is transitive, then '$OPp \rightarrow Pp$' is valid. Hence, if R is transitive and euclidean (see (n. 40) above), then '$Pp \leftrightarrow OPp$' is logically valid.

44 Alexy, 'On the Structure of Legal Principles' (n. 5), 300.

45 Alexy, 'Zum Begriff des Rechtsprinzips' (n. 4), 204.

(CON) For a set of norms N in a possible world W, N is consistent if and only if there is a possible world W^+ such that $I_N \subseteq W^+$.

Obviously, I_N is the *ideal situation* envisaged by the principles in N, and W^+ is an ideal world with respect to N, i.e., a deontic alternative to W. The ideal worlds from the viewpoint of a set of norms can be defined as

(IW) With regard to a set of norms N, a possible word W^+ is an ideal world if and only if $I_N \subseteq W^+$.

I will define the *ideal ought* in the following way:

(IO) With regard to a set of principles N, Op is an ideal ought if and only if $I_N \vdash p$ (p is a logical consequence of I_N).

I will propose an additional condition for possible worlds:

(C. $\vdash$) For every sentence p, if W is a possible world and $W \vdash p$, then $p \in W$.

(C. $\vdash$) says that a possible world is closed under the consequence relation '$\vdash$'. The adequacy of (IO) is easy to demonstrate. I will call 'Op' a *deontic consequence* of N if and only if p is a logical consequence of I_N.[46] (IO) says that every deontic consequence of N is an ideal ought. Every element in N is a deontic consequence of N and also an ideal ought.[47] If p is a logical consequence of I_N, even though p is not an element of I_N (i.e., 'Op' is not explicitly contained in N), it still follows that $Op \in W$. The reason is obvious: Since $I_N \subseteq W^+$ for every ideal world W^+ with respect to N in W, p is also a logical consequence of W^+ if $I_N \vdash p$.[48] According to (C. $\vdash$), this implies $p \in W^+$, i.e., p is true in every ideal world. It is thus quite natural to term not only norms in N but also obligations following from N 'ideal ought'. In other words, an ideal ought is an ought explicitly or implicitly contained in N.

With regard to the normative inconsistency, a set of norms N is *categorically inconsistent* if and only if I_N is not satisfiable. The definition of categorical normative inconsistency is as follows:

(INC) N is categorically inconsistent if and only if there is no possible world W such that $I_N \subseteq W$.

For example, the set of norms $\{Op, O\neg p\}$ is inconsistent in the categorical sense. However, as mentioned above, a set of principles as such is normally consistent. Let us consider a simple set which contains only two principles 'It ought to be the case that the freedom of speech (p) is protected (r)' and 'It ought to be the case that the violation of the right to personal privacy (q) is not protected ($\neg r$)'. It is clear to see that $\{O(p \rightarrow r), O(q \rightarrow \neg r)\}$ is consistent, because $\{p \rightarrow r, q \rightarrow \neg r\}$ is satisfiable. Only in a situation in which the exercise of the freedom of speech violates the right to personal privacy does normative inconsistency arise, because $\{p \rightarrow r, q \rightarrow \neg r\}$ cannot be satisfied in a world in which $p \wedge q$ is true. I will call this kind of normative inconsistency *conditional inconsistency*. Conditional normative

46 On the distinction between deontic and logical consequence, see Hintikka, 'Some Main Problems of Deontic Logic' (n. 34), 77–87.

47 This is due to the postulate that the consequence relation is *inclusive*: For every sentence p in S, $S \vdash p$.

48 This is because the classical consequence relation is *monotonic*: For two sets of sentences S and S', if $S \subseteq S'$ and $S \vdash p$, then $S' \vdash p$.

inconsistency is inconsistency *modulo* certain facts. I will call a sentence s which is neither tautological (logically valid) nor contradictory and contains no deontic operators a *fact-description*. A set of principles N comes into conflict in a given situation s if and only if the norm-contents I_N and the fact description s cannot be jointly satisfied. The conditional inconsistency of a set of norms N in a situation s can thus be defined in this manner:

(CINC) N is inconsistent *modulo* s if and only if $I_N \cup \{s\}$ is not satisfiable (there is no possible world W such that $I_N \cup \{s\} \subseteq W$).[49]

The conditional normative inconsistency can be defined in another way. Assume that a set of sentences S is inconsistent or unsatisfiable if and only if S implies a logical contradiction, i.e., $S \vdash \bot$. On this assumption, $I_N \cup \{s\}$ is inconsistent if and only if $I_N \cup \{s\} \vdash \bot$.[50] According to the deduction theorem of classical logic,[51] this implies that $I_N \vdash s \rightarrow \bot$. Since '$s \rightarrow \bot$' is logically equivalent to '$\neg s$', an alternative definition of conditional normative inconsistency is as follows:

(CINC*) N is inconsistent *modulo* s if and only if $I_N \vdash \neg s$.

According to (IW) and (C. $\vdash$), $I_N \vdash \neg s$ implies that $\neg s \in W^+$ for every ideal world W^+ with respect to N. In other words, s cannot be true in any ideal world. Let us call a possible world in which s is true an 's-world'. The observation above therefore amounts to the claim that an s-world cannot be an ideal world with respect to N if $\neg s$ follows from I_N. If the situation in which we find ourselves is an s-world, it is impossible to transform our actual world to an ideal one, and thus the ideal situation envisaged by the principles in N cannot be fully realized.[52] The best we can do in such a situation is to make the actual s-world approximate the ideal situation as much as possible; the ideal ought contained in N will be 'relativized to the actual and legal possibilities' and transformed into a 'real' ought. This is exactly what Alexy's 'optimization thesis' says.

Let K_N be the set of all logical consequences of I_N, i.e., $K_N = \{p \mid I_N \vdash p\}$. According to (IO), K_N can be regarded as the complete contents of the ideal ought contained in the set of principles N, in other words, K_N is the 'full' description of the ideal situation, and Op is an ideal ought if and only if $p \in K_N$. It is clear that, first, for every possible world W, W is an ideal world with respect to N if and only if $K_N \subseteq W$,[53] and, second, $I_N \vdash \neg s$ if and only if $\neg s \in K_N$. Correspondingly, the definition of conditional inconsistency can be modified into the following:

(CINC**) A set of norms N is inconsistent *modulo* s if and only if $\neg s \in K_N$.

If N is a set of principles and K_N is the full description of the ideal situation envisaged by N, then, according to Alexy's optimization thesis, what we should do in a conflict situation s is to make an s-world as close to K_N as possible, in other words, we have to construct worlds in which s is true, and which in other aspects

49 '$\cup$' stands for the union of sets. It is noteworthy that the categorical consistency can be defined by virtue of the conditional inconsistency if we allow that s can be a tautology.

50 It is easy to show in the above example that $r \wedge \neg r$ follows from $\{p \rightarrow r, q \rightarrow \neg r\}$ and $\{p \wedge q\}$.

51 The deduction theorem says that if $S \cup \{p\} \vdash q$, then $S \vdash p \rightarrow q$.

52 The reason is obvious: If $I_N \vdash \neg s$, then, according to (IO), $O\neg s$ is an ideal ought. If our actual world is an s-world, then it cannot be an ideal world because $O\neg s$ has been violated.

53 Here we assume that every ideal world satisfies the condition (C. $\vdash$).

resemble the ideal worlds as much as possible. Such 'almost ideal' worlds are not deontically perfect, because s is false in every ideal world, but they can be dubbed 'almost ideal,' because they preserve the elements in K_N to the greatest extent.

In my view, an 'almost ideal' world can be constructed in the following way. The first step is to form a subset of K_N which is maximally consistent with s, i.e., a maximal subset of K_N that fails to imply $\neg s$. To put it more technically:

(MAX) A set of sentences K is a maximal subset of K_N that fails to imply $\neg s$ if and only if

(i) K is a non-empty subset of K_N ($K \subseteq K_N$).

(ii) $\neg s$ is not a logical consequence of K ($\neg s \notin K$).

(iii) For every sentence p that is in K_N but not in K, if K were to be expanded by p, it would imply $\neg s$. (If $p \in K_N$ and $p \notin K$, then $K \cup \{p\} \vdash \neg s$).

Any set of sentences satisfying conditions (i), (ii), and (iii) can be called a 'sub-ideal situation' of K_N relative to s. It should be noted that there are in general several sub-ideal situations when a set of principles comes into conflict under certain circumstances. For instance, in the simple example above, there are at least two alternative sub-ideal situations under circumstances $p \wedge q$, one of which contains '$p \rightarrow r$' but not '$q \rightarrow \neg r$', the other of which contains '$q \rightarrow \neg r$' but not '$p \rightarrow r$'. The idea of weighing and balancing presupposes that some sub-ideal situations are 'better' than others.[54] To make it more precise, we can assume that there is a *preference relation* among the sub-ideal situations that can be used to pick out the best elements from them. Let '$\leq$' be such a preference relation, and '$K_N \perp \neg s$' stand for the set of all maximal subsets of K_N that fails to imply $\neg s$ (i.e., the set of all possible sub-ideal situations of K_N relative to s). For any two sets K, K^+ in $K_N \perp \neg s$, '$K \leq K^+$' is read as 'K^+ is *at least as good as* K'. 'K and K^+ are equally good' is defined as '$K \leq K^+$ and $K^+ \leq K$', and 'K^+ is strictly better than K' is defined as '$K \leq K^+$, but not $K^+ \leq K$'. Accordingly, K^+ is one of the 'best' sub-ideal situations if and only if K^+ is at least as good as all other sub-ideal situations. The set of the best elements of $K_N \perp \neg s$ is denoted by '$best(K_N \perp \neg s)$' and can be defined as follows:

(BEST) $best(K_N \perp \neg s) = \{K^+ \in K_N \perp \neg s \mid K \leq K^+ \text{ for all } K \in K_N \perp \neg s\}$

An 'almost ideal' world under circumstances s is achieved by virtue of the expansion of one of the 'best' sub-ideal situations K^+ by s, i.e., $K^+ \cup \{s\}$. A possible world W in which s is true is an 'almost ideal' s-world if and only if $K^+ \cup \{s\}$ is satisfiable in W. Let us call such possible worlds '*s-ideal worlds*'. An s-ideal world is a world in which s is true, but which otherwise is as 'ideal' as an s-world can possibly be. More technically:

(IW_S) With respect to a set of norms N, a possible world W is an s-ideal world if and only if $K^+ \cup \{s\} \subseteq W$, where $K^+ \in best(K_N \perp \neg s)$.

Now we are in a position to define the so called 'real ought' or 'definitive obligation'. Recall Alexy's account of ideal and real ought. Principles contain an ideal ought which is not yet relativized to actual and legal possibilities. An ideal ought

54 We may say that a principle prefers one sub-ideal situation to another if the former contains more the contents of the requirements of this principle.

will be transformed into a real ought if it is relativized to actual and legal possibilities. Therefore, a real ought is a 'relativized ideal ought' which states what ought to be done in an actual situation according to a set of principles. Whereas the actual possibilities are described by the fact-description of a given situation, the legal possibilities are determined by weighing and balancing all relevant principles in this situation. Suppose the actual situation in which we find ourselves is a possible world in which the ideal situation envisaged by a set of principles N cannot be totally realized, what should we do under these circumstances? Although the ideal worlds are ruled out, among the still achievable possible worlds there are some that are better than others. Therefore, we should make the best out the 'bad' circumstances, and must always try to make one of the best achievable worlds come true. Some sentences are true in all of these best, most ideal worlds. Therefore we have to make these sentences true if the actual world is to become approximately ideal. Such sentences are called *definitively obligatory* under the given circumstances. This suggests the following definition of 'real ought':

(RO) With regard to a set of principles N, p is obligatory ('Op' is a real ought) under circumstances s if and only if $K^{+}\cup\{s\} \vdash p$ for every $K^{+}\in best(K_N\perp\neg s)$.

Intuitively, if p follows from $K^{+}\cup\{s\}$, then p is true in every possible world satisfying $K^{+}\cup\{s\}$, namely, p is the case in every s-ideal world. If we want to transform an actual s-world into an 'almost ideal' world, we must bring about p. I will use the dyadic deontic operator '$O(\ldots/\ldots)$' to denote a real ought. '$O(p/s)$' may be read as 'p is obligatory under circumstances s'.[55] (RO) can be revised to a more concise definition of the truth-condition for conditional obligations, as follows:

(RO*) $O(p/s)$ is true if and only if p is true in every s-ideal world.[56]

Since s-ideal worlds, as said above, are achieved through expanding the best sub-ideal situations of K_N by s, and the best sub-ideal situations are determined by the preference relation '$\leq$' among the sub-ideal situations relative to s, it seems very reasonable to conclude that a real ought (or a definitive obligation) is the outcome of weighing and balancing.[57]

55 It is to be noticed that every monadic deontic sentence 'Op' can be translated into '$O(p/t)$', where 't' stands for any tautology, i.e., sentences true in every possible world. Thus, '$O(p/t)$' represents an unrelativized ought. This might explain why Alexy thought that it is unnecessary to introduce two different deontic operators to characterize 'ideal ought' and 'real ought' respectively.

56 Correspondingly, '$P(p/s)$' is true if and only if p is true in at least one s-ideal world. One might object that (RO) as well as (RO*) is not very adequate because they validate '$O(s/s)$'. However, as Bengt Hansson pointed out, one must think of obligation relative to circumstances s as obligation in a restricted universe: Of all possible worlds only s-worlds are now available; and the set of s-worlds plays the role of the universe. Therefore, what '$O(s/s)$' says is only that at least something is obligatory under circumstances s. See Bengt Hansson, 'An Analysis of Some Deontic Logics', in R. Hilpinen (ed.), *Deontic Logic: Introductory and Systematic Readings* (D. Reidel: Dordrecht, 1981), 144.

57 For the possible worlds semantics of dyadic deontic logics, see Hansson, 'An Analysis of Some Deontic Logics' (n. 56), 121–47. The construction presented here suggests that there is a close connection between the semantics for dyadic deontic logics and the AGM model of theory revision (see Carlos E. Alchourrón, Peter Gärdenfors and David Makinson, 'On the Logic of Theory Change: Partial Meet Contraction and Revision Functions' (1985) 50 *Jour-*

There is one further point that is worth noting. In the previous discussion it is assumed that the preference relation among the sub-ideal situations of K_N is not given beforehand, but has to be established in a given situation *s*. To some extent, this corresponds to Alexy's claim that the preference relation between principles is *conditional* and laid down in the context of a concrete case.[58] If so, the preference relation had better be indexed as '$\leq_s$', because the conditional preference relation between principles might change under different circumstances. But we can also imagine a non-indexed preference relation. Let us call a pair consisting of a set of principles and the preference relation $\langle N, \leq\rangle$ 'a system of principles' or, in Alexy's words, 'a theory of the relations of principles'.[59] A system of principles is *perfect* if and only if the preference relation $\leq$ can determine the best *s*-worlds for every possible case *s*, such that we can always determine what is definitively obligatory in every possible situation. However, as Alexy correctly argues, if $\langle N, \leq\rangle$ is perfect in this sense, it will not be a system of principles anymore, but rather a system of rules, because the legally and actually possible extent to which the principles are realized is already fixed definitively and completely in advance.[60] Therefore, if $\langle N, \leq\rangle$ is a genuine system of principles, it must have 'gaps': There must exist some cases in which the preference relation among the sub-ideal situations relative to these cases is not given beforehand, otherwise there would be no room for weighing and balancing in concrete cases. Recently, Alexy has proposed the so called 'Weight Formula' for establishing the preference relation in a concrete case.[61] Whether and how this formula can be incorporated into the approach presented here is an open question, and thus remains a matter to be investigated further.

V. Conclusions

In this paper I criticize Sieckmann's analysis of the logical structure of principles and propose an alternative approach which applies the Kripke-Hintikka semantics to explicate the notion of ideal and real ought, optimization requirements, and weighing and balancing. The main theses of this paper can be summarized as follows:

nal of Symbolic Logic 510–30). In fact, the construction of the sub-ideal situations and the *s*-ideal worlds is an application of the theory revision based on the partial meet contraction. For a detailed account, see Carlos E. Alchourrón, 'Philosophical Foundations of Deontic Logic and the Logic of Defeasible Conditionals' in J.-J. Ch. Meyer and R. J. Wieringa (eds.), *Deontic Logic in Computer Science: Normative System Specification* (John Wiley & Sons Ltd: Cichester, 1993), 65–83; Carlos E. Alchourrón, 'Detachment and Defeasibility in Deontic Logic' (1996) 57 *Studia Logica* 5–18.

58 Alexy, *A Theory of Constitutional Rights* (n. 3), 52.

59 Alexy, 'Zum Begriff des Rechtsprinzips' (n. 4), 208.

60 Alexy, 'Zum Begriff des Rechtsprinzips' (n. 4), 208.

61 Alexy, *A Theory of Constitutional Rights* (n. 3), 52, 407–14. See also Robert Alexy, 'On Balancing and Subsumption. A Structural Comparison' (2003) 16 *Ratio Juris* 443–8.

1. An ideal world with respect to a set of principles *N* is a possible world in which the ideal situation described by the norm-contents of *N* is fully realized.
2. *Op* is an ideal ought if and only if *p* is true in every ideal world. Every deontic consequence of a set of principles *N* is an ideal ought.
3. If *N* is inconsistent under circumstances *s*, the optimizing character of principles requires that the ideal situation envisaged by *N* shall be realized approximately, i.e., we have to bring about an *s*-ideal world, which is one of the best *s*-worlds that are closest to the ideal worlds.
4. With regard to *N*, *p* is definitively obligatory under circumstances *s* ($O(p/s)$) if and only if *p* is true in all *s*-ideal worlds.
5. The *s*-ideal worlds are determined by a preference relation among the possible sub-ideal situations relative to *s*. The so called 'weighing and balancing' comes into play in establishing such a preference relation if this is not given beforehand.