NCU Radio Astronomy – Homework 7 – June 15, 2008

1. Free-Free/Bremsstrahlung Continuum Emission and Recombination Line Emission in HII Regions

As introduced in the class, the Orion A, the strongest radio source in the Orion Constellation, is a HII region. The figure below gives the spectral energy distribution (SED) of Orion A at radio frequencies. Its flux density is observed to first rise with increasing frequency and then starts decreasing with increasing frequency. The "turnover" frequency (ν_0) can be roughly estimated by noting the frequency at which the linear extrapolation of the high and low frequency parts of the SED curve meet. At this frequency, the optical depth , τ_{ff}, of free-free emission is about unity. The relation of the turnover frequency, the election temperature, T_e, and the emission measure, $EM = N_e^2 \; L$ is given as (also see Equation (9.36) in "Tools of Radio Astronomy")

$$\nu_0[GHz] = 0.3045(T_e[K])^{-0.643}(EM[cm^{-6}\; pc])^{0.476} \tag{1}$$

We note that this relation applies to a *uniform density, uniform temperature* region; actual HII regions have gradients in both quantities. The above relation is therefore best only a first approximation.

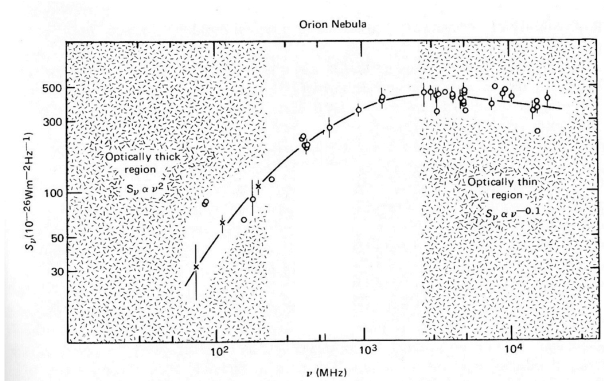

Fig. 2.2. Spectral flux density of the Orion Nebula plotted against frequency. The shaded regions mark the optically thick and thin regions of the spectrum. (Reprinted with permission by Gordon and Breach Science Publishers from: Terzian, Y. and Parrish A., *Astrophysical Letters*. Vol. 5(1970), pp. 261.

(a) What is the turnover frequency of Orion A based on the figure?

(b) Assuming an electron temperature $T_e = 8300K$, determine EM.

(c) The FWHP size of Orion A is $2.5'$ and Orion A is 500 pc from the Sun. What is the linear diameter, or the physical size of Orion A, assuming it is a uniform sphere?

(d) What is the electron density of the ionized gas in the Orion A HII region?

2. Dust Continuum Emission

The Orion hot core is a condensation of molecular gas and dust in Orion A. Physically, the core is located behind the Orion A HII region. Observations showed that the Orion hot core has an apparent brightness temperature of 0.58 K at 1.3 mm when observed with the JCMT 15" (FWHM) beam. The dust, and therefore gas column density can be derived from the observed dust continuum emission using Equation (9.7) in "Tools of Radio AStronomy",

$$N_H[cm^{-3}] = 1.93 \times 10^{24} \frac{S_\nu[Jy]}{\theta^2[arcsecond^2]} \frac{\lambda^4[mm^4]}{(Z/Z_\odot) b T_{dust}[K]} \quad (2)$$

where b=1.9, λ is the wavelength, and θ is the source FWHP size.

(a) If the Orion hot core is found to have a true angular size of 10", what is its true brightness temperature at 1.3 mm?

(b) What is the flux density (in Jy) of the Orion hot core at 1.3 mm?

(c) Assuming the dust in Orion hot core has a thermal temperature of 160 K, what is the dust optical depth at 1.3 mm?

(d) Assuming its metallicity is solar ($Z = Z_\odot$), find out the molecular gas (H_2) column density in Orion hot core.

3. Spectral Line Fundamentals

The variation of T_{ex} with the collisional rate, C_{21}, and the spontaneous decay rate, A_{21} for a two-level system can be inspected from the equation,

$$T_{ex} = T_K \left(\frac{T_0 C_{21} + T_b A_{21}}{T_0 C_{21} + T_K A_{21}} \right) \quad (3)$$

where $T_0 = h\nu/k$; T_K is the kinetic temperature, and T_b is the radiation field. Suppose the collisional rate C_{21} in given by $n < \sigma \ v >$ where the value of $< \sigma \ v >$ is $\sim 10^{-10}$. When $n < \sigma \ v >= A_{21}$ for the transition involved, this is referred to as the 'critical density', n_{crit} of the transition. As discussed in the class, the relative importance of A_{21} and C_{21} tells if a transition can be excited and further thermalized to be in LTE.

(a) For the 21 cm line, $A_{21} = 2.85 \times 10^{-15} s^{-1}$. Find n_{crit} for this transition.

(b) Estimate n_{crit} for the J= 1—0 transition of the molecule HCO^+ which has an Einstein A coefficient $A_{21} = 3 \times 10^{-5} s^{-1}$.

(c) Calculate n_{crit} for the J=1—0 transition of the carbon monoxide molecule, CO, with $A_{21} = 7.4 \times 10^{-8} s^{-1}$.

We note that, for neutral hydrogen, in most cases, only two levels are involved in the formation and excitation of the 21 cm line since N=2 level is 9 eV higher. Less secure is any result for multi-level systems. One can still obtain an order of magnitude estimate by modeling multiple level system as two-level systems.

3. Molecular Transitions

A typical giant molecular could (GMC) is thought to have a diameter of 30 pc, and total mass of $10^6 M_\odot$. Assume that GMC's have no small scale structure.

(a) What is the density of the GMC? Find out the column density of H_2 in this cloud.

(b) What is the column density through such a GMC?

(c) What is the FWHP width of a molecular line tracing this cloud if the cloud is in viral equilibrium?