NCU Radio Astronomy – Homework 7 – June 15, 2008

1. Free-Free/Bremsstrahlung Continuum Emission and Recombination Line Emission in HII Regions

As introduced in the class, the Orion A, the strongest radio source in the Orion Constellation, is a HII region. The figure below gives the spectral energy distribution (SED) of Orion A at radio frequencies. Its flux density is observed to first rise with increasing frequency and then starts decreasing with increasing frequency. The "turnover" frequency (ν_0) can be roughly estimated by noting the frequency at which the linear extrapolation of the high and low frequency parts of the SED curve meet. At this frequency, the optical depth , τ_{ff}, of free-free emission is about unity. The relation of the turnover frequency, the election temperature, T_e, and the emission measure, $EM = N_e^2\ L$ is given as (also see Equation (9.36) in "Tools of Radio Astronomy")

$$\nu_0[GHz] = 0.3045(T_e[K])^{-0.643}(EM[cm^{-6}\ pc])^{0.476} \tag{1}$$

We note that this relation applies to a *uniform density, uniform temperature* region; actual HII regions have gradients in both quantities. The above relation is therefore best only a first approximation.

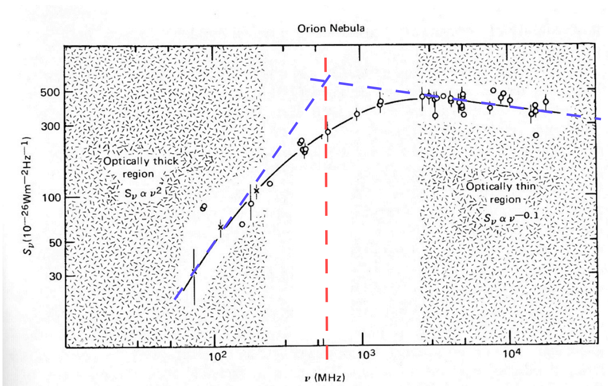

Fig. 2.2. Spectral flux density of the Orion Nebula plotted against frequency. The shaded regions mark the optically thick and thin regions of the spectrum. (Reprinted with permission by Gordon and Breach Science Publishers from: Terzian, Y. and Parrish A., *Astrophysical Letters.* Vol. 5(1970), pp. 261.

(a) What is the turnover frequency of Orion A based on the figure?

Ans: Based on the figure, the turnover frequency is about 600 MHz.

(b) Assuming an electron temperature $T_e = 8300K$, determine EM.

Ans:

$$\begin{aligned} 0.6[GHz] &= 0.3045(8300[K])^{-0.643}(EM[cm^{-6}\ pc])^{0.476} \\ EM &= 8.18 \times 10^5 cm^{-6}\ pc \end{aligned}$$

(c) The FWHP size of Orion A is 2.5′ and Orion A is 500 pc from the Sun. What is the linear diameter, or the physical size of Orion A, assuming it is a uniform sphere?

Ans:

$$\begin{aligned} 2.5' &= 2.5/(180 * 60) * \pi(radian) = D(pc)/500pc \\ D &= 0.364pc \end{aligned}$$

More sophisticated calculation will show that a "uniform sphere" with true diameter θ_s on the sky, when observed and resolved by a Gaussian beam, will in fact have a Gaussian FWHP size of $\theta_o = 0.67\ \theta_s$.

(d) What is the electron density of the ionized gas in the Orion A HII region?

Ans:

$$N_e[cm^{-3}] = \sqrt{EM[cm^{-6}\ pc/D[pc]} = \sqrt{8.18^5/0.364} = 1.5 \times 10^3$$

2. Dust Continuum Emission

The Orion hot core is a condensation of molecular gas and dust in Orion A. Physically, the core is located behind the Orion A HII region. Observations showed that the Orion hot core has an apparent brightness temperature of 0.58 K at 1.3 mm when observed with the JCMT 15" (FWHM) beam. The dust, and therefore gas column density can be derived from the observed dust continuum emission using Equation (9.7) in "Tools of Radio AStronomy",

$$N_H[cm^{-2}] = 1.93 \times 10^{24} \frac{S_\nu[Jy]}{\theta^2[arcsecond^2]} \frac{\lambda^4[mm^4]}{(Z/Z_\odot)bT_{dust}[K]} \tag{2}$$

where b=1.9, λ is the wavelength, and θ is the source FWHP size.

(a) If the Orion hot core is found to have a true angular size of 10", what is its true brightness temperature at 1.3 mm?

Ans: Based on previous homeworks, we know the observed (apparent) brightness temperature T_B and the true (source) brightness temperature T_0 is related by

$$T_B = T_0(\frac{\theta_s^2}{\theta_s^2 + \theta_B^2})$$

where θ_s is the true source (angular) size, and $theta_B$ is the telescope beam (angular) size. Hence, the true brightness temperature of the Orion hot core at 1.3 mm is

$$\begin{aligned} T_0 &= T_B(\frac{\theta_s^2 + \theta_B^2}{\theta_s^2}) \\ &= 0.58[K] * (15^2 + 10^2/10^2) \\ &= 1.885[K] \end{aligned}$$

(b) What is the flux density (in Jy) of the Orion hot core at 1.3 mm?

Ans: Using the Rayleigh-Jeans approximation, the flux density S of a Gaussian source can be found as in Equation (7.20) of "Tools of Radio Astronomy",

$$\begin{aligned} S &= \frac{2kT_B}{\lambda^2}\Omega_B \\ &= 2.65\frac{T_B(\theta_s^2 + \theta_B^2)}{\lambda^2} \\ &= 2.65\frac{T_0\ \theta_s^2}{\lambda^2} \end{aligned}$$

where S is in unit of Jy, k is the Boltzmann constant, T_B and T_0 are in Kelvin, λ is in unit of centimeters, and θ_s as well as θ_B are in arc minutes. Thus,

$$\begin{aligned} S[Jy] &= 2.65\frac{1.885 \times (10/60)^2}{0.13^2} \\ &= 8.2[Jy] \end{aligned}$$

(c) Assuming the dust in Orion hot core has a thermal temperature of 160 K, what is the dust optical depth at 1.3 mm?

Ans: At millimeter and submillimeter wavelengths, the dust is normally optically thin. Therefore we have

$$\begin{aligned} T_0 &= T_dust(1 - e^{-\tau_{dust}}) \\ &\approx T_dust\ \tau_{dust} \\ 1.885[K] &= 160[K]\ \tau_{dust} \\ \tau_{dust} &= 0.012 \end{aligned}$$

We dust optical depth τ_{dust} at 1.3 mm toward the Orion hot core is about 0.012, which satisfies the original optically-thin assumption.

(d) Assuming its metallicity is solar ($Z = Z_\odot$), find out the molecular gas (H_2) column density in Orion hot core.

Ans: The molecular gas (H_2) colum density is approximately

$$\begin{aligned} N_{H_2}[cm^{-2}] &= N_H[cm^{-2}]/2 \\ &= \frac{1.93}{2} \times 10^{24} \frac{S_\nu[Jy]}{\theta^2[arcsecond^2]} \frac{\lambda^4[mm^4]}{(Z/Z_\odot) b T_{dust}[K]} \\ &= 0.965 \times 10^{24} \frac{8.2}{10^2} \frac{1.3^4}{1 \times 1.9 \times 160} \\ &= 7.4 \times 10^{20}[cm^{-2}] \end{aligned}$$

In the Equation (9.7) given by "Tools of Radio Astronomy", the factor 10^{24} should in fact be 10^{27}.

3. Spectral Line Fundamentals

The variation of T_{ex} with the collisional rate, C_{21}, and the spontaneous decay rate, A_{21} for a two-level system can be inspected from the equation,

$$T_{ex} = T_K\left(\frac{T_0 C_{21} + T_b A_{21}}{T_0 C_{21} + T_K A_{21}}\right) \tag{3}$$

where $T_0 = h\nu/k$; T_K is the kinetic temperature, and T_b is the radiation field. Suppose the collisional rate C_{21} in given by $n < \sigma\ v >$ where the value of $< \sigma\ v >$ is $\sim 10^{-10} cm^3 s^{-1}$. When $n < \sigma\ v >= A_{21}$ for the transition involved, this is referred to as the 'critical density', n_{crit} of the transition. As discussed in the class, the relative importance of A_{21} and C_{21} tells if a transition can be excited and further thermalized to be in LTE.

We note that, for neutral hydrogen, in most cases, only two levels are involved in the formation and excitation of the 21 cm line since N=2 level is 9 eV higher. Less secure is any result for multi-level systems. One can still obtain an order of magnitude estimate by modeling multiple level system as two-level systems.

(a) For the 21 cm line, $A_{21} = 2.85 \times 10^{-15} s^{-1}$. Find n_{crit} for this transition.

Ans: For the 21 cm line, $n_{crit} = \frac{2.85 \times 10^{-15} s^{-1}}{10^{-10} cm^3 s^{-1}} = 2.85 \times 10^{-5} cm^{-3}$

(b) Estimate n_{crit} for the J = 1—0 transition of the molecule HCO^+ which has an Einstein A coefficient $A_{21} = 3 \times 10^{-5} s^{-1}$.

Ans: For the HCO^+ J = 1—0 transition, $n_{crit} = \frac{3\times10^{-5}s^{-1}}{10^{-10}cm^3s^{-1}} = 3\times10^5 cm^{-3}$

(c) Calculate n_{crit} for the J = 1—0 transition of the carbon monoxide molecule, CO, with $A_{21} = 7.4\times10^{-8}s^{-1}$.

Ans: For the CO J = 1—0 transition, $n_{crit} = \frac{7.4\times10^{-8}s^{-1}}{10^{-10}cm^3s^{-1}} = 740cm^{-3}$

3. Molecular Transitions

A typical giant molecular could (GMC) is thought to have a diameter of 30 pc, and total mass of $10^6 M_\odot$. Assume that GMC's have no small scale structure.

(a) What is the density of the GMC? Find out the density of H_2 in this cloud.

Ans: Assuming the GMC is spherical and composed by only molecular hydrogen (H_2), the average density is

$$\begin{aligned}
n[cm^{-3}] &= (10^6[M_\odot]/m_{H_2})/(\frac{4}{3}\pi(15\ pc)^3) \\
&= (10^6\times1.99\times10^{33}[g]/(2\times1.673\times10^{-24}[g])/(\frac{4}{3}\pi(15\times3.086\times10^{18}[cm])^3) \\
&= 1.4\times10^3[cm^{-3}]
\end{aligned}$$

(b) What is the column density through such a GMC?

Ans: The column density through the center of such a GMC is

$$\begin{aligned}
N[cm^{-2}) &= \int n[cm^{-3}]dl[cm] \\
&\approx n[cm^{-3}]\times D[cm] \\
&= 1.4\times10^3[cm^{-3}]\times(30\times3.086\times10^{18}[cm]) \\
&= 1.3\times10^{23}[cm^{-2}]
\end{aligned}$$

(c) What is the FWHP width of a molecular line tracing this cloud if the cloud is in virial equilibrium?

Ans: In virial equilibrium, all material in a system is gravitationally bound. Assuming the line-width of a molecular line tracing the cloud is dominated by random motions bounded by the gravity, the line-widths thus reflect the velocity of such gravitationally bound motion. The velocity dispersion in such a system can be found as the following,

$$\begin{aligned}
\bar{v^2} &= \frac{GM}{2R} \\
&= \frac{3}{8ln2}\Delta v_{1/2}^2
\end{aligned}$$

where$M$ is the mass of the whole system (the mass of the GMC in our case), R is the size of the system (the radius of the GMC), and $\Delta v_{1/2}$ is the FTHP line-width. More details can be found in "Tools of Radio Astronomy" Section 12.8.1. According to Equation (12.72),

$$\begin{aligned}
\frac{M}{M_\odot} &= 250(\frac{\Delta v_{1/2}}{km\ s^{-1}})^2(\frac{R}{pc}) \\
\frac{\Delta v_{1/2}}{km\ s^{-1}} &= \sqrt{\frac{M/M_\odot}{250R/pc}} \\
&= \sqrt{\frac{10^6}{250\times15}} \\
&= 16.3
\end{aligned}$$