

Radio Astronomy Fundamentals and EM Wave Properties

- Blackbody Radiation and Brightness Temperature
- Radiative Transfer
- Maxwell's Equations

Radio Astronomy Fundamentals

- BlackBody Radiation and Brightness Temperature

$$B_\nu(T) = \frac{2hv^3}{c^2} \frac{1}{e^{hv/kT} - 1}$$

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/k\lambda T} - 1}$$

$$B(T) = \int B_\nu(T) d\nu = \int B_\lambda(T) d\lambda$$

$$= \frac{2h}{c^2} \int_0^\infty \frac{v^3}{e^{hv/kT} - 1} dv$$

$$= \frac{2h}{c^2} \left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1} dx$$

$$= \sigma T^4$$

$$\sigma = \frac{2\pi^4 k^4}{15c^2 h^3}$$

$$\nu_{max}(GHz) = 58.789 \left(\frac{T}{K}\right)$$

$$\lambda_{max}(cm) \left(\frac{T}{K}\right) = 0.28978$$

Radio Astronomy Fundamentals

- BlackBody Radiation and Brightness Temperature

$$B_v(T) = \frac{2hv^3}{c^2} \frac{1}{e^{hv/kT} - 1}$$

Rayleigh – Jeans Law ($hv \ll kT$)

$$B_v^{RJ}(T) = \frac{2v^2}{c^2} kT$$

$$T = \frac{c^2}{2v^2k} B_v^{RJ}(T)$$

Check unit!

Brightness Temperature

$$\begin{aligned} T_b &= \frac{c^2}{2v^2k} B(T) \\ &= \frac{\lambda^2}{2k} B(T) \end{aligned}$$

Think about 1. effective temperature, 2. color temperature...

Radio Astronomy Fundamentals

- BlackBody Radiation and Brightness Temperature

$$dE = I_v \cos\theta \, dt \, dA \, d\Omega \, dv$$

$$dW = I_v \cos\theta \, dA \, d\Omega \, dv$$

dE=infinitesimal energy

dW=infinitesimal power

dt=infinitesimal time interval

dA=infinitesimal surface area

dΩ=infinitesimal solid angle

dv=infinitesimal bandwidth

θ=the angle between the normal to dA

and the direction to dΩ

I_v=specific intensity or brightness **Check unit!**

rate of energy transport, along a particular direction,
per unit area, per unit solid angle, and per unit frequency

Radio Astronomy Fundamentals

- BlackBody Radiation and Brightness Temperature

$$dW_1 = dW_2$$

$$S_\nu = \int_{\Omega_s} I_\nu(\theta, \phi) \cos\theta d\Omega$$

1 Jy=1 Jansky

$$= 10^{-26} W m^{-2} Hz^{-1}$$

$$= 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}$$

$$dW_1 = I_{\nu 1} d\sigma_1 d\Omega_1 dv$$

$$dW_2 = I_{\nu 2} d\sigma_2 d\Omega_2 dv$$

$$d\Omega_1 = d\sigma_2 / R^2$$

$$d\Omega_2 = d\sigma_1 / R^2$$

$$dW_1 = I_{\nu 1} d\sigma_1 d\sigma_2 / R^2 dv$$

$$dW_2 = I_{\nu 2} d\sigma_2 d\sigma_1 / R^2 dv$$

$$I_{\nu 1} = I_{\nu 2}$$

Radio Astronomy Fundamentals

- Radiative Transfer

- change of specific intensity described by the equation of transfer (emission, absorption, scattering)

emission

$$dI_\nu = \varepsilon_\nu \, ds$$

absorption

$$dI_\nu = -\kappa_\nu I_\nu \, ds$$

$$dI_\nu/ds = -\kappa_\nu I_\nu + \varepsilon_\nu$$

$$dI_\nu/ds = \varepsilon_\nu$$

$$I_\nu(s) = I_\nu(s_0) + \int_{s_0}^s \varepsilon_\nu(s') \, ds'$$

$$dI_\nu/ds = -\kappa_\nu I_\nu$$

$$I_\nu(s) = I_\nu(s_0) \exp\left\{-\int_{s_0}^s \kappa_\nu(s') \, ds'\right\}$$

Radio Astronomy Fundamentals

- Radiative Transfer

no emission/absorption

T.E.

$$dI_\nu/ds=0$$

$$I_\nu = \epsilon_\nu / \kappa_\nu$$

$$= B_\nu(T)$$

$$= \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

no emission/absorption

in general

source term

$$S_\nu(T) = \epsilon_\nu / \kappa_\nu$$

Radio Astronomy Fundamentals

- Radiative Transfer

$$dI_\nu/ds = -\kappa_\nu I_\nu + \varepsilon_\nu$$

$$d\tau_\nu = \kappa_\nu ds$$

$$dI_\nu/d\tau_\nu = -I_\nu + S_\nu$$

$$\frac{dI_\nu}{d\tau_\nu} e^{-\tau_\nu} = -I_\nu e^{-\tau_\nu} + S_\nu e^{-\tau_\nu}$$

$$\frac{d(I_\nu e^{-\tau_\nu})}{d\tau_\nu} = S_\nu e^{-\tau_\nu}$$

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S_\nu(\tau') d\tau'_\nu$$

Radio Astronomy Fundamentals

- Radiative Transfer

$$\begin{aligned} \text{If } S_\nu(\tau'_\nu) &= S_\nu(s) = S_\nu = \text{const.} \\ \Rightarrow I_\nu(\tau_\nu) &= I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu}) \end{aligned}$$

$$\begin{aligned} \text{If } \tau_\nu &\ll 1 & \text{If } \tau_\nu &\gg 1 \\ \Rightarrow I_\nu(\tau_\nu) &= I_\nu(0)(1 - \tau_\nu) + S_\nu \tau_\nu & \Rightarrow I_\nu(\tau_\nu) &= S_\nu \end{aligned}$$

1. see the formulation in the R-J limit
2. see atmosphere opacity...

EM Wave Properties

- Maxwell's Equations

Electromagnetic Wave Properties

- Maxwell's Equations
 - electric field
 - electric displacement
 - magnetic field intensity
 - magnetic induction
 - electric current density
 - electric charge density

$$D = \epsilon E$$

$$B = \mu H$$

$$J = \sigma E$$

dielectric constant

magnetic permeability

specific conductivity

$$\nabla \cdot D = 4\pi\rho$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t}$$

$$\nabla \times H = \frac{4\pi}{c} J + \frac{1}{c} \frac{\partial D}{\partial t}$$

Electromagnetic Wave Properties

- Maxwell's Equations
 - energy density

$$\begin{aligned} u &= \frac{1}{8\pi} (E \cdot D + B \cdot H) \\ &= \frac{1}{8\pi} (\epsilon E^2 + \frac{B^2}{\mu}) \\ &= U_E + U_B \end{aligned}$$

- poynting vector

$$S = \frac{c}{4\pi} E \times H$$

- conservation of energy

$$\frac{\partial u}{\partial t} + \nabla \cdot S = -E \cdot J$$

Electromagnetic Wave Properties

- Maxwell's Equations
 - wave equations

$$\nabla^2 E = \frac{\epsilon\mu}{c^2} \frac{\partial^2 E}{\partial t^2} + \frac{4\pi\sigma\mu}{c^2} \frac{\partial E}{\partial t}$$

$$\nabla^2 H = \frac{\epsilon\mu}{c^2} \frac{\partial^2 H}{\partial t^2} + \frac{4\pi\sigma\mu}{c^2} \frac{\partial H}{\partial t}$$

- case I: vacuum (non-conducting, no net volume charge)

$$\sigma=0$$

$$\epsilon=1$$

$$\mu=1$$

$$\nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 H = \frac{1}{c^2} \frac{\partial^2 H}{\partial t^2}$$

Electromagnetic Wave Properties

- Maxwell's Equations
 - case I: vacuum (cont.)
 - general plane waves

$$E = E_0 e^{i(k \cdot r - \omega t)}$$

$$B = B_0 e^{i(k \cdot r - \omega t)}$$

$$E = E_0 e^{i(kx \pm \omega t)}$$

$$B = B_0 e^{i(kx \pm \omega t)}$$

$$ik \cdot E_0 = 0 \text{ and } ik \cdot B_0 = 0$$

$$k \perp E_0 \text{ and } k \perp B_0$$

$$ik \times E_0 = \frac{i\omega}{c} B_0$$

and

$$ik \times B_0 = -\frac{i\omega}{c} E_0$$

$$E_0 \perp B_0$$

$$k^2 = \frac{\omega^2}{c^2}$$

$$\omega = ck$$

$$\begin{aligned}\phi &= kx \pm \omega t \\ v_{ph} &= \frac{\omega}{k} \\ &= c\end{aligned}$$

Electromagnetic Wave Properties

- Maxwell's Equations
 - case I: vacuum (cont.)
 - general plane waves

$$\frac{|E|}{|H|} = \sqrt{\frac{\mu}{\epsilon}} = 1$$

$$|S| = \frac{c}{4\pi} \sqrt{\frac{\mu}{\epsilon}} E^2 \\ = \frac{c}{4\pi} E^2$$

$$u = \frac{1}{8\pi} (E \cdot D + B \cdot H) \\ = \frac{1}{8\pi} (\epsilon E^2 + \frac{B^2}{\mu}) \\ = \frac{1}{8\pi} (\epsilon E^2 + \mu H^2) \\ = \frac{\epsilon}{4\pi} E^2 \\ = \frac{1}{4\pi} E^2$$

$$\frac{|S|}{u} = \frac{c}{\sqrt{\epsilon \mu}} \\ = c$$

Electromagnetic Wave Properties

- Maxwell's Equations
 - case 2: non-vacuum non-conducting media

$$\sigma = 0$$

$$\epsilon \neq 1 \text{ (or) } \mu \neq 1$$

$$\nabla^2 E = \frac{\epsilon\mu}{c^2} \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 H = \frac{\epsilon\mu}{c^2} \frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 E = \frac{1}{v^2} \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 H = \frac{1}{v^2} \frac{\partial^2 H}{\partial t^2}$$

$$v = \frac{c}{\sqrt{\epsilon\mu}}$$

A simple modification of the phase and group velocity
as compared to the vacuum case

Electromagnetic Wave Properties

- Maxwell's Equations
 - case 3: dissipative media

$$[k^2 - \left(\frac{\epsilon\mu}{c^2}\omega^2 + i\frac{4\pi\sigma\mu\omega}{c^2}\right)]E = 0$$

$$[k^2 - \left(\frac{\epsilon\mu}{c^2}\omega^2 + i\frac{4\pi\sigma\mu\omega}{c^2}\right)]H = 0$$

$$\Rightarrow k^2 = \frac{\epsilon\mu\omega^2}{c^2} \left(1 + i\frac{4\pi\sigma}{\omega\epsilon}\right)$$

$$k = a + ib$$

$$E = E_0 e^{-bx} e^{i(ax - \omega t)}$$

exponential damping

Electromagnetic Wave Properties

- Maxwell's Equations
 - case 3: dissipative media
 - tenuous plasma - dispersion measure

$$m_e \frac{d\mathbf{v}}{dt} = m_e \frac{d^2 \mathbf{r}}{dt^2}$$
$$= -eE_0 e^{-i\omega t}$$

$$\omega^2 = \frac{4\pi Ne^2}{m_e}$$

$$\Rightarrow \mathbf{v} = \frac{e}{im_e \omega} E_0 e^{-i\omega t}$$
$$= -i \frac{e}{m_e \omega} \mathbf{E}$$

$$k^2 = \frac{\omega^2}{c^2} \left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

$$\Rightarrow \mathbf{J} = - \sum_{\alpha} e \mathbf{v}_{\alpha}$$
$$= -Ne\mathbf{v}$$
$$= i \frac{Ne^2}{m_e \omega} \mathbf{E}$$
$$= \sigma \mathbf{E}$$

$$\mathbf{v} = \frac{c}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}}$$

$$\mathbf{v}_g = \frac{d\omega}{dk}$$
$$= c \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

Electromagnetic Wave Properties

- Maxwell's Equations
 - case 3: dissipative media
 - tenuous plasma - dispersion measure

$$\begin{aligned}\tau_v &= \int_0^L \frac{dl}{v_g} \\ &\cong \frac{1}{c} \int_0^L \left\{ 1 + \frac{1}{2} \left(\frac{v_p}{v} \right)^2 \right\} dl \\ &= \frac{1}{c} \int_0^L \left\{ 1 + \frac{e^2}{2\pi m_e v^2} N(l) \right\} dl \\ &= \frac{L}{c} + \frac{e^2}{2\pi c m_e v^2} \int_0^L N(l) dl\end{aligned}$$

$$\begin{aligned}\Delta\tau &= \tau_{v_1} - \tau_{v_2} \\ &= \frac{e^2}{2\pi c m_e} \left[\frac{1}{v_1^2} - \frac{1}{v_2^2} \right] \int_0^L N(l) dl\end{aligned}$$

Homework 2a:

What is the typical range of “dispersion measure” due to ISM in the Galaxy?

How is it compared to that due to the atmosphere?

Electromagnetic Wave Properties

- polarization and Stokes parameters
 - consider at an arbitrary position ($r=0$)

$$\begin{aligned} E &= E_0 e^{-i\omega t} \\ &= (\hat{x}E_1 + \hat{y}E_2)e^{-i\omega t} \end{aligned}$$

$$\begin{aligned} E_1 &= A_1 e^{i\phi_1} \text{ and } E_2 = A_2 e^{i\phi_2} \\ E_x &= A_1 \cos(\omega t - \phi_1) \text{ and } E_y = A_2 \cos(\omega t - \phi_2) \end{aligned}$$

- draw figure

$$\begin{aligned} E_{x'} &= A_0 \cos\beta \cos\omega t \text{ and } E_{y'} = -A_0 \sin\beta \sin\omega t \\ -\pi/2 &\leq \beta \leq \pi/2 \end{aligned}$$

$$\left(\frac{E_{x'}}{A_0 \cos\beta} \right)^2 + \left(\frac{E_{y'}}{A_0 \sin\beta} \right)^2 = 1$$

Electromagnetic Wave Properties

- polarization and Stokes parameters

$$E_x = A_0 (\cos\beta \cos\chi \cos\omega t + \sin\beta \sin\chi \sin\omega t)$$

$$E_y = A_0 (\cos\beta \sin\chi \cos\omega t - \sin\beta \cos\chi \sin\omega t)$$

$$A_1 \cos\phi_1 = A_0 \cos\beta \cos\chi$$

$$A_1 \sin\phi_1 = A_0 \sin\beta \sin\chi$$

$$A_2 \cos\phi_2 = A_0 \cos\beta \sin\chi$$

$$A_2 \sin\phi_2 = -A_0 \sin\beta \cos\chi$$

$$I \equiv A_1^2 + A_2^2 = A_0^2$$

$$Q \equiv A_1^2 - A_2^2 = A_0^2 \cos 2\beta \cos 2\chi$$

$$U \equiv 2A_1 A_2 \cos(\phi_1 - \phi_2) = A_0^2 \cos 2\beta \sin 2\chi$$

$$V \equiv 2A_1 A_2 \sin(\phi_1 - \phi_2) = A_0^2 \sin 2\beta$$

$$A_0 = \sqrt{I}$$

$$\sin 2\beta = \frac{V}{I}$$

$$\tan 2\chi = \frac{U}{Q}$$

$$I^2 = Q^2 + U^2 + V^2$$

Electromagnetic Wave Properties

- wave again, at the presence of a magnetic field

$$\begin{aligned} m_e \frac{d\mathbf{v}}{dt} &= m_e \frac{d^2\mathbf{r}}{dt^2} \\ &= -e(\mathbf{E} + \frac{d\mathbf{r}}{dt} \times \mathbf{B}) \end{aligned}$$

- plane wave propagating in the z-direction

$$\begin{aligned} \ddot{\mathbf{r}}_x + \frac{e}{m} B \dot{\mathbf{r}}_y &= -\frac{e}{m} \mathbf{E}_x \\ \ddot{\mathbf{r}}_y - \frac{e}{m} B \dot{\mathbf{r}}_x &= -\frac{e}{m} \mathbf{E}_y \end{aligned}$$

$$\begin{aligned} \mathbf{r}_{\pm} &= \mathbf{r}_x \pm i \mathbf{r}_y \\ \mathbf{E}_{\pm} &= \mathbf{E}_x \pm i \mathbf{E}_y \end{aligned}$$

$$\ddot{\mathbf{r}}_{\pm} \mp i \frac{e}{m} B \dot{\mathbf{r}}_{\pm} = -\frac{e}{m} \mathbf{E}_{\pm}$$

$$E_x = \frac{1}{2}(E_+ + E_-) \text{ and } E_y = \frac{1}{2i}(E_+ - E_-)$$

Electromagnetic Wave Properties

- wave again, in the presence of a “strong” magnetic field

$$E_{\pm} = A e^{i(k_{\pm}z - \omega t)}$$

$$r_{\pm} = r_0 e^{i(k_{\pm}z - \omega t)}$$

$$r_{\pm}(-\omega^2 \mp \frac{e}{m}B\omega) = -\frac{e}{m}E_{\pm}$$

$$\dot{r}_{\pm} = \frac{i\frac{e}{m}}{-\omega^2 \mp \frac{e}{m}B\omega} \omega E_{\pm}$$

$$|J| = -N e \dot{r}_{\pm}$$

$$= i \frac{Ne^2}{m(\omega \pm \frac{e}{m}B)} E_{\pm}$$

$$= \sigma_{\pm} E_{\pm}$$

$$\omega_c = \frac{e}{m}B$$

$$v_c = \frac{e}{2\pi m}B$$

$$k_{\pm}^2 = \frac{\omega^2}{c^2} \left(1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_c)}\right)$$

$$\sigma_{\pm} = i \frac{Ne^2}{m(\omega \pm \frac{e}{m}B)}$$

Electromagnetic Wave Properties

- wave again, at the presence of a magnetic field

$$\Delta\psi = \frac{1}{2}(k_+ - k_-)\Delta z$$

$$\omega \gg \omega_p \text{ and } \omega \gg \omega_c$$

$$\begin{aligned} &\simeq \frac{\omega_p^2 \omega_c}{2c^2 \omega^2} \Delta z \\ &= \frac{2\pi Ne^3 B}{m^2 c^2 \omega^2} \Delta z \\ &= \frac{e^3}{2\pi m^2 c^2} \frac{1}{\nu^2} \int_0^L B_{||}(z) N(z) dz \end{aligned}$$

Faraday Rotation!

Homework 2b:

Express the above equation in more sensible units and check the typical values for this effect in a reference (please cite)

Electromagnetic Wave Properties

- polarization and Stokes parameters
 - quasi-monochromatic waves
 - slow varying amplitudes and phases

$$E_1(t) = A_1(t)e^{i\phi_1(t)} \text{ and } E_2(t) = A_2(t)e^{i\phi_2(t)}$$

- some finite bandwidth and coherence time
- Stokes parameters

$$I^2 \geq Q^2 + U^2 + V^2$$

- superposition principle of Stokes parameters and degree of polarization

$$\begin{aligned} p &\equiv \frac{I_{pol}}{I} \\ &= \frac{\sqrt{Q^2 + U^2 + V^2}}{I} \end{aligned}$$

Homework 2

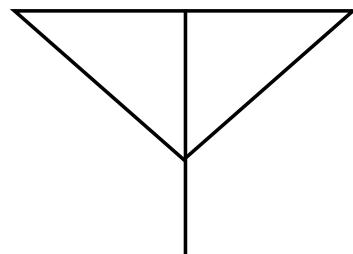
- I.1 : A cable has an optical depth \mathcal{T} of 0.5 and a temperature of 300 K. A signal of peak temperature 1K is connected to the input of this cable. Use the equation of transfer we learned at class, find the temperature at the output end of the cable.
- I.2 : Repeat the above exercise (I.1), but assume the optical depth of the cable is 0.001.
- I.3 : Repeat the above exercise again, but assume the optical depth of the able is back to 0.5, but the single has a peak temperature of 10000 K.
- I.4 : A plan electromagnetic wave perpendicularly approaches a surface with conductivity σ . The wave penetrates to a depth of δ . Consider the EM wave equation and take

$$\sigma \gg \epsilon/4\pi$$

The solution to this equation is an exponentially decay wave. Use this to estimate and express the penetration depth. Assuming for copper $\sigma = 10^{17} s^{-1}$ and $\mu \approx 1$, what is the penetration depth for a wave at a frequency of 10^{10} Hz .

- I.5 : What is the typical range of “dispersion measure” due to ISM in the Galaxy (Please estimate by yourself or cite a reference)? How is it compared to that due to the atmosphere?
- I.6 : Express the Faraday Rotation formula in more sensible units and check the typical values for this effect in a reference (Please cite).

Antenna



RF

Front End

Radio Telescope Fundamentals

- direct detection
- amplification (LNA) → detection

usually broadband
(incoherent)

- amplification (LNA) → inserting Local Oscillator (heterodyne system)

coherent
usually narrowband

IF

Back End

detection with
spectral capability