

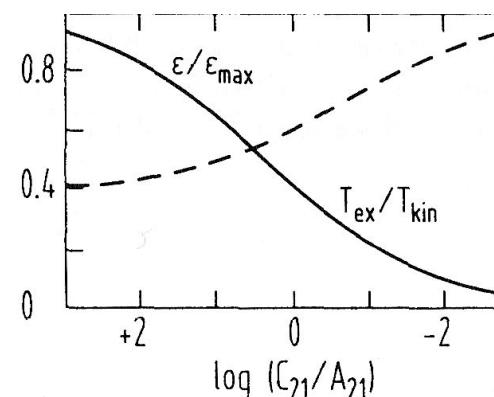
# Radio Astronomy - Line Emission

- two-level (radiative + collisional)

$$\varepsilon_v = \frac{h\nu_0}{4\pi} \frac{n_l(C_{lu} + M_{lu})}{(A_{ul} + C_{ul} + M_{ul})} \phi(v)$$

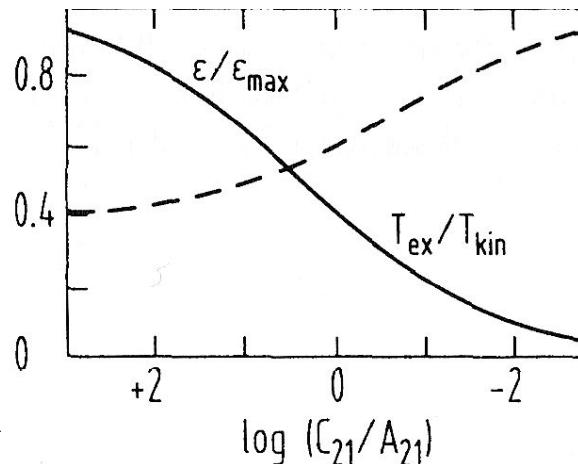
$$\varepsilon_v = \frac{h\nu_0 g_u n_l}{4\pi g_l} \frac{\left(\frac{I_v}{2hv_0^3/c^2} + \frac{C_{ul}}{A_{ul}} e^{-h\nu_0/kT_K}\right)}{\left(1 + \frac{I_v}{2hv_0^3/c^2} + \frac{C_{ul}}{A_{ul}}\right)} \phi(v)$$

$$\begin{aligned} \frac{n_u}{n_l} &= \frac{g_u}{g_l} e^{-h\nu_0/kT_{ex}} \\ \frac{C_{lu}}{C_{ul}} &= \frac{g_u}{g_l} e^{-h\nu_0/kT_K} \quad T_{ex} = T_K \frac{T_b A_{ul} + T_0 C_{ul}}{T_K A_{ul} + T_0 C_{ul}} \\ \frac{I_v}{I_v + \frac{2hv_0^3}{c^2}} &= e^{-h\nu_0/kT_b} \quad (T_{ex}, T_K, T_b \gg h\nu_0/k) \end{aligned}$$



# Radio Astronomy - Line Emission

- two-level (radiative + collisional)



LTE

$$I = B(T_b) = B(T)$$

$$\frac{n_u}{n_l} = \frac{g_u}{g_l} e^{-hv_0/kT_{\text{ex}}} = \frac{g_u}{g_l} e^{-hv_0/kT}$$

$$T_b = T_{\text{ex}} = T_K = T$$

in general  
Non-LTE

$$T_b < T_{\text{ex}} < T_K$$

(but,) no masering

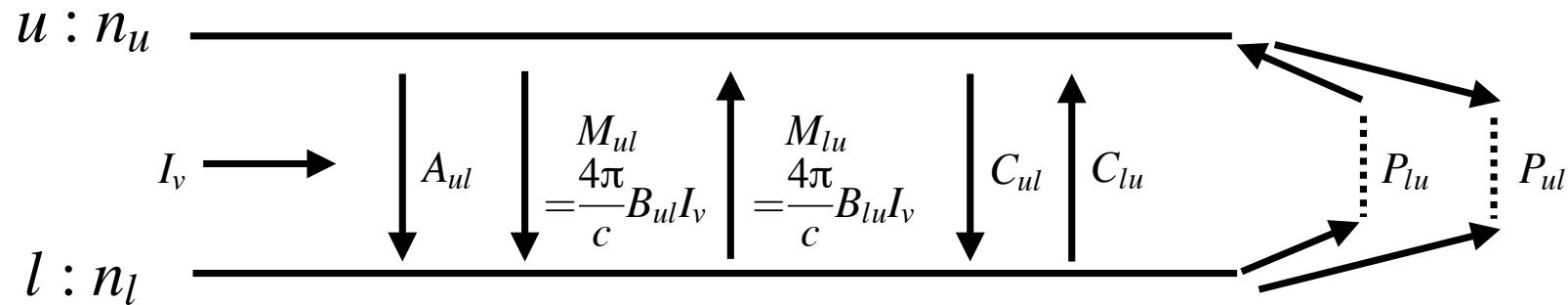
What is critical density?

# Radio Astronomy - Line Emission

- three-level (radiative + collisional) or “two-level” + “pumping”

$$\frac{dI_v}{ds} = -\kappa_v I_v + \varepsilon_v$$

$$\frac{dI_v}{ds} = \frac{hv_0}{4\pi} \left[ (n_u - n_l) B \frac{4\pi}{c} I_v + n_u A \right] \phi(v)$$



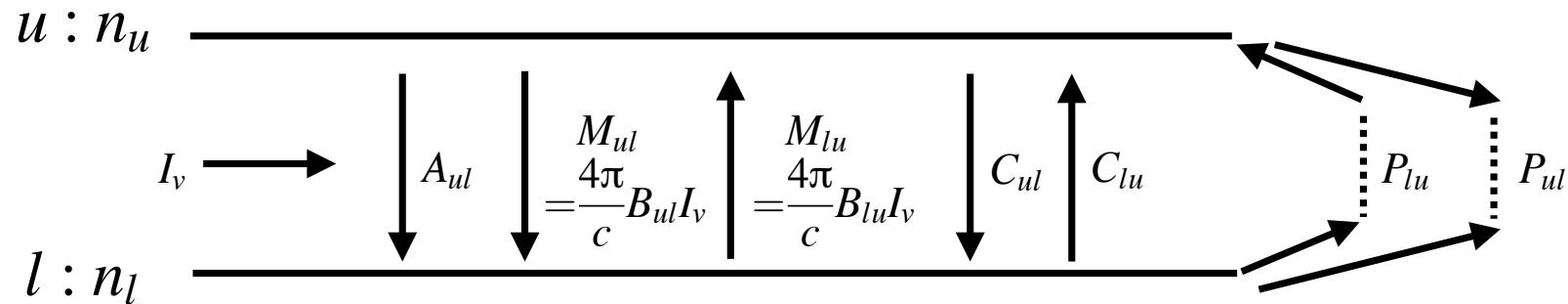
**LTE**  $\frac{n_u}{n_l} = \frac{g_u}{g_l} e^{-hv_0/kT_K}$

# Radio Astronomy - Line Emission

- three-level (radiative + collisional) or “two-level” + “pumping”

$$\frac{dI_v}{ds} = -\kappa_v I_v + \varepsilon_v$$

$$\frac{dI_v}{ds} = \frac{h\nu_0}{4\pi} \left[ (n_u - n_l) B \frac{4\pi}{c} I_v + n_u A \right] \phi(v)$$



**Non-LTE**     $n_u(A_{ul} + C_{ul} + M_{ul} + P_{ul}) = n_l(C_{lu} + M_{lu} + P_{lu})$

# Radio Astronomy - Line Emission

- Steady State [Statistical Equilibrium]

$$n_u(A_{ul} + C_{ul} + M_{ul} + P_{ul}) = n_l(C_{lu} + M_{lu} + P_{lu})$$

For simplification, assume

$$\begin{aligned}C_{ul} &\approx C_{lu} \approx C \\M_{ul} &\approx M_{lu} \approx M \\A &\ll C, A \ll M\end{aligned}$$

$$\frac{n_u}{n_l} = \frac{P_{lu} + M + C}{P_{ul} + M + C}$$

For more simplification, assume  $g_u = g_l$

in a special case

$$\begin{aligned}(n_u - n_l)|_{M=C=0} &= (n_u + n_l) \frac{P_{lu} - P_{ul}}{P_{lu} + P_{ul}} \\&= n \frac{P_{lu} - P_{ul}}{P_{lu} + P_{ul}} \\&\equiv \Delta n_0\end{aligned}$$

in a more general case

$$\Delta n = \frac{\Delta n_0}{1 + \frac{2(C+M)}{P}}$$

# Radio Astronomy - Line Emission

- Population Inversion

$$\Delta n = \frac{\Delta n_0}{1 + \frac{2(C+M)}{P}}$$

If  $\Delta n_0 > 0 \Rightarrow \Delta n > 0$

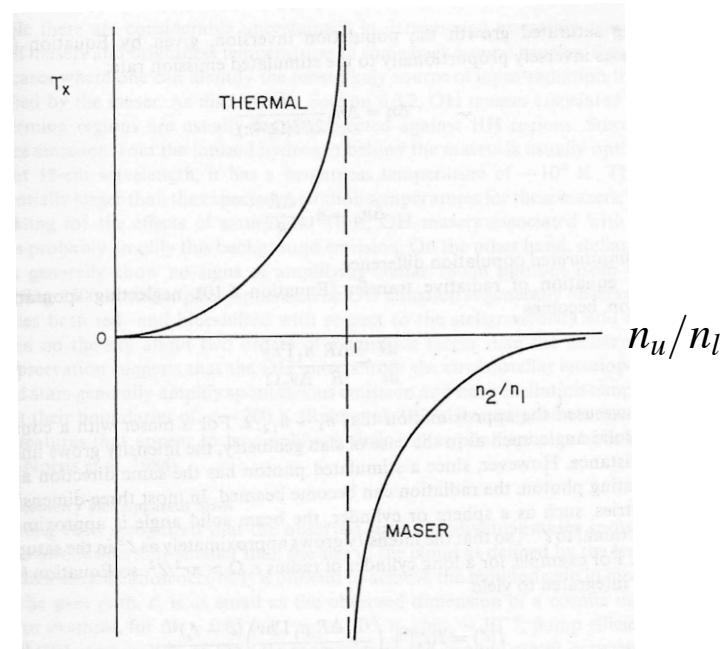
$$\frac{n_u}{n_l} = e^{-hv_0/kT_{ex}} > 1$$

↓

$$T_{ex} < 0$$

excitation temperature

$T_{ex}$



# Radio Astronomy - Line Emission

- Amplification [in transfer equation]

$$\frac{dI_v}{ds} = \frac{\alpha I_v}{1 + I_v/I_s} + \varepsilon$$
$$\alpha = \frac{hv_0}{c} B \frac{\Delta n_0}{1 + \frac{2C}{P}} \phi(v)$$
$$I_s = \frac{cP}{2B\Omega_m} \left(1 + \frac{2C}{P}\right)$$
$$\varepsilon = \frac{hv_0}{4\pi} n_u A \phi(v)$$

Unsaturated  
exponential amplification

Saturated  
linear amplification

$$I_{v_0} = I_0 e^{\alpha L} + \frac{\varepsilon}{\alpha_0} (e^{\alpha_0 L} - 1)$$

$$I_{v_0} = I_0 + (\alpha_0 I_s + \varepsilon) L$$

$$T_b = T_c e^{\alpha L} + |T_{ex}| (e^{\alpha_0 L} - 1)$$

# Radio Astronomy - Line Emission

- Line width [in transfer equation] : unsaturated case

$$I_{v_0} = I_0 e^{\alpha L} + \frac{\epsilon}{\alpha_0} (e^{\alpha_0 L} - 1)$$

$$T_b = T_c e^{\alpha L} + |T_{ex}| (e^{\alpha_0 L} - 1)$$

consider background amplification

$$T_b(v) = T_c e^{\alpha(v)L}$$

$$\alpha(v) = \alpha_0 \exp\left(-\frac{(v-v_0)^2}{2\sigma_0^2}\right)$$

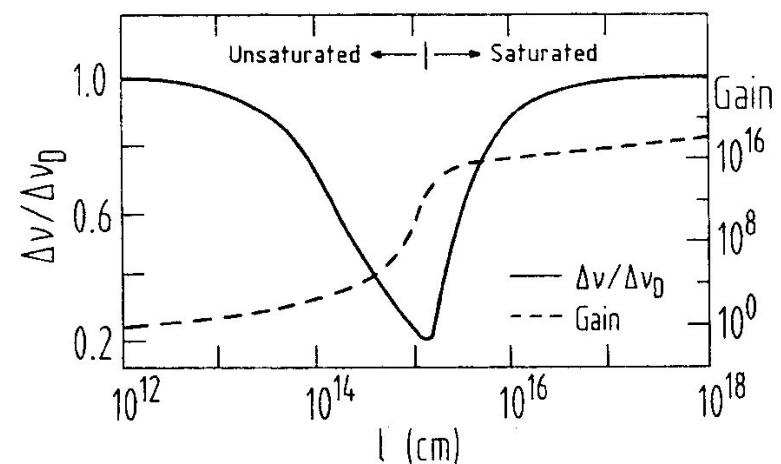
$$T_b(v) = T_c e^{\alpha_0 \exp\left(-\frac{(v-v_0)^2}{2\sigma_0^2}\right) L}$$

$$\alpha(v) \approx \alpha_0 \left(1 - \frac{(v-v_0)^2}{2\sigma_0^2}\right)$$

$$\begin{aligned} T_b(v) &= [T_c e^{\alpha_0 L}] e^{-\frac{\alpha_0 L}{2\sigma_0^2} (v-v_0)^2} \\ &= T_{v_0} e^{-\frac{\alpha_0 L}{2\sigma_0^2} (v-v_0)^2} \end{aligned}$$

line-width and path length

$$\sigma = \frac{\sigma_0}{\alpha_0 L}$$



# Radio Astronomy - Masers

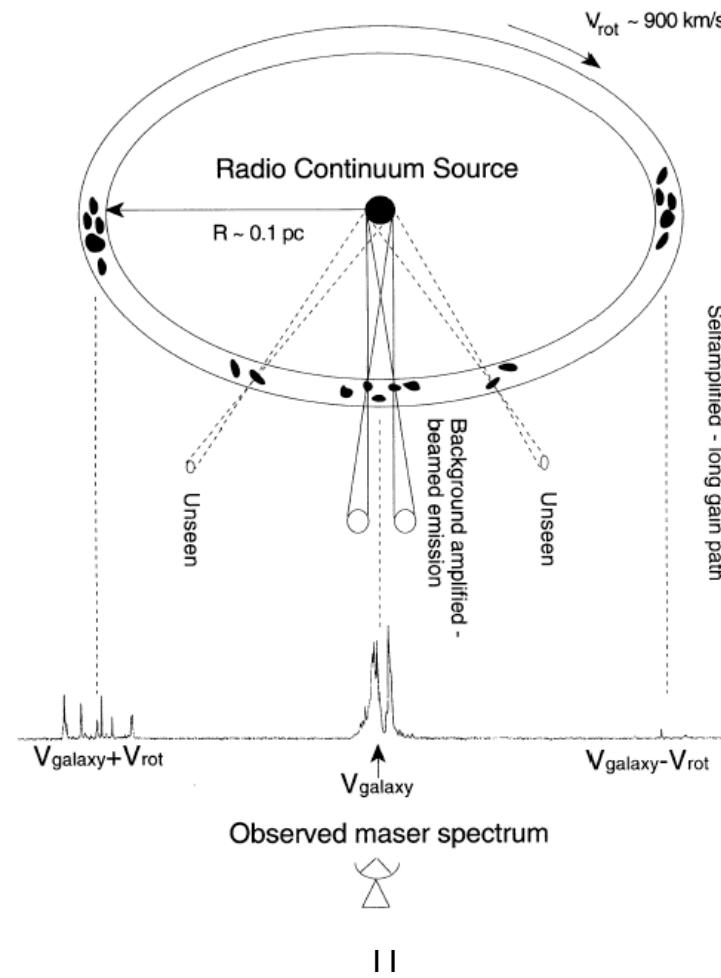
- Masers
  - physical condition required
    - population inversion : still mostly poor understood
      - collisional pumping
      - radiative pumping
  - physical information provided
    - density/temperature (required for pumping)
    - accurate velocity (required for pumping)
      - kinematic information on “small” parcel of gas
    - accurate (relative) positional information
      - proper motion measurements - distance

# Radio Astronomy - Masers

- Maser
  - species
    - OH, H<sub>2</sub>O, SiO, H<sub>2</sub>CO, CH<sub>3</sub>OH, HCN, NH<sub>3</sub>, H-recomb
  - locations
    - Galactic
      - ISM : star forming regions, HII regions
        - OH, H<sub>2</sub>O, SiO, H<sub>2</sub>CO, CH<sub>3</sub>OH, NH<sub>3</sub>, H-recomb
      - stellar : circumstellar envelopes around evolved stars
        - SiO, OH, H<sub>2</sub>O, HCN
      - solar : comets
    - Extragalactic
      - “megamaser” (~1e6 times brighter than galactic counterparts), or even “gigamaser” for OH
        - H<sub>2</sub>O in NGC 4945 (Santos and Lepine 1979)
        - OH in Arp 220 (Baan et al. 1982)
        - circumnuclear disk
          - amplification of radio continuum
          - OH (FIR?), H<sub>2</sub>CO (FIR/Radio?), H<sub>2</sub>O (collisional?)
        - star formation?

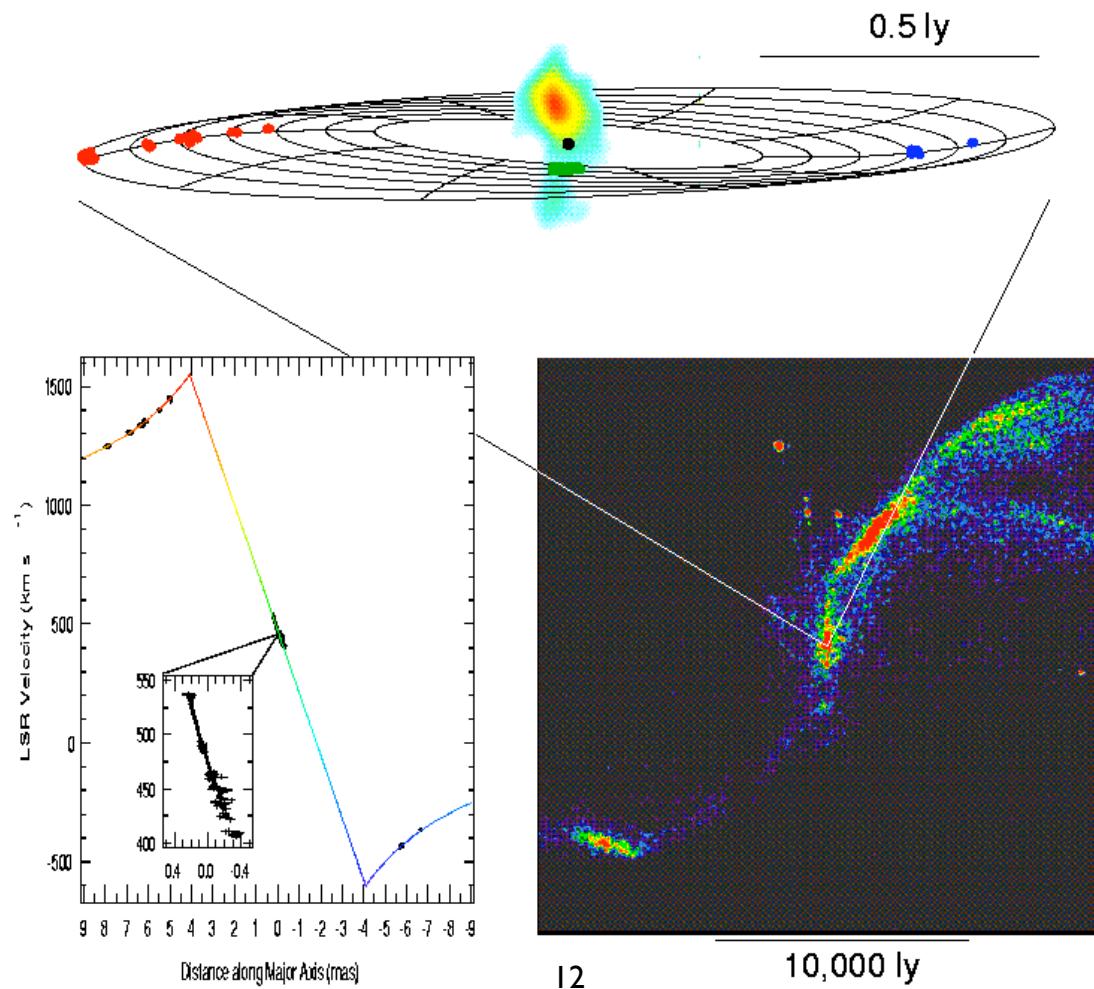
# Radio Astronomy - Masers

- Megamasers : SMBH in NGC 4258 (Syfert-2 AGN)



# Radio Astronomy - Masers

- Megamasers : SMBH in NGC 4258 (Syfert-2 AGN)



# Radio Astronomy - The 21 cm Line

- The 21 cm line
  - first radio wavelengths spectral line detected
  - prediction : van de Hulst and Oort
  - detection : Ewen and Purcell
  - Hyperfine splitting - interaction between the magnetic moment of the proton and the magnetic moment of the orbiting electron at the ground state  ${}^2S_{1/2}$
- Einstein A ( $2.86888 \times 10^{-15}$ /s) due to
  - small (magnetic) dipole transition
  - low frequency
- Rest Frequency :  $1.420405751786 \pm 0.00000000030$  MHz

# Radio Astronomy - The 21 cm Line

- The 21 cm line
  - excitation, general called “spin”, temperature

$$\begin{aligned}\frac{N_u}{N_l} &= \frac{g_u}{g_l} \exp\left(\frac{-hv_0}{kT_e}\right) \\ &= 3 \exp\left(\frac{-hv_0}{kT_s}\right)\end{aligned}$$

- recall

$$\begin{aligned}T_s &= T_K \frac{T_B A_{ul} + T_0 C_{ul}}{T_K A_{ul} + T_0 C_{ul}} &= \frac{T_B + y T_K}{1 + y} \\ T_0 &= \frac{hv_0}{k} = 0.0682 K & y = \frac{hv}{kT_K} \frac{C_{ul}}{A_{ul}} \\ && \text{Purcell \& Field (1956)}\end{aligned}$$

$$\begin{aligned}&= \frac{T_K + y' T_B}{1 + y'} \\ y' &= \frac{kT_K A_{ul}}{hv} \frac{C_{ul}}{C_{ul}} \\ && \text{Kulkarni \& Heiles (1988)}\end{aligned}$$

# Radio Astronomy - The 21 cm Line

- The 21 cm line
  - absorption coefficient, optical depth, and column density

$$\kappa_v = \frac{3c^2}{32\pi} \frac{1}{v_{ul}} A_{ul} n_H \frac{h}{kT_s} \phi(v)$$

$$d\tau(v) = -\kappa_v(s, v) ds$$

$$d\tau\left(\frac{v}{km s^{-1}}\right) = -5.4873 \cdot 10^{-19} \left(\frac{n_H(s)}{cm^{-3}}\right) \left(\frac{T_s(s)}{K}\right)^{-1} \left(\frac{\phi(v)}{km^{-1}s}\right) d\left(\frac{s}{cm}\right)$$

$$\tau(v) = \int -\kappa_v(s, v) ds$$

$$\tau\left(\frac{v}{km s^{-1}}\right) = -5.4873 \cdot 10^{-19} \left(\frac{T_s}{K}\right)^{-1} \left(\frac{\phi(v)}{km^{-1}s}\right) \int \left(\frac{n_H(s)}{cm^{-3}}\right) d\left(\frac{s}{cm}\right)$$

$$= -5.4873 \cdot 10^{-19} \left(\frac{T_s}{K}\right)^{-1} \left(\frac{\phi(v)}{km^{-1}s}\right) \frac{N_H}{cm^{-2}}$$

$$\int \tau(v) dv = -5.4873 \cdot 10^{-19} \left(\frac{T_s}{K}\right)^{-1} \frac{N_H}{cm^{-2}} / \text{int}\left(\frac{\phi(v)}{km^{-1}s}\right) dv$$

$$= -5.4873 \cdot 10^{-19} \left(\frac{T_s}{K}\right)^{-1} \frac{N_H}{cm^{-2}}$$

$$\Downarrow$$

$$\frac{N_H}{cm^{-2}} = 1.8224 \cdot 10^{18} \left(\frac{T_s}{K}\right) \int \tau(v) d\left(\frac{v}{km s^{-1}}\right)$$

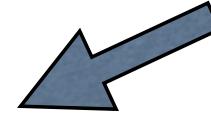
$$\tau = -\ln\left(1 - \frac{T_L}{T_s - T_{BG}}\right)$$

$$T_s \gg T_{BG}$$

$$\approx -\ln\left(1 - \frac{T_L}{T_s}\right)$$

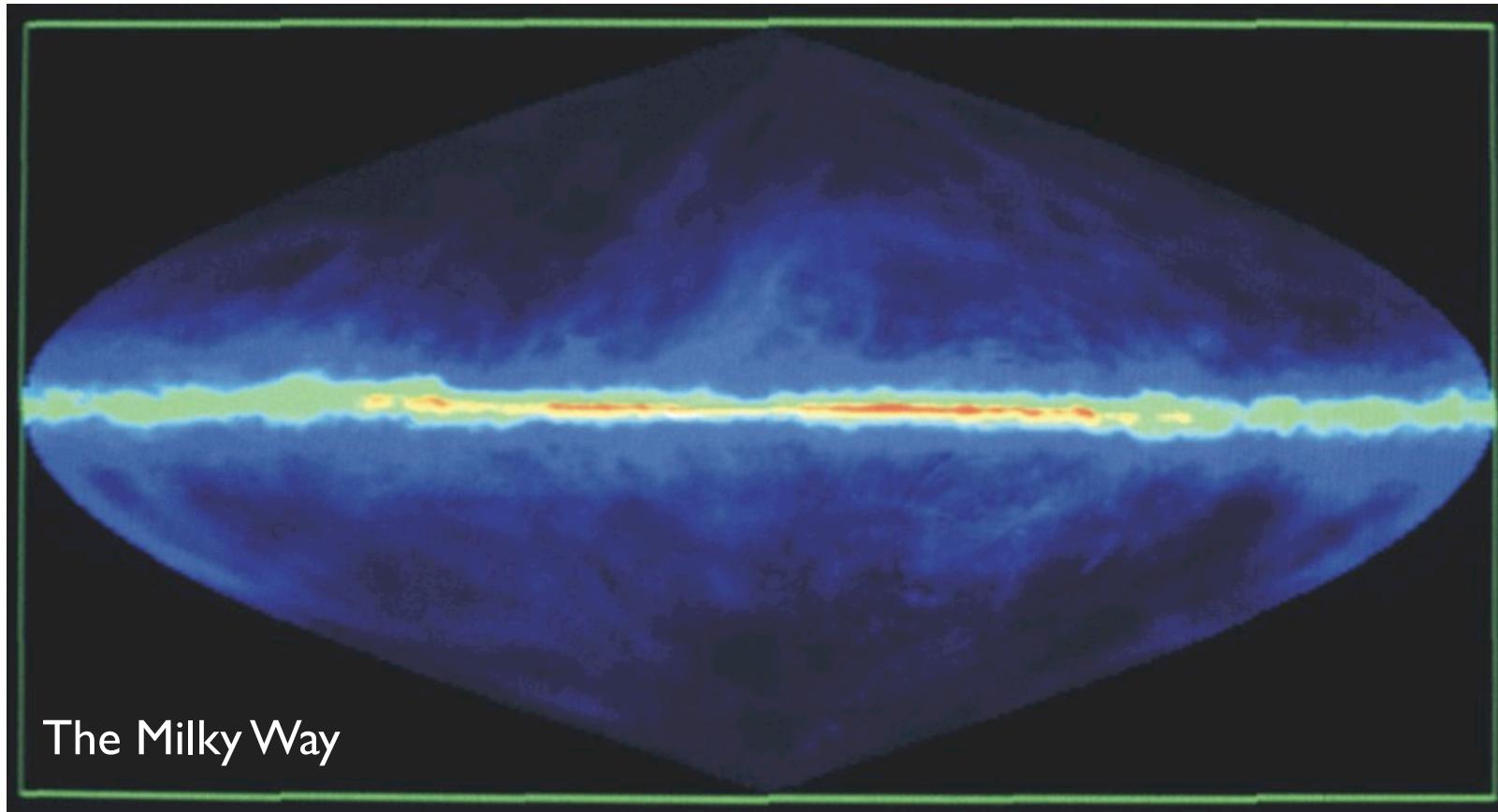
$$T_L \ll T_s \text{ or } \tau \ll 1$$

$$\approx \frac{T_L}{T_s}$$



# Radio Astronomy - The 21 cm Line

- HI Clouds and the Galactic Structure



# Radio Astronomy - The 21 cm Line

- HI Clouds and the Galactic Structure

- Interstellar Medium (ISM)
  - gas heating and cooling

$$\frac{d}{dt} \left( \frac{3}{2} N k T_k \right) = \Gamma - \Lambda$$

- heating (ionization by radiation, high-energy particles [cosmic rays], collisions of gas clouds or streams)
    - cooling (heavy element spectral emission)
  - four component model (in pressure balance?) - the swiss cheese picture (with undetermined filling factors)
    - CNM (cold neutral medium)
      - narrow 21cm absorption
      - $T < 50\text{K}$ ,  $n > 1-10 \text{ cm}^{-3}$
    - WNM (warm neutral medium)
      - ubiquitous wide 21cm emission
      - $T > 200\text{K}$
    - WIM (warm ionized medium)
      - widespread Halpha emission
      - $T \sim 10^4\text{K}$
    - HIM (hot ionized medium)
      - diffuse soft X-ray emission and OVI absorption
      - $T \sim 10^6\text{K}$

# Radio Astronomy - The 21 cm Line

- HI in the extragalactic studies
  - extragalaxies
    - major tool for kinematic studies, such as rotation curve since :
      - tracing relative more extended regions since :
        - HI - neutral atomic gas
        - low excitation/density requirement
      - spectral (line) feature - kinematic (velocity) probe
    - findings :
      - HI distribution not centered
      - flat rotation curve
      - tidal interaction features

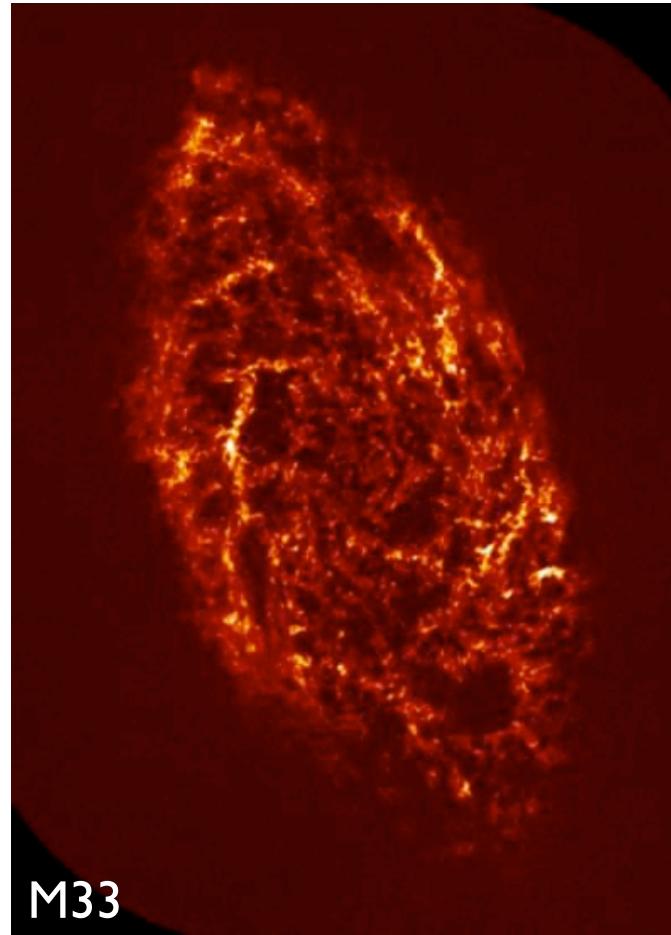
# Radio Astronomy - The 21 cm Line

- HI in the extragalactic studies



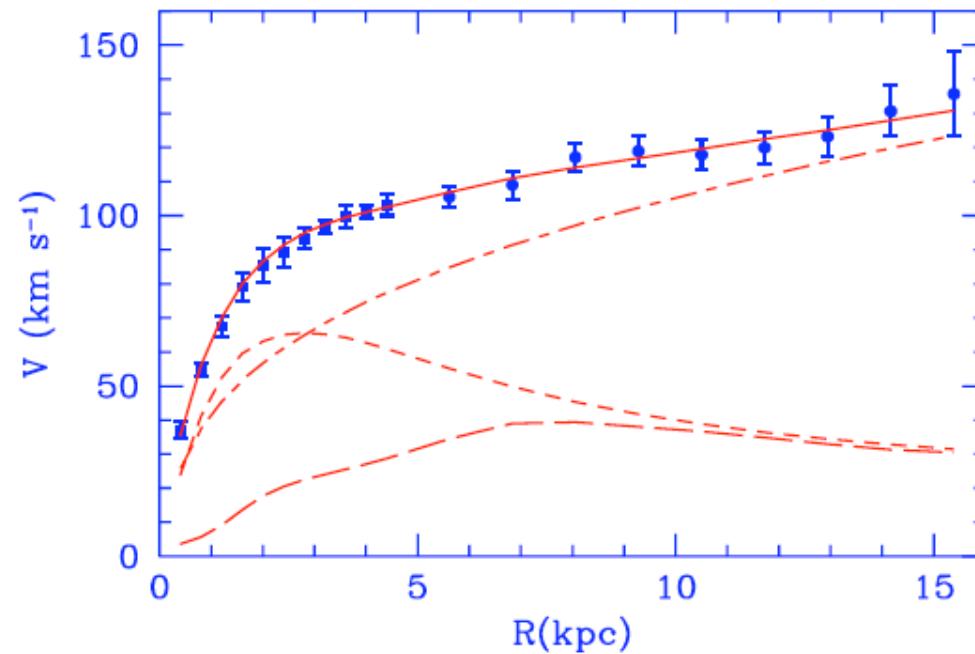
# Radio Astronomy - The 21 cm Line

- HI in the extragalactic studies



# Radio Astronomy - The 21 cm Line

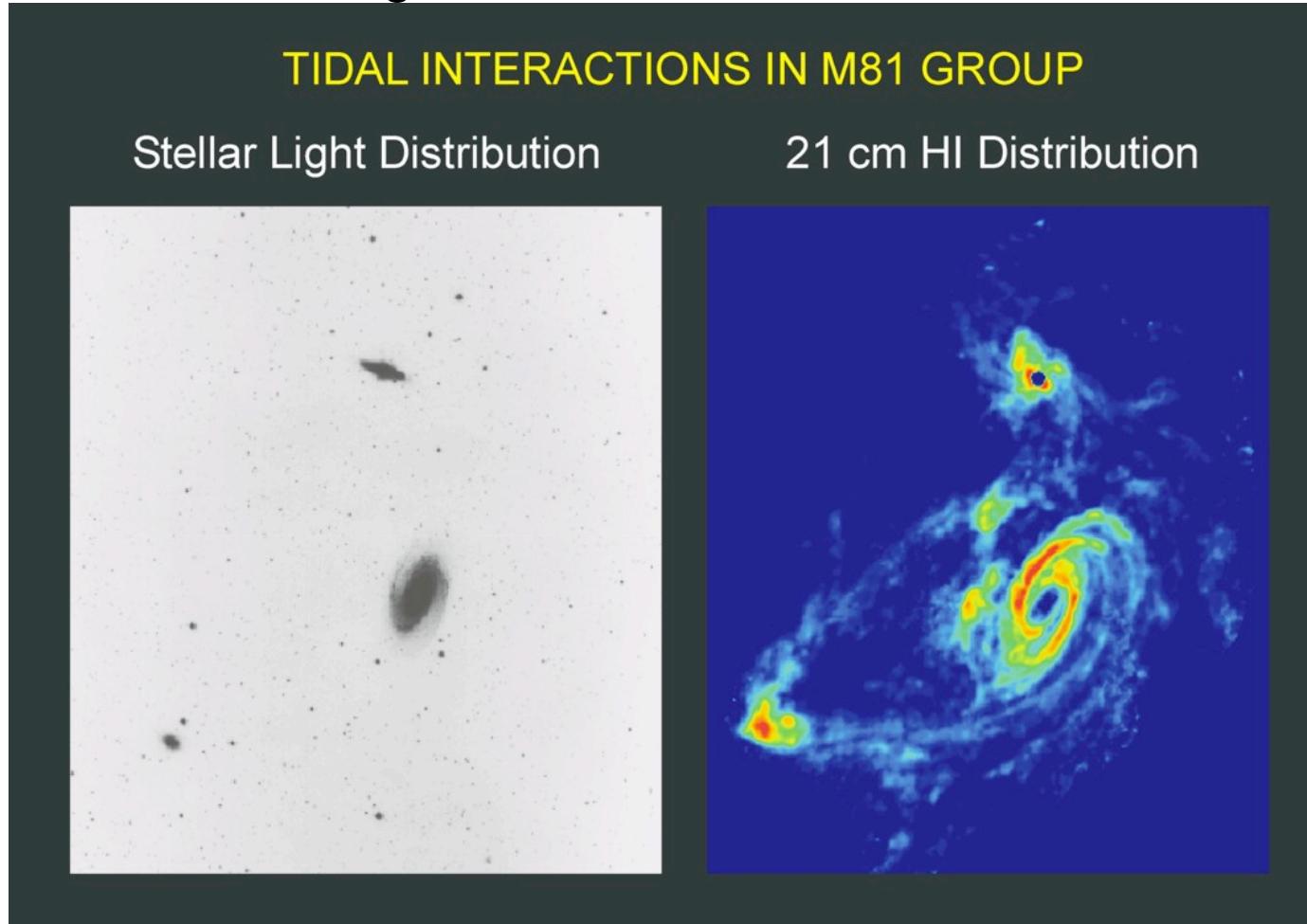
- HI in the extragalactic studies



M33 rotation curve (points) compared with the best fit model (continuous line). Also shown the halo contribution (dashed-dotted line), the stellar disk (short dashed line) and the gas contribution (long dashed line) [Corbelli and Salucci 2000, MNRAS, 311, 441]

# Radio Astronomy - The 21 cm Line

- HI in the extragalactic studies



# Radio Astronomy - The 21 cm Line

- HI in the extragalactic studies

