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A Simple Analytic Approximation Approach for Estimating the True Random Effects and True Fixed Effects Stochastic Frontier Models

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Abstract

This paper derives an analytic approximation formula for the likelihood function of the true random effects stochastic frontier model of Greene (2005) with a time span $T = 2$. Gaussian quadrature procedure and simulation-based method is not required for the closed-form approach. Combining the analytic formula with a pairwise likelihood estimator (PLE), we easily can estimate the true random effects stochastic frontier models with $T > 2$. This analytic approximation approach is also applicable to the true fixed effects stochastic frontier model of Greene (2005) after the fixed effects parameters are eliminated from the pairwise differencing or first differencing operators. The Monte Carlo simulations confirm the promising performance of the analytic methodology under all the configurations generated from the true random effects and true fixed effects stochastic frontier models in this paper. The proposed method is applied to the World Health Organization's (WHO) panel data on national health care systems.

Key words: True random effects; true fixed effects; panel stochastic frontier model

JEL classifications: C33; C4

1 Introduction

This paper considers the estimation problems of the true random effects stochastic frontier analysis (TRESFA) and the closed related true fixed effects SFA (TFESFA, hereafter) models proposed by Greene (2005). These panel SFA models preserve the feature of the stochastic frontier model of Aigner, Lovell, and Schmidt (ALS, 1977) and accommodates heterogeneity which cannot be shared with the random effects model of Pitt and Lee (1981), in particular, the efficiency is no more a time-invariant variable in the TRESFA and TFESFA frameworks.

Both TRESFA and TFESFA models are useful workhorses for efficiency estimation provided that the associated estimation is not costly. However, the full likelihood function of the TRESFA model does not admit a close-form presentation, and the maximuum likelihood estimation (MLE) of the TFESFA model might be subject to the incidental parameters problems when the number of cross units, N, are huge.

This paper first shows that the full likelihood function of the TRESFA model does admit a close-form analytic approximation when the time span $T = 2$. The analytic method makes the associated likelihood estimation easy and stable. When comparing to the quadrature-based MLE suggested by Butler and Moffitt (1982), we find that the bias and root of mean-squared-error (RMSE) performance of the analytic methodology is very much similar to that from the quadrature approach. This is expected, because the MLE performed by both quadrature-based and analytic methods are full MLE when $T = 2$. The simulations thus confirm the accuracy of the analytic formula in approximating the likelihood of the TRESFA models when $T = 2$.

Another contribution of this paper is to investigate whether the TRESFA model can be estimated with the Gaussian quadrature procedure with $T \geq 2$. Greene (2005) does not use the quadrature method for his proposed models, instead, he employs a simulation-based method to generate the random effects so as to form the approximate likelihood function.

In fact, both Gaussian quadrature and simulated maximum likelihood estimator are alternative approximation methods for evaluating the full likelihood function of the TRESFA models. Theoretically, the simulated MLE works for the TRESFA model if the number of random draws approaches a huge number. The trade-off is that the computational cost might be huge.

Using Monte Carlo experiment, this paper documents that the quadrature method cannot be used for the TRESFA model without modification. Table 4 of this paper clearly reveals that the RMSE from estimating some of the parameters of the TRESFA models is not only sizable but also opposite to what normal asymptotic theory would suggest, i.e., the RMSE does not decrease with an increasing value of T when the within group correlation (generated by the presence of random effects) is strong. This finding is also similar to that observed in Borjas and Sueyoshi (1994) that numerical difficulties might occur if one applies the quadrature technique to the probit models with structural group effects where the number of individuals in a group is large.

This paper also demonstrates that the TRESFA models with $T > 2$ is easily estimated by combining the pairwise likelihood estimator (PLE) in Besag (1975) and Heagerty and Lele (1998) and the analytic approximation. The associated computational burdens from the PLE are still mild, because each pairwise likelihood function is effectively approximated without using numerical-integral or simulation-based procedures. Furthermore, the pairwise nature of the proposed method naturally takes care of the unbalanced panel data as long as two periods of observations are available for that cross unit. The simulations conducted in this paper also confirm a satisfactory performance of the analytic approach under the general setting.

This analytic approximation approach is also applicable to the TFESFA model after the fixed effects parameters are eliminated from the pairwise differencing or first differencing operators. The simulation results in the following Table 9 reveal the usefulness of the analytic formula for the TFESFA models.

The coverage of the developed methodology is not confined to the panel data scenario. The analytic formula also works for the models based on grouped cross-sectional or clustering data with different number of individuals in each group, because the panel random effects model share an identical structure with the correlated grouped data frequently encountered in the economics and biomedical sciences. See Baltagi (2001) and Hsiao (2003) for discussions of clustering models.

The remaining parts of this paper are arranged as follows: Section 2 presents the major theoretical finding of this paper, and the analytic approximation formula for evaluating the likelihood function of the TRESFA model when $T = 2$. The idea of pairwise likelihood principle is also revealed in this section. Section 3 displays the approximation formula for the TFESFA model under pairwise differencing transformation. In Section 4 we illustrate the finite sample performance of the proposed methods for both TRESFA and TFESFA models. We also apply the proposed method to the World Health Organization's (WHO) panel data on national health care systems in Section 5. We conclude in Section 6.

2 A true random effects stochastic frontier model

Consider the following true random effects stochastic frontier models:

$$y_{it} = \alpha + x_{it}^{\top}\beta + w_i + v_{it} - Su_{it}, \quad i = 1, 2, \dots, N \quad \text{and} \quad t = 1, \dots, T, \tag{1}$$

where y_{it} is the performance of firm i in period t, x_{it} is the vector of inputs or input prices, w_i is the random firm specific effect, and $S = 1$ or -1, depending on the context under investigation. v_{it} and u_{it} are the symmetric and one sided components to the stochastic frontier model proposed by ALS (1977):

$$v_{it} \sim N\left[0, \sigma_v^2\right], \qquad u_{it} = |U_{it}| \text{ where } U_{it} \sim N\left[0, \sigma_u^2\right] \perp v_{it}. \tag{2}$$

Define $\varepsilon_{it} = v_{it} - Su_{it}$ and rewrite the model in (1) as:

$$y_{it} = \alpha + x_{it}^{\top}\beta + w_i + \varepsilon_{it}, \tag{3}$$

the above TRESFA model is a usual random effects model with a time varying component ε_{it} which has the asymmetric distribution as the key feature of the SFA model.

In order to evaluate the likelihood function of the TRESFA model, we note that

$$f(y_{it}|w_i) = \frac{2}{\sigma}\,\phi\left(\frac{\varepsilon_{it}}{\sigma}\right)\Phi\left(\frac{-S\lambda\varepsilon_{it}}{\sigma}\right), \qquad \varepsilon_{it} = y_{it} - \alpha - x_{it}^\top\beta - w_i = Z_{it} - w_i, \tag{4}$$

where $\sigma^2 = \sigma_u^2 + \sigma_v^2$, $\phi(.)$ and $\Phi(.)$ are the probability density function (pdf) and the cumulative distribution function (cdf) of a standard normal distribution, respectively, and $\lambda = \sigma_u/\sigma_v$.

Conditional on w_i, the T observations for firm i are independent with each other, thus, the joint density of the T observations is:

$$f(y_{i1},\ldots,y_{iT}|w_i) = \prod_{t=1}^{T}\frac{2}{\sigma}\,\phi\left(\frac{\varepsilon_{it}}{\sigma}\right)\Phi\left(\frac{-S\lambda\varepsilon_{it}}{\sigma}\right). \tag{5}$$

It follows that the unconditional joint density is:

$$L^i = \int_{w_i}\prod_{t=1}^{T}\frac{2}{\sigma}\,\phi\left(\frac{\varepsilon_{it}}{\sigma}\right)\Phi\left(\frac{-S\lambda\varepsilon_{it}}{\sigma}\right)g(w_i)dw_i, \tag{6}$$

where $g(.)$ denotes some probability density function.

Assume that w_i is generated as a normal distribution with a variance σ_w^2 as regularly imposed in the literature, we derive from (6) that

$$L^i = \int_{-\infty}^{\infty}\prod_{t=1}^{T}\frac{2}{\sigma}\,\phi\left(\frac{\varepsilon_{it}}{\sigma}\right)\Phi\left(\frac{-S\lambda\varepsilon_{it}}{\sigma}\right)\frac{1}{\sigma_w}\phi\left(\frac{w_i}{\sigma_w}\right)dw_i. \tag{7}$$

The above likelihood function can be evaluated with the Gaussian quadrature procedure suggested by Butler and Moffitt (1982). However, Gaussian quadrature never has been used for the TRESFA model. Instead, Greene (2005) adopts a simulation-based method to generate the random effects so as to form the approximate likelihood function. Theoretically, the simulated MLE works for the TRESFA model if the number of random draws approaches a huge number. The trade-off is that the computational cost might be huge. The common feature shares with the Gaussian quadrature and the simulated MLE is that both approaches are alternative ways to approximate the full likelihood function of the TRESFA models.

Using simulations, Table 4 of this paper shows that the RMSE from estimating the parameters of the TRESFA models based on quadrature method could be sizable, and the changing pattern of the RMSE is opposite to what normal asymptotic theory would suggest. This paper thus paves a new way for estimating the TRESFA models with an easy-to-implement analytic approximation formula. As will be shown later, the performance of the analytic formula is promising and almost equivalent to that from using the quadrature approach when $T = 2$. We will also show that the proposed analytic method works extremely well for the cases $T > 2$ by combining the pairwise likelihood principle. Moreover, the implementation of MLE based on the proposed formula is very stable according to the Monte Carlo experiment conducted in this paper.

2.1 An analytic formula for TRESFA with $T = 2$

This subsection illustrates the major idea behind the analytic approximation approach. Consider the TRESFA model with $T = 2$, we see from (7) that

$$L^i_{s,t} = \frac{4}{\sigma^2}\int_{-\infty}^{\infty} \Phi\left(\frac{-S\lambda\varepsilon_{is}}{\sigma}\right)\Phi\left(\frac{-S\lambda\varepsilon_{it}}{\sigma}\right)\phi\left(\frac{\varepsilon_{is}}{\sigma}\right)\phi\left(\frac{\varepsilon_{it}}{\sigma}\right)\frac{1}{\sigma_w}\phi\left(\frac{w_i}{\sigma_w}\right)dw_i. \tag{8}$$

Define

$$A^* = \frac{4}{\sqrt{8\pi^3}\sigma^2\sigma_w}, \tag{9}$$

then we rewrite $L^i_{s,t}$ in (8) as

$$L^i_{s,t} = A^*\int_{-\infty}^{\infty} e^{-\nu^2/2\sigma_w^2} e^{(-Z_{is}^2+2Z_{is}\nu-\nu^2)/2\sigma^2} e^{(-Z_{it}^2+2Z_{it}\nu-\nu^2)/2\sigma^2}\Phi\left(P_1\nu+Q_1\right)\Phi\left(P_2\nu+Q_2\right)d\nu, \tag{10}$$

where

$$P_1 = \frac{S\lambda}{\sigma},\quad Q_1 = \frac{-S\lambda Z_{is}}{\sigma},\quad P_2 = \frac{S\lambda}{\sigma},\quad Q_2 = \frac{-S\lambda Z_{it}}{\sigma}, \tag{11}$$

and $Z_{it} = y_{it} - \alpha - x_{it}^{\top}\beta$ as defined in (4) which is observable conditional the values of α and β.

The likelihood function in (10) belongs to a subcase of the following more general integral function:

$$L_{s,t}^{i} = A^{*} I_{i,s,t}, \tag{12}$$

where

$$I_{i,s,t} = \int_{-\infty}^{\infty} e^{-q_1\nu^2+q_2\nu+q_3} \Phi\left(P_1\nu + Q_1\right) \Phi\left(P_2\nu + Q_2\right) d\nu, \qquad q_1 > 0, \tag{13}$$

and

$$q_1 = \frac{1}{\sigma^2} + \frac{1}{2\sigma_w^2}, \quad q_2 = (Z_{is} + Z_{it})/\sigma^2, \quad q_3 = -(Z_{is}^2 + Z_{it}^2)/2\sigma^2. \tag{14}$$

The major task of this paper is to derive an analytic formula for the function $I_{i,s,t}$ which might be of interest in its own right, and is kept as the focus of this subsection.

To avoid any difficult implementation of numerical integral, we derive a closed-form approximation for the function in (13). For expositional purposes, we first present the result in (7.1.1) of Abramowitz and Stegun (1970) to define the error function as:

$$erf\left(z\right) \;=\; \frac{2}{\sqrt{\pi}} \int_{0}^{z} e^{-t^2} dt = 2\int_{0}^{\sqrt{2}z} \phi\left(t\right) dt, \tag{15}$$

where

$$erf\left(-x\right) = -erf\left(x\right). \tag{16}$$

The error function is closely related to the cdf of a standard normal distribution.

By (13), we observe that:

$$\begin{aligned} I_{i,s,t} &= \int_{-\infty}^{\infty} e^{-q_1\nu^2+q_2\nu+q_3} \left[\int_{-\infty}^{P_1\nu+Q_1} \phi\left(\zeta\right) d\zeta\right] \left[\int_{-\infty}^{P_2\nu+Q_2} \phi\left(\zeta\right) d\zeta\right] \phi\left(\nu\right) d\nu \\ &= \int_{-\infty}^{\infty} e^{-q_1\nu^2+q_2\nu+q_3} \left[\frac{1}{2} + \frac{1}{2} erf\left(\frac{P_1\nu + Q_1}{\sqrt{2}}\right)\right] \left[\frac{1}{2} + \frac{1}{2} erf\left(\frac{P_2\nu + Q_2}{\sqrt{2}}\right)\right] \phi\left(\nu\right) d\nu. \end{aligned}$$

If the condition $-Q_1/P_1 < -Q_2/P_2$ holds, then ν in (13) is located in three mutually exclusive segments:

$$\nu \in \left(-\infty, -\frac{Q_1}{P_1}\right), \quad \text{or} \quad \nu \in \left[-\frac{Q_1}{P_1}, -\frac{Q_2}{P_2}\right), \quad \text{or} \quad \nu \in \left[-\frac{Q_2}{P_2}, \infty\right), \tag{17}$$

such that the value of $I_{i,s,t}$ is evaluated as:

$$\begin{aligned} I_{i,s,t} &= \int_{-\infty}^{-Q_1/P_1} e^{-q_1\nu^2+q_2\nu+q_3}\left[\frac{1}{2}+\frac{1}{2}erf\left(\frac{P_1\nu+Q_1}{\sqrt{2}}\right)\right]\left[\frac{1}{2}+\frac{1}{2}erf\left(\frac{P_2\nu+Q_2}{\sqrt{2}}\right)\right]\phi\left(\nu\right)d\nu \\ &+ \int_{-Q_1/P_1}^{-Q_2/P_2} e^{-q_1\nu^2+q_2\nu+q_3}\left[\frac{1}{2}+\frac{1}{2}erf\left(\frac{P_1\nu+Q_1}{\sqrt{2}}\right)\right]\left[\frac{1}{2}+\frac{1}{2}erf\left(\frac{P_2\nu+Q_2}{\sqrt{2}}\right)\right]\phi\left(\nu\right)d\nu \\ &+ \int_{-Q_2/P_2}^{\infty} e^{-q_1\nu^2+q_2\nu+q_3}\left[\frac{1}{2}+\frac{1}{2}erf\left(\frac{P_1\nu+Q_1}{\sqrt{2}}\right)\right]\left[\frac{1}{2}+\frac{1}{2}erf\left(\frac{P_2\nu+Q_2}{\sqrt{2}}\right)\right]\phi\left(\nu\right)d\nu \\ &= A_1+B_1+C_1. \end{aligned} \tag{18}$$

Following the idea in Tsay et al. (2009), we approximate the values of A_1, B_1, and C_1 by approximating the value of the error functions appearing in (18) with a nonlinear function. Particularly, Tsay et al. (2009) show that $erf(x)$ can be well approximated with a function of the form $g(x) = 1 - e^{c_1x+c_2x^2}$ if $x \geq 0$:

$$erf(x \geq 0) \sim 1 - e^{c_1x+c_2x^2}, \quad c_1 = -1.0950081470333, \quad \text{and} \quad c_2 = -0.75651138383854. \tag{19}$$

The values of c_1 and c_2 are chosen to make $1 - e^{c_1x+c_2x^2}$ as close to $erf(x)$ as possible. The simulations conducted in Tsay et al. (2009) clearly reveal the power of $1 - e^{c_1x+c_2x^2}$ in approximating $erf(x)$. See Tsay et al. (2009) for the details.

Technically, after substituting the approximating function in (19) for the error functions in (18), we find the integrand in each of A_1, B_1, and C_1 of (18) as a form of $e^{-\left(a\nu^2+2b\nu+c\right)}$. Using (7.4.32) of Abramowitz and Stegun (1970):

$$\int e^{-\left(kx^2+2mx+n\right)}dx = \frac{1}{2}\sqrt{\frac{\pi}{k}}e^{\frac{m^2-kn}{k}}erf\left(\sqrt{k}x+\frac{m}{\sqrt{k}}\right)+C, \quad k \neq 0,$$

where C denotes some finite constant, we obtain an analytic approximation formula for $I_{i,s,t}$.

Theorem 1. *Given that the condition* $-Q_1/P_1 < -Q_2/P_2$ *holds, and defining* $a_1 = P_1$, $b_1 = Q_1$, $a_2 = P_2$, *and* $b_2 = Q_2$, $I_{i,s,t}$ *in (18) can be approximated by* $I^{app}_{i,s,t}(a_1, b_1; a_2, b_2)$ *as such:*

$$
\begin{aligned}
I^{app}_{i,s,t}(a_1, b_1; a_2, b_2) &= \frac{1}{8}\left[1 - sign(a_1)\right]\left[1 - sign(a_2)\right]\sqrt{\frac{\pi}{q_1}}\, e^{\frac{q_2^2/4+q_1q_3}{q_1}} \\
&\times \left[erf\left(\sqrt{q_1}\,\frac{-b_1}{a_1} - \frac{q_2}{2\sqrt{q_1}}\right) + 1\right] \\
&+ \frac{1}{8}\, sign(a_1)\left[1 - sign(a_2)\right]\sqrt{\frac{\pi}{\eta_1}}\, e^{\frac{\eta_3^2-\eta_1\eta_5}{\eta_1}} \\
&\times \left[erf\left(\sqrt{\eta_1}\,\frac{-b_1}{a_1} + \frac{\eta_3}{\sqrt{\eta_1}}\right) + 1\right] \\
&+ \frac{1}{8}\left[1 - sign(a_1)\right] sign(a_2)\sqrt{\frac{\pi}{\eta_2}}\, e^{\frac{\eta_4^2-\eta_2\eta_6}{\eta_2}} \\
&\times \left[erf\left(\sqrt{\eta_2}\,\frac{-b_1}{a_1} + \frac{\eta_4}{\sqrt{\eta_2}}\right) + 1\right] \\
&+ \frac{1}{8}\, sign(a_1) sign(a_2)\sqrt{\frac{\pi}{\eta_1+\eta_2-q_1}}\, e^{\frac{(\eta_3+\eta_4+q_2/2)^2-(\eta_1+\eta_2-q_1)(\eta_5+\eta_6+q_3)}{\eta_1+\eta_2-q_1}} \\
&\times \left[erf\left(\sqrt{\eta_1+\eta_2-q_1}\,\frac{-b_1}{a_1} + \frac{\eta_3+\eta_4+q_2/2}{\sqrt{\eta_1+\eta_2-q_1}}\right) + 1\right] \\
&+ \frac{1}{8}\left[1 + sign(a_1)\right]\left[1 - sign(a_2)\right]\sqrt{\frac{\pi}{q_1}}\, e^{\frac{q_2^2/4+q_1q_3}{q_1}} \\
&\times \left[erf\left(\sqrt{q_1}\,\frac{-b_2}{a_2} - \frac{q_2}{2\sqrt{q_1}}\right) - erf\left(\sqrt{q_1}\,\frac{-b_1}{a_1} - \frac{q_2}{2\sqrt{q_1}}\right)\right] \\
&+ \frac{1}{8}\left[1 + sign(a_1)\right] sign(a_2)\sqrt{\frac{\pi}{\eta_2}}\, e^{\frac{\eta_4^2-\eta_2\eta_6}{\eta_2}} \\
&\times \left[erf\left(\sqrt{\eta_2}\,\frac{-b_2}{a_2} + \frac{\eta_4}{\sqrt{\eta_2}}\right) - erf\left(\sqrt{\eta_2}\,\frac{-b_1}{a_1} + \frac{\eta_4}{\sqrt{\eta_2}}\right)\right] \\
&- \frac{1}{8}\, sign(a_1)\left[1 - sign(a_2)\right]\sqrt{\frac{\pi}{\eta_1}}\, e^{\frac{\eta_7^2-\eta_1\eta_8}{\eta_1}} \\
&\times \left[erf\left(\sqrt{\eta_1}\,\frac{-b_2}{a_2} + \frac{\eta_7}{\sqrt{\eta_1}}\right) - erf\left(\sqrt{\eta_1}\,\frac{-b_1}{a_1} + \frac{\eta_7}{\sqrt{\eta_1}}\right)\right] \\
&- \frac{1}{8}\, sign(a_1) sign(a_2)\sqrt{\frac{\pi}{\eta_1+\eta_2-q_1}}\, e^{\frac{(\eta_7+\eta_4+q_2/2)^2-(\eta_1+\eta_2-q_1)(\eta_8+\eta_6+q_3)}{\eta_1+\eta_2-q_1}}
\end{aligned}
$$

$$
\begin{aligned}
\times \quad & erf\left(\sqrt{\eta_1+\eta_2-q_1}\,\frac{-b_2}{a_2}+\frac{\eta_7+\eta_4+q_2/2}{\sqrt{\eta_1+\eta_2-q_1}}\right)\\
+ \quad & \frac{1}{8}\,sign(a_1)sign(a_2)\sqrt{\frac{\pi}{\eta_1+\eta_2-q_1}}\,e^{\frac{(\eta_7+\eta_4+q_2/2)^2-(\eta_1+\eta_2-q_1)(\eta_8+\eta_6+q_3)}{\eta_1+\eta_2-q_1}}\\
\times \quad & erf\left(\sqrt{\eta_1+\eta_2-q_1}\,\frac{-b_1}{a_1}+\frac{\eta_7+\eta_4+q_2/2}{\sqrt{\eta_1+\eta_2-q_1}}\right)\\
+ \quad & \frac{1}{8}\left[1+sign(a_1)\right]\left[1+sign(a_2)\right]\sqrt{\frac{\pi}{q_1}}\,e^{\frac{q_2^2/4+q_1q_3}{q_1}}\\
\times \quad & \left[1-erf\left(\sqrt{q_1}\,\frac{-b_2}{a_2}-\frac{q_2}{2\sqrt{q_1}}\right)\right]\\
- \quad & \frac{1}{8}\,\left[1+sign(a_1)\right]sign(a_2)\sqrt{\frac{\pi}{\eta_2}}\,e^{\frac{\eta_9^2-\eta_2\eta_{10}}{\eta_2}}\\
\times \quad & \left[1-erf\left(\sqrt{\eta_2}\,\frac{-b_2}{a_2}+\frac{\eta_9}{\sqrt{\eta_2}}\right)\right]\\
- \quad & \frac{1}{8}\,sign(a_1)\left[1+sign(a_2)\right]\sqrt{\frac{\pi}{\eta_1}}\,e^{\frac{\eta_7^2-\eta_1\eta_8}{\eta_1}}\\
\times \quad & \left[1-erf\left(\sqrt{\eta_1}\,\frac{-b_2}{a_2}+\frac{\eta_7}{\sqrt{\eta_1}}\right)\right]\\
+ \quad & \frac{1}{8}\,sign(a_1)sign(a_2)\sqrt{\frac{\pi}{\eta_1+\eta_2-q_1}}\,e^{\frac{(\eta_7+\eta_9+q_2/2)^2-(\eta_1+\eta_2-q_1)(\eta_8+\eta_{10}+q_3)}{\eta_1+\eta_2-q_1}}\\
\times \quad & \left[1-erf\left(\sqrt{\eta_1+\eta_2-q_1}\,\frac{-b_2}{a_2}+\frac{\eta_7+\eta_9+q_2/2}{\sqrt{\eta_1+\eta_2-q_1}}\right)\right],
\end{aligned}
$$

where

$$
\begin{aligned}
\eta_1 &= \frac{2q_1-c_2a_1^2}{2};\\
\eta_2 &= \frac{2q_1-c_2a_2^2}{2};\\
\eta_3 &= \frac{-\sqrt{2}c_2a_1b_1+c_1sign(a_1)a_1-\sqrt{2}q_2}{2\sqrt{2}};\\
\eta_4 &= \frac{-\sqrt{2}c_2a_2b_2+c_1sign(a_2)a_2-\sqrt{2}q_2}{2\sqrt{2}};
\end{aligned}
$$

$$
\begin{aligned}
\eta_5 &= \frac{-\sqrt{2}c_2b_1^2 + 2c_1 sign(a_1)b_1 - 2\sqrt{2}q_3}{2\sqrt{2}}; \\
\eta_6 &= \frac{-\sqrt{2}c_2b_2^2 + 2c_1 sign(a_2)b_2 - 2\sqrt{2}q_3}{2\sqrt{2}}; \\
\eta_7 &= \frac{-\sqrt{2}c_2a_1b_1 - c_1 sign(a_1)a_1 - \sqrt{2}q_2}{2\sqrt{2}}; \\
\eta_8 &= \frac{-\sqrt{2}c_2b_1^2 - 2c_1 sign(a_1)b_1 - 2\sqrt{2}q_3}{2\sqrt{2}}; \\
\eta_9 &= \frac{-\sqrt{2}c_2a_2b_2 - c_1 sign(a_2)a_2 - \sqrt{2}q_2}{2\sqrt{2}}; \\
\eta_{10} &= \frac{-\sqrt{2}c_2b_2^2 - 2c_1 sign(a_2)b_2 - 2\sqrt{2}q_3}{2\sqrt{2}},
\end{aligned}
$$

and P_1, Q_1, P_2, and Q_2 are defined in (11).

The derivation of $I^{app}_{i,s,t}(a_1, b_1; a_2, b_2)$ in Theorem 1 can be requested upon the authors. Note that Theorem 1 is related to the condition $-Q_1/P_1 < -Q_2/P_2$, while on the other hand, if $-Q_1/P_1 \geq -Q_2/P_2$ holds, by symmetry, the likelihood function is calculated with the following Theorem 2.

Theorem 2. *Given that the condition $-Q_1/P_1 \geq -Q_2/P_2$ holds, then $I_{i,s,t}$ in (18) can be approximated by $I^{app}_{i,s,t}(a_1, b_1; a_2, b_2)$ in Theorem 1 with $a_1 = P_2$, $b_1 = Q_2$, $a_2 = P_1$, and $b_2 = Q_1$.*

With Theorems 1 and 2, the approximate joint likelihood function for firm i at periods s and t is computed as:

$$
\widetilde{L}^i_{s,t} = A^* I\left\{\frac{-Q_1}{P_1} < \frac{-Q_2}{P_2}\right\} I^{app}_{i,s,t}(a_1 = P_1, b_1 = Q_1; a_2 = P_2, b_2 = Q_2)
$$

$$+ \quad A^* I \left\{ \frac{-Q_1}{P_1} \geq \frac{-Q_2}{P_2} \right\} I^{app}_{i,s,t}(a_1 = P_2, b_1 = Q_2; a_2 = P_1, b_2 = Q_1), \tag{20}$$

where $I\{.\}$ is the indicator function taking the value one if the statement in the bracket is true and zero otherwise. We are now in a position to derive the log-likelihood function for the entire sample.

Theorem 3. *When $T = 2$, the log-likelihood function for the true random effects stochastic frontier model in (1) for firm i at time s and t is approximated as:*

$$\ln L^{app}_{s,t} = \sum_{i=1}^{N} \ln \widetilde{L}^{i}_{s,t},$$

where $\widetilde{L}^{i}_{s,t}$ is defined in (20).

It is clear that Gaussian quadrature method or simulation-based procedure are not needed when computing $\ln L^{app}_{s,t}$, because the error function $erf(.)$ can be directly calculated with a standard statistic package. As a consequence, the evaluation of the analytic formula is so straightforward that the interested researcher can easily conduct the maximum likelihood estimator based on the proposed method.

2.2 Dealing with TRESFA with $T > 2$

The analytic procedure of evaluating the log-likelihood function of the model in (1) is not feasible when T is large, but we can easily circumvent this difficulty by using the pairwise likelihood principle. The pairwise strategy has been used for the pseudolikelihood methods of Besag (1974) under spatial data, and Heagerty and Lele (1998) for binary spatial data. General results concerning the consistency and asymptotic normality of the PLE can be derived along the lines of the classical proofs in Arnold and Strauss (1991) and Renard et al. (2004).

In this paper we pool all pairwise log-likelihood functions under various combinations of time s and time t for each unit i as:

$$\ln L \sim \sum_{i=1}^{N} \sum_{t \neq s} \ln \widetilde{L}_{s,t}^{i} = \sum_{t \neq s} \ln L_{s,t}^{app}. \tag{21}$$

Clearly, the cost of conducting PLE is limited, because the computational cost of $\widetilde{L}_{s,t}^{i}$ is mild. Furthermore, the PLE can handle the unbalanced panel data naturally in that the pairwise principle is designed to compute all the possible pairwise combinations across time span.

Once all the parameters of the TRESFA model are in hand, we observe from (1) that

$$\varepsilon_{i,t} = w_i + v_{i,t} - S u_{i,t}. \tag{22}$$

At time t, we note that

$$\varepsilon_{i,t} = e_{i,t} - S u_{i,t}, \quad e_{i,t} \sim N[0, \sigma_w^2 + \sigma_v^2], \quad u_{i,t} \sim |N[0, \sigma_u^2]|, \tag{23}$$

so we can use the method of Jondrow et al. (1982) to extract the inefficiency part of the composite error as $E(u_{i,t}|\varepsilon_{i,t})$ for firm i at time t. To rank the efficiency level across firms, we might use the average of $E(u_{i,t}|\varepsilon_{i,t})$ across t.

3 A true fixed effects stochastic frontier model

This section considers the estimation of the true fixed effects SFA models as of the form in (3):

$$y_{it} = \alpha + x_{it}^{\top}\beta + w_i + \varepsilon_{it} = \alpha_i + x_{it}^{\top}\beta + \varepsilon_{it}. \tag{24}$$

The major advantage of the fixed effects setting over the random effects counterpart is that w_i in (24) is not assumed to be independent of the regressors $x_{i,t}$ anymore. On the other hand, if we try to eliminate the impacts of fixed effects on the estimation of the remaining parameters by using the pairwise differencing or the first differencing operators, then we cannot estimate the parameters related to time-invariant regressors. For ease of exposition, from now on, we assume $x_{i,t}$ is time-varying throughout this paper.

To extract more information from the TFESFA data, we suggest using a pairwise differencing operator as follows:

$$y_{i,s} - y_{i,t} = (x_{i,s} - x_{i,t})^\top \beta + (\varepsilon_{i,s} - \varepsilon_{i,t}). \quad s \neq t. \tag{25}$$

For a data with a time span T, the possible pairwise difference is $T(T-1)/2$.

The coefficient β and the parameters characterizing the composite error $\varepsilon_{i,t}$ can be estimated with likelihood-based estimator if the pdf of $\varepsilon_{i,s} - \varepsilon_{i,t}$ is known. To fulfill this purpose, we observe that the joint density function of $\varepsilon_{i,s}$ and $\varepsilon_{i,t}$ is the product of their individual densities because of their mutual independence,

$$f(\varepsilon_{i,s}, \varepsilon_{i,t}) = \frac{4}{\sigma^2} \phi\left(\frac{\varepsilon_{i,s}}{\sigma}\right) \Phi\left(\frac{-S\lambda\varepsilon_{i,s}}{\sigma}\right) \phi\left(\frac{\varepsilon_{i,t}}{\sigma}\right) \Phi\left(\frac{-S\lambda\varepsilon_{i,t}}{\sigma}\right). \tag{26}$$

Using (26) and making the transformation

$$h^i_{s,t} = \varepsilon_{i,s} - \varepsilon_{i,t}, \tag{27}$$

the joint density function of $\varepsilon_{i,t}$ and $h^i_{s,t}$ is

$$f(\varepsilon_{i,t}, h^i_{s,t}) = \frac{4}{\sigma^2} \phi\left(\frac{h^i_{s,t} + \varepsilon_{i,t}}{\sigma}\right) \Phi\left(\frac{-S\lambda(h^i_{s,t} + \varepsilon_{i,t})}{\sigma}\right) \phi\left(\frac{\varepsilon_{i,t}}{\sigma}\right) \Phi\left(\frac{-S\lambda\varepsilon_{i,t}}{\sigma}\right). \tag{28}$$

Thus, the marginal density function of $h^i_{s,t}$ is computed by

$$f(h^i_{s,t}) = \frac{4}{\sigma^2} \int_{-\infty}^{\infty} \phi\left(\frac{h^i_{s,t} + \varepsilon_{i,t}}{\sigma}\right) \Phi\left(\frac{-S\lambda(h^i_{s,t} + \varepsilon_{i,t})}{\sigma}\right) \phi\left(\frac{\varepsilon_{i,t}}{\sigma}\right) \Phi\left(\frac{-S\lambda\varepsilon_{i,t}}{\sigma}\right) d\varepsilon_{i,t}. \tag{29}$$

Without loss of clarity, we can write above marginal density function as:

$$f(h) = \frac{4}{\sigma^2} \int_{-\infty}^{\infty} \phi\left(\frac{h + \varepsilon}{\sigma}\right) \Phi\left(\frac{-S\lambda(h + \varepsilon)}{\sigma}\right) \phi\left(\frac{\varepsilon}{\sigma}\right) \Phi\left(\frac{-S\lambda\varepsilon}{\sigma}\right) d\varepsilon, \tag{30}$$

The integral function in (30) admits the following familiar form:

$$f(h) = \frac{2}{\pi\sigma^2} \int_{-\infty}^{\infty} e^{(-h^2 - 2h\nu - \nu^2)/2\sigma^2} e^{(-\nu^2)/2\sigma^2} \Phi\left(P_1\nu + Q_1\right) \Phi\left(P_2\nu\right) d\nu, \tag{31}$$

or

$$f(h) = \frac{2}{\pi\sigma^2} \int_{-\infty}^{\infty} e^{-\nu^2/\sigma^2 - h\nu/\sigma^2 - h^2/2\sigma^2} \Phi\left(P_1\nu + Q_1\right) \Phi\left(P_2\nu\right) d\nu, \tag{32}$$

where

$$P_1 = \frac{-S\lambda}{\sigma}, \quad Q_1 = \frac{-S\lambda h}{\sigma}, \quad P_2 = \frac{-S\lambda}{\sigma}. \tag{33}$$

That is, the function in (32) can be recast as:

$$f(h) = \frac{2}{\pi\sigma^2} \int_{-\infty}^{\infty} e^{-q_1\nu^2 + q_2\nu + q_3} \Phi\left(P_1\nu + Q_1\right) \Phi\left(P_2\nu\right) d\nu, \qquad q_1 > 0, \tag{34}$$

and

$$q_1 = \frac{1}{\sigma^2}, \quad q_2 = \frac{-h}{\sigma^2}, \quad q_3 = \frac{-h^2}{2\sigma^2}. \tag{35}$$

By appealing the results in Theorems 1, 2, and 3, we can estimate the parameters (β, σ_u, and σ_v) of the TFESFA model by pooling all the possible pairwise differencing combinations within each firm i.

Pairwise differencing makes the direct estimation of the fixed effect, α_i, impossible. However, it can be estimated as:

$$\widehat{\alpha}_{i,TFE} = \overline{y}_i - \overline{x}_i^\top \widehat{\beta}_{TFE} + \sqrt{\frac{2}{\pi}}\,\widehat{\sigma}_u, \tag{36}$$

where $\overline{y}_i$ and $\overline{x}_i$ are within group sample means of the dependent variable and the stochastic regressors, respectively, while $\widehat{\beta}_{TFE}$ denotes the coefficient estimate from the first stage estimation based on our analytic formula. $\widehat{\alpha}_{i,TFE}$ can be used to rank the relative efficiency level across firms via the Schmidt and Sickles' (1984) method, i.e., the firm specific inefficiency is measured as a deviation from the benchmark level:

$$\widehat{u}_{i,TFE} = \max_i(\widehat{\alpha}_{i,TFE}) - \widehat{\alpha}_{i,TFE} \geq 0. \tag{37}$$

The accuracy of $\widehat{\alpha}_{i,TFE}$ depends on the magnitude of T. It is clear that our method is a two-stage approach as compared to the one-step simulated MLE procedure of Greene (2005) who proposes using brute force computation to estimate all the fixed effects parameters along with the other ones. Nevertheless, the estimate of β using the pairwise differencing method is not affected by the incidental parameters problems especially when N is huge.

4 Monte Carlo experiment

This section investigates the finite sample performance of the MLE (or PLE) for both true random effects and true fixed effects SFA models. Following Olson et al. (1980, p. 76), we consider a set of experiments with two-regressors model:

$$y_{it}^l = \alpha + \beta_1 x_{it}^l + w_i^l + v_{it}^l - u_{it}^l, \quad u_{it} = |U_{it}| \tag{38}$$

where x_{it}'s are generated from standard normal distribution, and l denotes the l-th replication of the data.

All the programs are written in GAUSS. Two hundred additional values are generated in order to obtain random starting values. The optimization algorithm used to implement the MLE is the quasi-Newton algorithm of Broyden, Fletcher, Goldfarb, and Shanno (BFGS) contained in the GAUSS MAXLIK library. The maximum number of iterations for each replication is 100.

4.1 Gaussian quadrature versus analytic approximation for TRESFA model

It is argued in this paper that the TRESFA model can be estimated with the Gaussian quadrature procedure, thus, it serves as a benchmark for the proposed analytic approximation method. Moreover, when $T = 2$, the full likelihood function can be evaluated with both methods, so we can assess the accuracy of the analytic formula via the case $T = 2$. The simulation results are contained in Table 1 and Table 2 where the true parameter values are:

$$\xi = (\alpha, \beta_1, \sigma_u, \sigma_v, \sigma_w)^\top, \quad \alpha = \beta_1 = 1, \quad \sigma_w = 1, \tag{39}$$

and

$$\{\sigma_u = 2, \sigma_v = 1\}, \quad \text{or} \quad \{\sigma_u = 0.5, \sigma_v = 0.25\}, \quad \text{or} \quad \{\sigma_u = 0.4, \sigma_v = 0.2\}. \tag{40}$$

Note that we estimate the parameters ξ in (39) with the following transformation function:

$$\xi = (\alpha, \beta_1, \sigma_u, \sigma_v, \sigma_w)^\top = \kappa(\widetilde{\xi}), \tag{41}$$

where

$$\widetilde{\xi} = (\alpha, \beta_1, \ln(\sigma_u), \ln(\sigma_v), \ln(\sigma_w))^\top, \tag{42}$$

are the parameters really estimated when conducting the MLE. In order to create a realistic simulation scheme, the inverse function of the preceding transformation function calculated at the true parameter value plus an extra (5×1) random vector generated from $N[0, 1/3]$ is used as the initial values for the MLE procedure, i.e., the initial value of $\widetilde{\xi}$ is:

$$\widetilde{\xi}_0 = \kappa(\xi)^{-1} + N[0, 1/3]. \tag{43}$$

In order to maintain the accuracy of quadrature approach, we use 40 points to evaluate the Gaussian quadrature. The first 1000 replication of normal convergence are recorded for numerical analysis.

The results indicate that the size performance based on the analytic formula is very similar to that of quadrature method for all configurations considered in Table 1. We also find that the bias from both estimation methods is very small. Furthermore, Table 2 demonstrates that the finite sample performance of the MLE based on both approaches are very satisfactory, because the RMSE from these methods decrease with the increasing sample size N, indicating that both estimators possesses well-defined asymptotic behaviors. All these findings support the good performance of the analytic formula and the quadrature method in dealing with the TRESFA model when $T = 2$, even though it was mentioned by Greene (2005) that close-form solution does not exist for this interesting model.

One interesting observation from Lee (2000) is that the typical numerical-integral procedure suggested by Butler and Moffitt (1982) for the random effects probit model becomes biased when the correlation coefficient within each unit (ρ) is relatively large. Since the random effects w_i might induce a high correlation coefficient within firm or cluster i of the

TRESFA model, accordingly, we might find it unsatisfactory in applying the quadrature approach to the TRESFA models under certain circumstances. To shed more light on this estimation issue, we investigate the performance of the quadrature method in Table 3 under the setup that ρ is relatively large, i.e.,

$$\{\sigma_u = 0.4, \sigma_v = 0.2\}. \tag{44}$$

The correlation between individuals within a group is $\rho \approx 0.9106$ under the desgin in (44). Note that we use $N = 1000$ and 200 replications for the experiments in Table 3. The choice of $N = 1000$ is to ensure that the performance of the quadrature method thus derived is not due to the small sample size. The use of 200 replications is reasonable when the number of cross units is relatively large.

Before discussing the results in Table 3, we emphasize here that, when $T > 2$, the estimation based on quadrature method is still equivalent to the full MLE of the TRESFA models. The RMSE from the PLE with the analytic formula is thus expected to be larger than those from the MLE generated by the quadrature approach. This conjecture, however, is not borne out in Table 3. Indeed, the PLE is found to possess a well-defined asymptotic behavior, because the associated RMSE decreases with the increasing sample size T. On the contrary, we cannot observe a similar pattern from the quadrature method. This observation leads us to further check the performance of the quadrature procedure when T is relatively large.

Table 4 displays the performance of the quadrature method under the same data-generating processes (DGP) used in Table 3, but T is raised to be 10, 20, and 30, respectively. First, we find the bias performance from the quadrature method remains satisfactory, however, the resulting RMSE from estimating the parameters σ_v and σ_w do not decrease with an increasing value of T. This finding is opposite to what normal asymptotic theory would imply, nevertheless, it is in line with the finding of Borjas and Sueyoshi (1994) that some numerical difficulties occurs when they apply the quadrature technique to study probit models with structural group effects where the number of individuals in a group was large. In other words, the quadrature method cannot be used for the TRESFA model without modification, especially when T is large.

4.2 PLE with analytic formula for TRESFA models

This subsection demonstrates that the PLE based on the proposed analytic formula is useful to estimate the TRESFA model by exploring more experiments with different choice of $\lambda = \sigma_u/\sigma_v$. Table 5 considers the model with the following parameter values:

$$\xi = (\alpha, \beta_1, \sigma_u, \sigma_{v,\sigma_w})^\top, \quad \alpha = \beta_1 = 1, \quad \sigma_u = \sigma_w = 1, \quad \sigma_v = \{1/2, 1, 3/2\}. \tag{45}$$

The result show that the analytic formula is computationally efficient in that it easily handles the simulations with a sample size of 800 and 1000 replications. Table 5 also reveals that the performance of the MLE improves with the value λ. This phenomenon is typically found in the stochastic frontier literature, because the identification of σ_u hinges on the asymmetric distribution of the composite error. When λ is smaller, the larger noise generated from the symmetric disturbance v_{it} will make the identification of the model more difficult and will result in a larger RMSE of the estimators.

We further investigate the performance of PLE by considering 3 additional choice of σ_w:

$$\sigma_w = \{0.5, 0.7, 1.3\}, \tag{46}$$

under 3 different value of T:

$$T = \{2, 4, 8\}, \tag{47}$$

and

$$\alpha = \beta_1 = 1, \quad \sigma_u = 1, \quad \sigma_v = 0.5. \tag{48}$$

The simulations are contained in Tables 6, 7, and 8. As expected, the PLE works well for these TRESFA model. The changing patterns found in these tables resemble closely to those found in Tables 1, 2, and 5, i.e., the performance of the PLE improves with the increase of T or N.

4.3 Analytic formula for TFESFA models

This subsection investigates the finite sample performance of the PLE based on pairwise differencing and our analytic formula under the following TFESFA model:

$$\{\sigma_u = 1, \quad \sigma_v = 0.5\}, \quad \text{or} \quad \{\sigma_u = 2, \quad \sigma_v = 1\}, \quad \sigma_w = 1.5, \quad \alpha = \beta_1 = 1, \tag{49}$$

with 3 different value of T:

$$T = \{2, 4, 8\}. \tag{50}$$

As discussed previously, pairwise differencing makes the estimation of time-invariant parameters infeasible. Nevertheless, provided that $x_{i,t}$ is time-varying and w_i is mean zero, we can estimate the global intercept as:

$$\widehat{\alpha}_{TFE} = \overline{y} - \overline{x}^{\top}\widehat{\beta}_{TFE} + \sqrt{\frac{2}{\pi}}\,\widehat{\sigma}_u. \tag{51}$$

The last item at the right side of the above equality derives from the population mean of a half-normal distribution, and $\overline{y}$ and $\overline{x}$ denotes the sample average of the dependent variable and independent variables, respectively. The simulation findings are contained in Table 9.

The major feature of the results in Table 9 is similar to what we observe for the TRESFA models. The analytic formula work very well for the TFESFA model in that the bias is small, and the RMSE of the PLE improves with the value of T across different model specifications as well.

5 Empirical application to WHO health attainment

This section applies the TRESFA model to the data used in Evans et al. (2000a,b) (ETML) and Greene (2003) about the World Health Organization's (WHO) panel data on national health care systems. The WHO 2000 report is a worldwide assessment of the effectiveness of health care delivery. This study contains a rankings about the productive efficiency of the health care systems of 191 countries based on the observations 1993-1997. As clearly pointed out in ETML (2000a,b) and Greene (2003), the rankings were produced using a form of the "fixed effects" stochastic frontier methodology proposed by Schmidt and Sickles (1984).

Greene (2003) emphasizes that one criticism of the fixed effects methodology used for the WHO 2000 report is that the model fails to distinguish between cross country heterogeneity unrelated to inefficiency and the inefficiency itself. This findings motivates Greene (2003) to propose the TRESFA model and TFESFA models. Thus, it is natural for us to apply the proposed methodology to the WHO data.

Two measures of health care attainment were analyzed for the WHO panel data, disability adjusted life expectancy (DALE) and a composite measure of health care delivery (COMP). Because several econometric studies have placed the first (DALE) study under narrower scrutiny, we also use DALE for empirical study.

It is shown in Gravelle et al. (GJJS) (2002a,b) that 51 out of 191 countries are observed for only one year (1997). The results from the method of Schmidt and Sickles (1984) are based on 140 countries only. Thus, the findings of this section are generated from 700 observations spanning 1993-1997 of these 140 countries, because the method of Schmidt and Sickles (1984) is used as the benchmark for comparison.

The production function considered in Schmidt and Sickles (1984) is denoted

$$y_{it} = \alpha + x_{it}^{\top}\beta + v_{it} - u_i, \quad i = 1, 2, \dots, N \quad \text{and} \quad t = 1, \dots, T, \tag{52}$$

i.e., σ_w is assumed to be zero, and the level of inefficiency is time-invariant. The preceding model is rewritten as:

$$y_{it} = (\alpha - u_i) + x_{it}^{\top}\beta + v_{it} = \alpha_i + x_{it}^{\top}\beta + v_{it}. \tag{53}$$

Under the condition that $v_{i,t}$ is uncorrelated with other components of the model, Schmidt and Sickles (1984) suggest using within estimator for the model in (53), and the country specific inefficiency is measured as a deviation from the benchmark level:

$$\widehat{u}_i = \max_i(\widehat{\alpha}_i) - \widehat{\alpha}_i \geq 0. \tag{54}$$

The advantage of Schmidt and Sickles' (1984) method is that no parametric assumption of the disturbance term is required. Nevertheless, their model does not distinguish between heterogeneity and inefficiency.

Table 10 presents the descriptive statistics used for this section. Following ETML (2000a,b) and Greene (2003), the variables HEXP, EDUC, and the square of EDUC are included in the explanatory variables. To single out that the random effects model specification is capable of dealing with time-invariant regressors, we include the variables TROPICS and GDPC into the TRESFA models. The results are contained in Table 11.

We find that the sign of the estimates related to health expenditure, education, and the square of education remains robust across the method of Schmidt and Sickles (1984) and ours. Indeed, the magnitude of these estimates are found to be close to each other for both methods. In addition, the variable EDUC remains significant at the 5% level across different estimation methods. The major difference lies on the finding that the square of EDUC are highly significant for the likelihood-method only.

Table 11 also shows that the estimated σ_w is much larger than the estimated σ_u and σ_v, indicating that heterogeneity is strongly evident in the WHO's panel data. This finding is closely related to the the observations in GJJS (2002) that 99.8% of the variation in the log of the DALE variable is between, rather than within the groups (countries). Nevertheless, this message cannot be obtained from the method of Schmidt and Sickle (1984), because their model do not allow the existence of heterogeneity.

6 Conclusions

We consider the estimation issues of the true random effects and the true fixed effects SFA models of Greene (2005). Since the estimation based on quadrature method is equivalent to the full MLE of the TRESFA model, we first evaluate the effectiveness of Gaussian quadrature via Monte Carlo experiments. The simulations reveal that the performance of the quadrature procedure is not reliable when T is relatively large and the within group correlation is strong as found in the panel probit literature.

On the other hand, the proposed analytic approximation method can easily deal with the models of Greene (2005). The novelty of the proposed method is that the MLE can be implemented without resorting to a numerical integral or a simulation-based technique. Furthermore, the analytic strategy can be easily carried out with standard statistics packages, and its implementation for the likelihood estimation is found to be stable for the experiments conducted in this paper. The simulations confirm the promising performance of the analytic method for both the TRESFA and the TFESFA models. Since the structure of panel data is identical to that of clustering or grouped cross-sectional data, our methodology is useful to the studies based on this type of data as well.

This paper also applies the TRESFA model to the data used in Evans et al. (2000a,b) and Greene (2003) about the World Health Organization's (WHO) panel data on national health care systems. The estimates are robust across the method of Schmidt and Sickles (1984) and ours, revealing the potentials of using our methodology for the WHO panel health data.

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Table 1. Maximum Likelihood Estimation for the True Random Effects SFA Model: $\alpha = \beta_1 = 1$, $\sigma_w = 1$, and $T = 2$

N	Bias (Analytic)					Bias (Quadrature)				
	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
					$\sigma_u = 2$ and $\sigma_v = 1$					
100	0.0994	-0.0046	0.1266	0.0131	0.0350	0.1010	-0.0045	0.1291	0.0100	0.0339
200	0.0419	0.0003	0.0487	0.0080	0.0149	0.0430	0.0002	0.0506	0.0056	0.0149
400	0.0108	-0.0006	0.0102	0.0081	0.0075	0.0122	-0.0006	0.0126	0.0055	0.0075
800	0.0006	-0.0006	-0.0003	0.0098	0.0038	0.0021	-0.0006	0.0022	0.0071	0.0038
					$\sigma_u = 0.5$ and $\sigma_v = 0.25$					
100	0.1075	-0.0026	0.1372	-0.0059	0.0086	0.0834	-0.0026	0.1094	0.0088	0.0061
200	0.0755	-0.0001	0.0928	0.0003	0.0043	0.0698	-0.0003	0.0847	0.0026	0.0030
400	0.0456	0.0000	0.0551	0.0012	0.0021	0.0492	0.0000	0.0580	-0.0048	0.0019
800	0.0224	-0.0003	0.0264	0.0019	0.0010	0.0251	-0.0004	0.0299	-0.0019	0.0010
					$\sigma_u = 0.4$ and $\sigma_v = 0.2$					
100	0.0932	-0.0012	0.1181	-0.0071	0.0088	0.0600	-0.0013	0.0804	0.0067	0.0050
200	0.0720	-0.0003	0.0896	-0.0033	0.0041	0.0676	-0.0005	0.0846	-0.0116	0.0026
400	0.0495	0.0000	0.0605	-0.0018	0.0023	0.0669	0.0000	0.0816	-0.0226	0.0020
800	0.0226	-0.0002	0.0274	0.0016	0.0012	0.0489	-0.0002	0.0609	-0.0230	0.0014

Notes: All the results are based on 1000 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (39), and (40).

Table 2. Maximum Likelihood Estimation for the True Random Effects SFA Model: $\alpha = \beta_1 = 1$, $\sigma_w = 1$, and $T = 2$

N	RMSE (Analytic)					RMSE (Quadrature)				
	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
					$\sigma_u = 2$ and $\sigma_v = 1$					
100	0.4784	0.1272	0.5834	0.3016	0.1974	0.4804	0.1273	0.5865	0.2962	0.1926
200	0.3035	0.0902	0.3653	0.2117	0.1328	0.3033	0.0902	0.3650	0.2106	0.1328
400	0.1917	0.0622	0.2246	0.1483	0.0913	0.1920	0.0622	0.2250	0.1480	0.0913
800	0.1307	0.0440	0.1515	0.1038	0.0617	0.1309	0.0440	0.1519	0.1034	0.0617
					$\sigma_u = 0.5$ and $\sigma_v = 0.25$					
100	0.2488	0.0385	0.2857	0.1256	0.0759	0.2507	0.0402	0.2750	0.1240	0.0757
200	0.2041	0.0278	0.2402	0.1139	0.0541	0.2078	0.0288	0.2399	0.1066	0.0541
400	0.1558	0.0189	0.1841	0.0935	0.0393	0.1583	0.0192	0.1854	0.0820	0.0393
800	0.1096	0.0135	0.1297	0.0665	0.0262	0.1109	0.0135	0.1308	0.0621	0.0263
					$\sigma_u = 0.4$ and $\sigma_v = 0.2$					
100	0.2121	0.0311	0.2357	0.1026	0.0731	0.2109	0.0340	0.2153	0.0926	0.0734
200	0.1796	0.0224	0.2076	0.0942	0.0525	0.1826	0.0229	0.1987	0.0748	0.0526
400	0.1469	0.0153	0.1709	0.0805	0.0384	0.1551	0.0153	0.1753	0.0634	0.0383
800	0.1008	0.0109	0.1184	0.0591	0.0256	0.1158	0.0110	0.1322	0.0510	0.0257

Notes: All the results are based on 1000 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (39), and (40).

Table 3. Maximum Likelihood Estimation for the True Random Effects SFA Model
$\alpha = \beta_1 = 1$, $\sigma_u = 0.4$, $\sigma_v = 0.2$, $\sigma_w = 1$, and $N = 1000$

	Bias					RMSE				
$T = 3$	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
					Analytic					
	0.0066	0.0001	0.0122	-0.0017	-0.0007	0.0557	0.0071	0.0615	0.0347	0.0228
					Quadrature					
	0.0175	0.0000	0.0234	-0.0172	-0.0016	0.0600	0.0073	0.0379	0.0241	0.0230
$T = 4$										
					Analytic					
	0.0117	0.0001	0.0101	-0.0028	0.0010	0.0526	0.0056	0.0486	0.0313	0.0215
					Quadrature					
	0.0224	0.0000	0.0169	-0.0181	-0.0021	0.0822	0.0056	0.0265	0.0216	0.0221

Notes: All the results are based on 200 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (39), and (44).

Table 4. MLE for the True Random Effects SFA Model using Gaussian Quadrature
$\alpha = \beta_1 = 1$, $\sigma_u = 0.4$, $\sigma_v = 0.2$, $\sigma_w = 1$, and $N = 1000$

	Bias					**RMSE**				
	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
$T = 10$	-0.0047	-0.0001	0.0071	-0.0266	-0.0331	0.2567	0.0033	0.0125	0.0272	0.0708
$T = 20$	-0.0065	-0.0001	0.0036	-0.0316	-0.0970	0.4619	0.0023	0.0077	0.0318	0.1678
$T = 30$	0.0201	0.0000	0.0017	-0.0331	-0.1245	0.5267	0.0020	0.0060	0.0333	0.2116

Notes: All the results are based on 200 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (39), and (44).

Table 5. PLE for the True Random Effects SFA Model:
$\alpha = \beta_1 = 1$, $\sigma_w = 1$, and $T = 2$

N	Bias					RMSE				
	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
					$\sigma_u = 1$ and $\sigma_v = 1/2$					
100	0.1224	-0.0023	0.1546	0.0086	0.0133	0.3730	0.0720	0.4498	0.2154	0.0968
200	0.0707	-0.0003	0.0861	0.0068	0.0054	0.2766	0.0515	0.3339	0.1728	0.0680
400	0.0290	-0.0004	0.0335	0.0052	0.0026	0.1776	0.0350	0.2116	0.1225	0.0492
800	0.0138	-0.0007	0.0158	0.0003	0.0022	0.1101	0.0251	0.1292	0.0817	0.0343
					$\sigma_u = 1$ and $\sigma_v = 1$					
100	0.1235	-0.0049	0.1572	0.0525	0.0155	0.5405	0.1026	0.6625	0.2390	0.1359
200	0.1341	-0.0001	0.1658	0.0126	0.0089	0.4664	0.0715	0.5767	0.1663	0.0939
400	0.0979	-0.0011	0.1222	0.0049	0.0043	0.3930	0.0495	0.4846	0.1309	0.0668
800	0.0758	-0.0009	0.0920	0.0006	0.0021	0.3241	0.0357	0.3993	0.1060	0.0476
					$\sigma_u = 1$ and $\sigma_v = 3/2$					
100	0.0558	-0.0045	0.0746	0.0907	0.0319	0.6781	0.1352	0.8311	0.2698	0.2197
200	0.0961	0.0003	0.1180	0.0476	0.0162	0.5925	0.0940	0.7308	0.1979	0.1420
400	0.0986	-0.0006	0.1208	0.0279	0.0084	0.5296	0.0657	0.6545	0.1559	0.0988
800	0.1085	-0.0012	0.1327	0.0141	0.0032	0.4764	0.0462	0.5855	0.1264	0.0684

Notes: All the results are based on 1000 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38) and (45).

Table 6. PLE for the True Random Effects SFA Model:
$\alpha = \beta_1 = 1$, $\sigma_u = 1$, $\sigma_v = 1/2$, and $\sigma_w = 1/2$

	Bias					**RMSE**				
$T = 2$	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
$N = 50$	0.0976	-0.0056	0.1267	0.0174	0.0380	0.3494	0.0883	0.4232	0.2061	0.1518
$N = 100$	0.0562	-0.0023	0.0713	0.0069	0.0166	0.2527	0.0636	0.3085	0.1557	0.0991
$N = 150$	0.0275	-0.0011	0.0324	0.0073	0.0088	0.1773	0.0505	0.2141	0.1248	0.0774
$T = 4$										
$N = 50$	0.0468	0.0021	0.0564	0.0048	0.0209	0.2375	0.0598	0.2756	0.1396	0.0866
$N = 100$	0.0213	-0.0001	0.0263	0.0003	0.0110	0.1503	0.0417	0.1728	0.0966	0.0592
$N = 150$	0.0089	-0.0013	0.0117	0.0022	0.0087	0.1089	0.0340	0.1237	0.0754	0.0489
$T = 8$										
$N = 50$	0.0145	-0.0007	0.0161	0.0049	0.0169	0.1486	0.0429	0.1572	0.0892	0.0686
$N = 100$	0.0042	-0.0006	0.0038	0.0050	0.0101	0.1009	0.0306	0.1040	0.0616	0.0465
$N = 150$	0.0027	-0.0004	0.0012	0.0046	0.0063	0.0783	0.0248	0.0825	0.0497	0.0372

Notes: All the results are based on 1000 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (46), (47), and (48).

Table 7. PLE for the True Random Effects SFA Model:
$\alpha = \beta_1 = 1$, $\sigma_u = 1$, $\sigma_v = 1/2$, and $\sigma_w = 0.7$

	Bias					**RMSE**				
$T = 2$	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
$N = 50$	0.1338	-0.0054	0.1741	0.0112	0.0291	0.3965	0.0931	0.4773	0.2253	0.1292
$N = 100$	0.0850	-0.0026	0.1074	0.0081	0.0133	0.3088	0.0677	0.3769	0.1847	0.0893
$N = 150$	0.0460	-0.0015	0.0556	0.0092	0.0070	0.2271	0.0538	0.2738	0.1533	0.0718
$T = 4$										
$N = 50$	0.0691	0.0023	0.0837	0.0048	0.0208	0.2885	0.0620	0.3313	0.1641	0.0958
$N = 100$	0.0326	-0.0001	0.0404	-0.0007	0.0111	0.1854	0.0431	0.2101	0.1152	0.0666
$N = 150$	0.0149	-0.0011	0.0195	0.0014	0.0091	0.1345	0.0353	0.1495	0.0893	0.0549
$T = 8$										
$N = 50$	0.0240	-0.0008	0.0275	0.0047	0.0187	0.1875	0.0444	0.1942	0.1058	0.0844
$N = 100$	0.0091	-0.0006	0.0093	0.0046	0.0112	0.1268	0.0316	0.1269	0.0733	0.0571
$N = 150$	0.0056	-0.0003	0.0042	0.0045	0.0071	0.0962	0.0256	0.0965	0.0590	0.0459

Notes: All the results are based on 1000 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (46), (47), and (48).

Table 8. PLE for the True Random Effects SFA Model:
$\alpha = \beta_1 = 1$, $\sigma_u = 1$, $\sigma_v = 1/2$, and $\sigma_w = 1.3$

	Bias					**RMSE**				
$T = 2$	α	β_1	σ_u	σ_v	σ_w	α	β_1	σ_u	σ_v	σ_w
$N = 50$	0.2123	-0.0049	0.2789	0.0056	0.0321	0.5087	0.1019	0.5953	0.2672	0.1578
$N = 100$	0.1564	-0.0026	0.1971	0.0058	0.0146	0.4224	0.0745	0.5030	0.2356	0.1110
$N = 150$	0.1187	-0.0012	0.1455	0.0078	0.0081	0.3615	0.0592	0.4301	0.2156	0.0890
$T = 4$										
$N = 50$	0.1318	0.0029	0.1612	0.0028	0.0259	0.4108	0.0641	0.4509	0.2130	0.1450
$N = 100$	0.0811	0.0001	0.1014	-0.0057	0.0143	0.3052	0.0443	0.3405	0.1607	0.1021
$N = 150$	0.0451	-0.0009	0.0580	-0.0032	0.0116	0.2286	0.0364	0.2509	0.1272	0.0836
$T = 8$										
$N = 50$	0.0626	-0.0008	0.0743	0.0013	0.0268	0.3098	0.0453	0.3046	0.1448	0.1404
$N = 100$	0.0296	-0.0007	0.0330	0.0015	0.0159	0.2045	0.0322	0.1970	0.1036	0.0958
$N = 150$	0.0183	-0.0002	0.0177	0.0023	0.0103	0.1559	0.0259	0.1411	0.0834	0.0771

Notes: All the results are based on 1000 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (46), (47), and (48).

Table 9. PLE for the True Fixed Effects SFA Model
$\alpha = \beta_1 = 1$, $\sigma_w = 1.5$, and $N = 1000$

	Bias				RMSE			
	α	β_1	σ_u	σ_v	α	β_1	σ_u	σ_v
				$\sigma_u = 1$ and $\sigma_v = 0.5$				
$T = 2$	0.1445	-0.0003	0.1760	-0.0225	0.3447	0.0235	0.4250	0.1860
$T = 4$	0.0434	0.0003	0.0517	-0.0138	0.1613	0.0135	0.1996	0.0928
$T = 8$	0.0271	0.0008	0.0334	-0.0144	0.1015	0.0093	0.1140	0.0585
				$\sigma_u = 2$ and $\sigma_v = 1$				
$T = 2$	0.2688	0.0003	0.3325	-0.0426	0.6567	0.0469	0.8182	0.3670
$T = 4$	0.0847	0.0005	0.1029	-0.0275	0.3169	0.0269	0.3973	0.1855
$T = 8$	0.0533	0.0016	0.0669	-0.0288	0.1861	0.0186	0.2282	0.1169

Notes: All the results are based on 200 replications. Bias is computed as the true parameter values minus the average estimated values. The simulated data are defined in (38), (49), and (50). The estimate for α is based on the formula in (51).

Table 10. Descriptive Statistics for Variables, 1997 Observations

	Mean	**Std. Dev.**
Variable		
Dale (Disability adjusted life expectancy)	56.83	12.29
HEXP (Health expenditures per capita in 1997 PPP$)	445.37	616.36
EDUC (Average year of schooling)	6.00	2.62
TROPICS (Dummy variable for tropical location)	0.508	0.501
DGPC (Per capita GDP in 1997 PPP$)	6609.4	7614.8

Notes: Data are taken from Greene (2003).

Table 11. Health Care Outcome Analysis

	Schmidt-Sickles' (1984) Method	TRESFA
Variable		
HEXP	0.1342 (3.433)	0.1177 (1.331)
EDUC	2.2111 (4.897)	4.1358(7.128)
EDUC^2	-0.0344 (0.859)	-0.1801(3.238)
DGPC	-	0.4708 (3.308)
TROPICS	-	-3.7600 (2.602)
Constant	-	37.8239 (2.221)
σ_u	-	0.0009 (0.000)
σ_v	-	0.4795 (24.164)
σ_w	-	6.6862 (16.282)

Notes: Data are defined in Table 10. DALE is the dependent variable. HEXP is divieded by 100, and GDPC is divided by 1000 before estimation. The number in parathesis denotes the absolute value of t ratio statistic. The t ratio of the TRESFA model is computed from the covariance matrix estimators outlined in (5) and (6) of Kuk and Nott (2000).

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