QUANTUM BLOWUPS AND x-REGULARITY

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ABSTRACT

In this talk I will discuss quantum cohomology of blowups based on a joint project with Yuan-Pin Lee and Hui-Wen Lin.

Let $\phi : Y = \mathrm{Bl}_Z X$ be the blowup of complex projective manifold X along the smooth center $Z \subset X$. We study analytic continuations of big quantum cohomology ring $QH(Y)$ along the ϕ-extremal ray variable q^ℓ.

Under $H(Y) = \phi^* H(X) \oplus K$ where $K = \ker \phi_*$, denote by $QH(Y)_X$ the restriction of $(QH(Y), *_t)$ along $t \in \phi^* H(X)$. We claim that

(i) $QH(Y)_X$ is meromorphic in $x := 1/q^\ell$.
(ii) K deforms uniquely to a quantum ideal $\widetilde{K}$ in $QH(Y)_X$ near $x = 0$.
(iii) The quotient ring $QH(Y)_X/\widetilde{K}$ is regular in x, and its restriction to $x = 0$ is canonically isomorphic to $QH(X)$.

All of them follow from the techniques developed in our earlier works on flops and the following new ingredient called the x-regularity conjecture:

$$\beta > \phi^* \phi_* \beta \quad \Longrightarrow \quad \langle \phi^* \alpha_1, \ldots, \phi^* \alpha_n \rangle^Y_{0,n,\beta} = 0.$$

I will present our progress towards proving this conjecture in the first non-trivial case: namely the blowup of 3-folds along $(-1,-1)$ curves.

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